Reflectance Models (BRDF)

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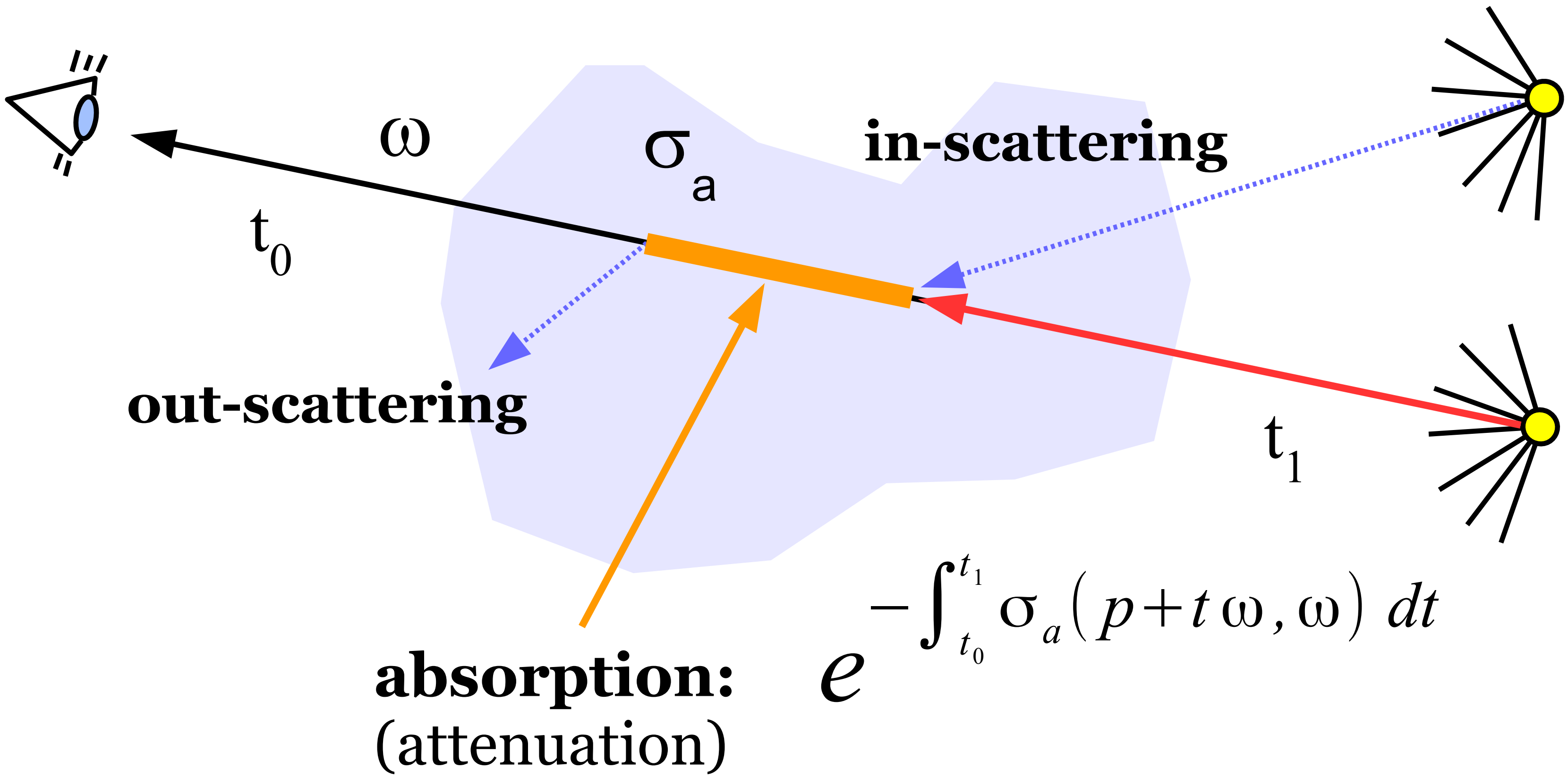
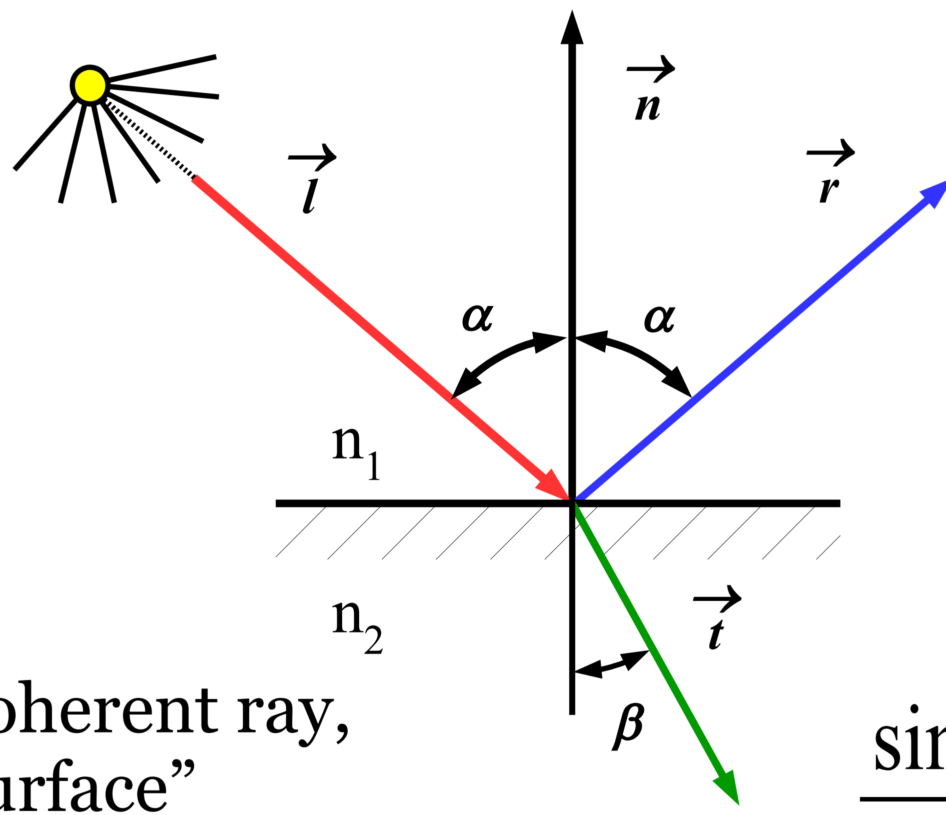
Light travels through media



$$e^{-\int_{t_0}^{t_1} \sigma_a(p+t\,\omega,\,\omega)\,dt}$$

Absorption is simple, ~~**scattering**~~ is very complicated

Light hits object surface (ideal)



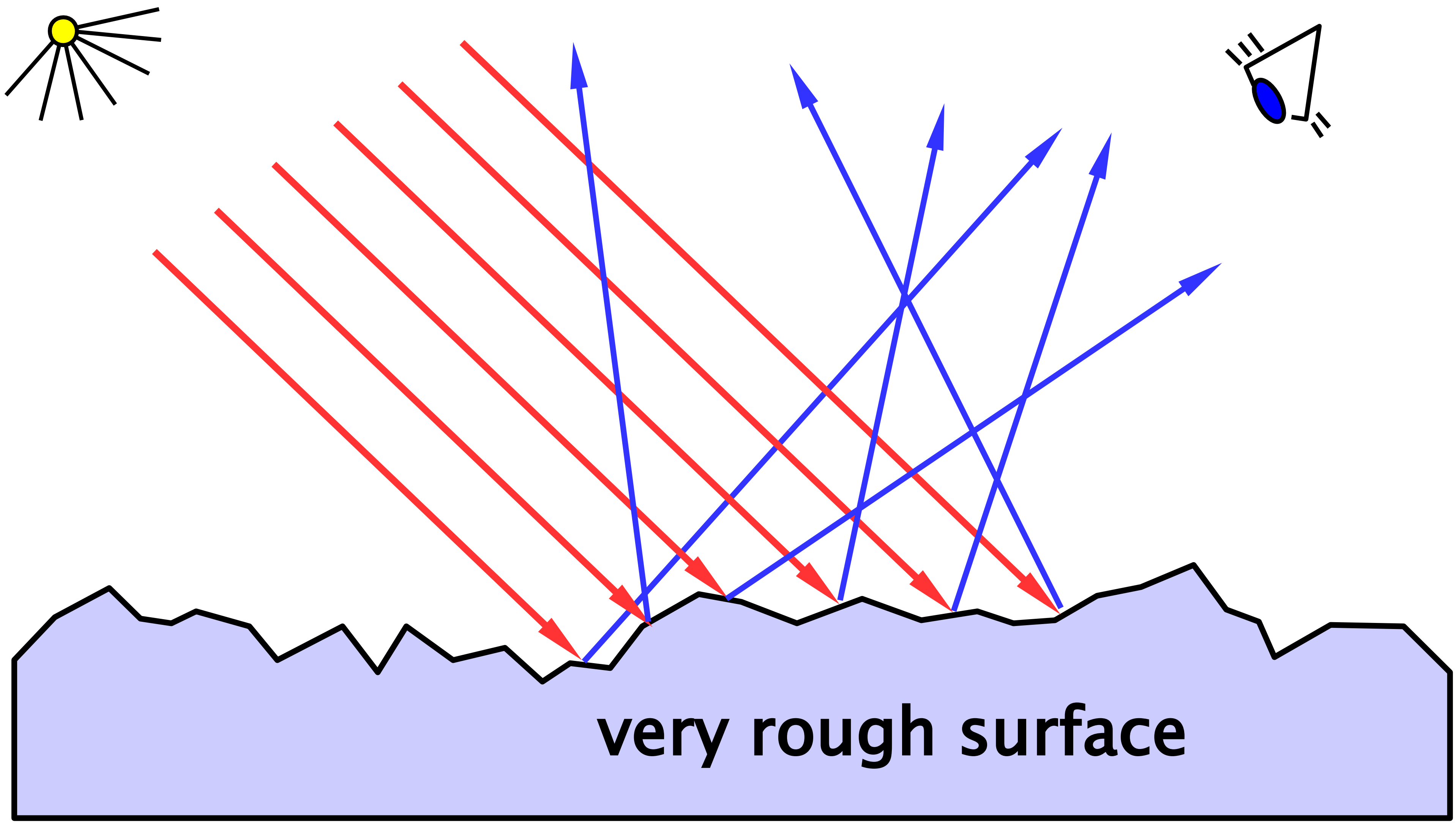
$$r = 2n(n \cdot l) - l$$

"Ideal coherent ray,
ideal surface"

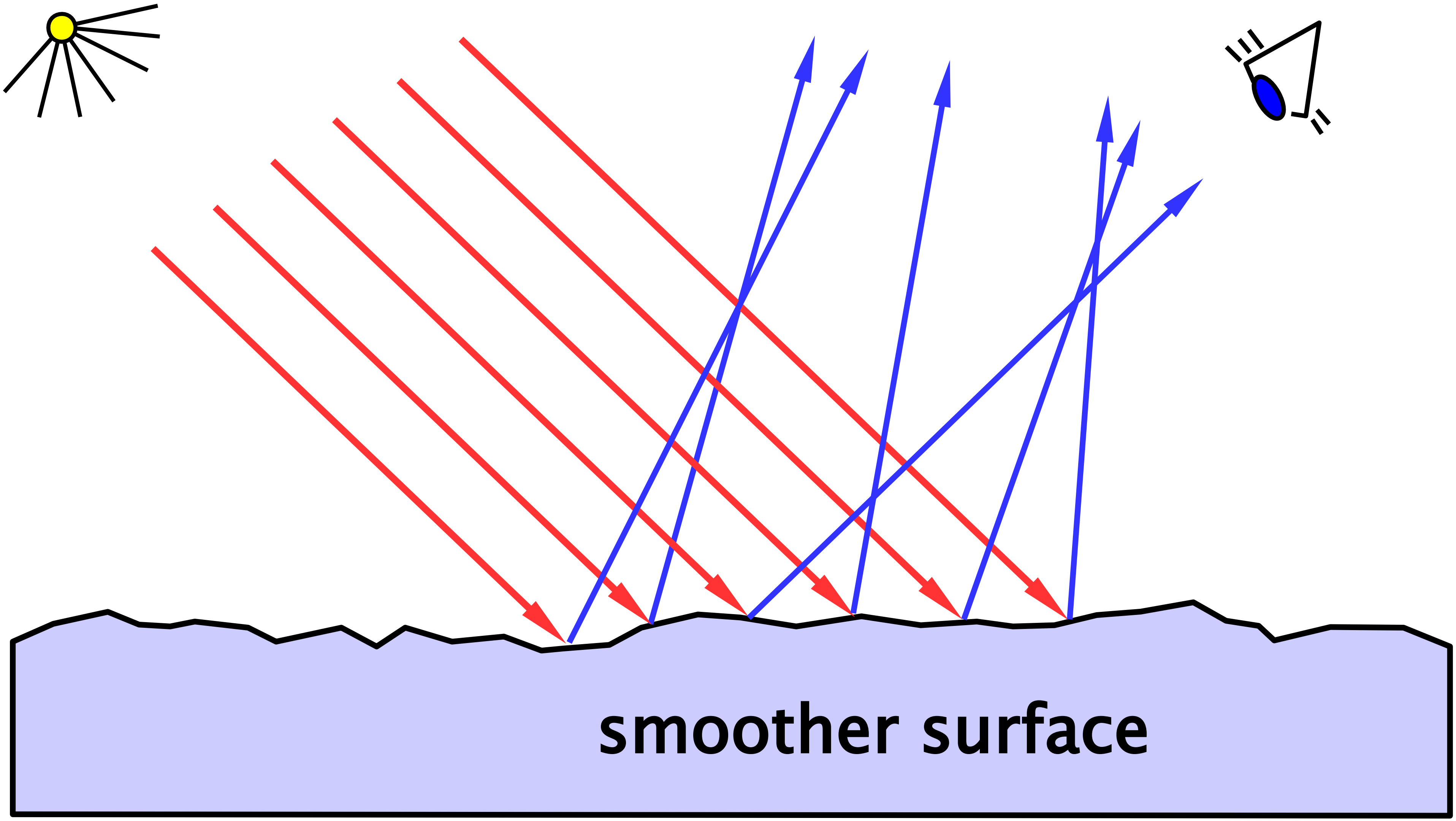
$$\frac{\sin\beta}{\sin\alpha} = \frac{n_1}{n_2} = n_{12}$$

Fresnel equations

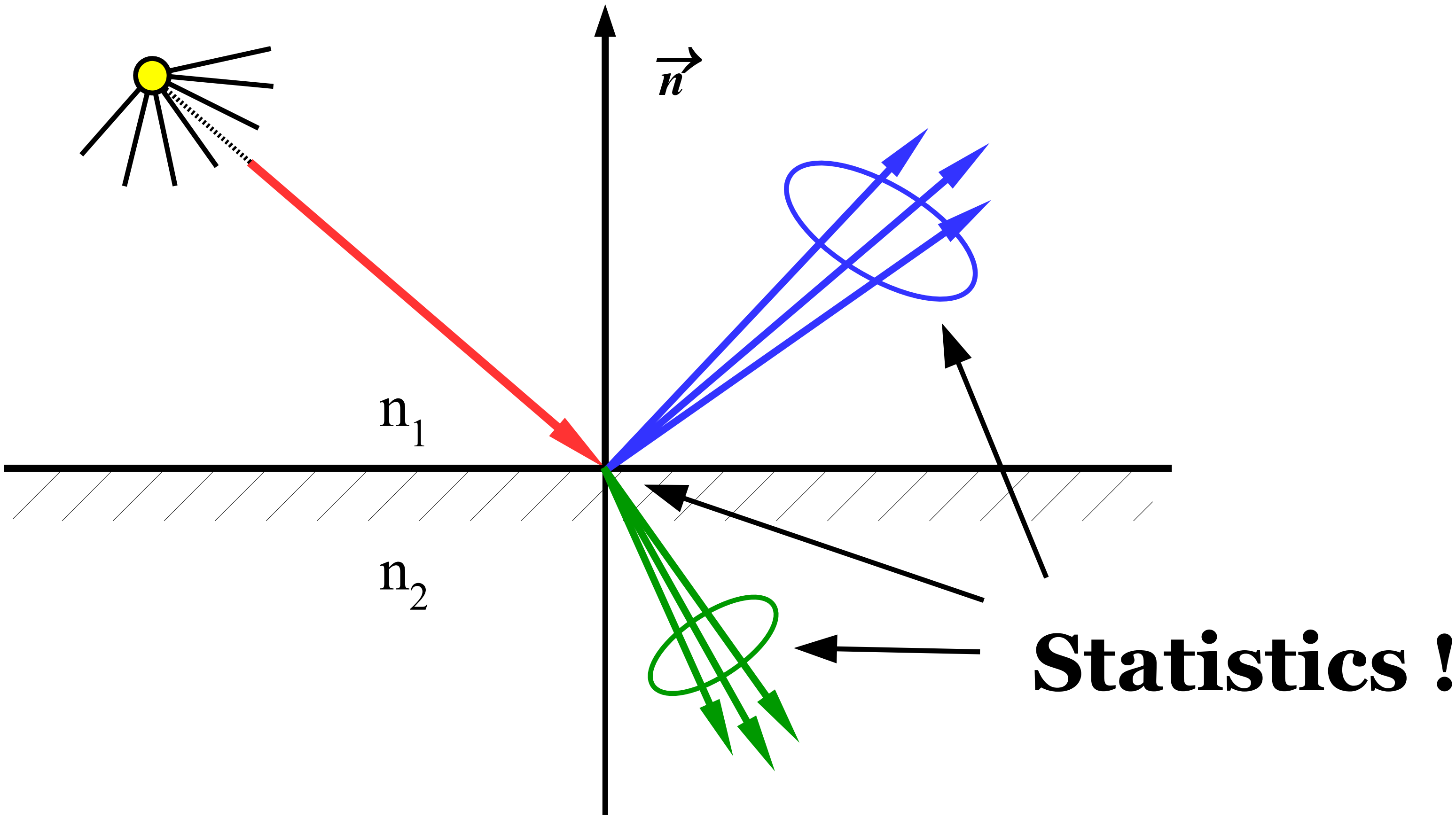
Real surface (microscopic view)



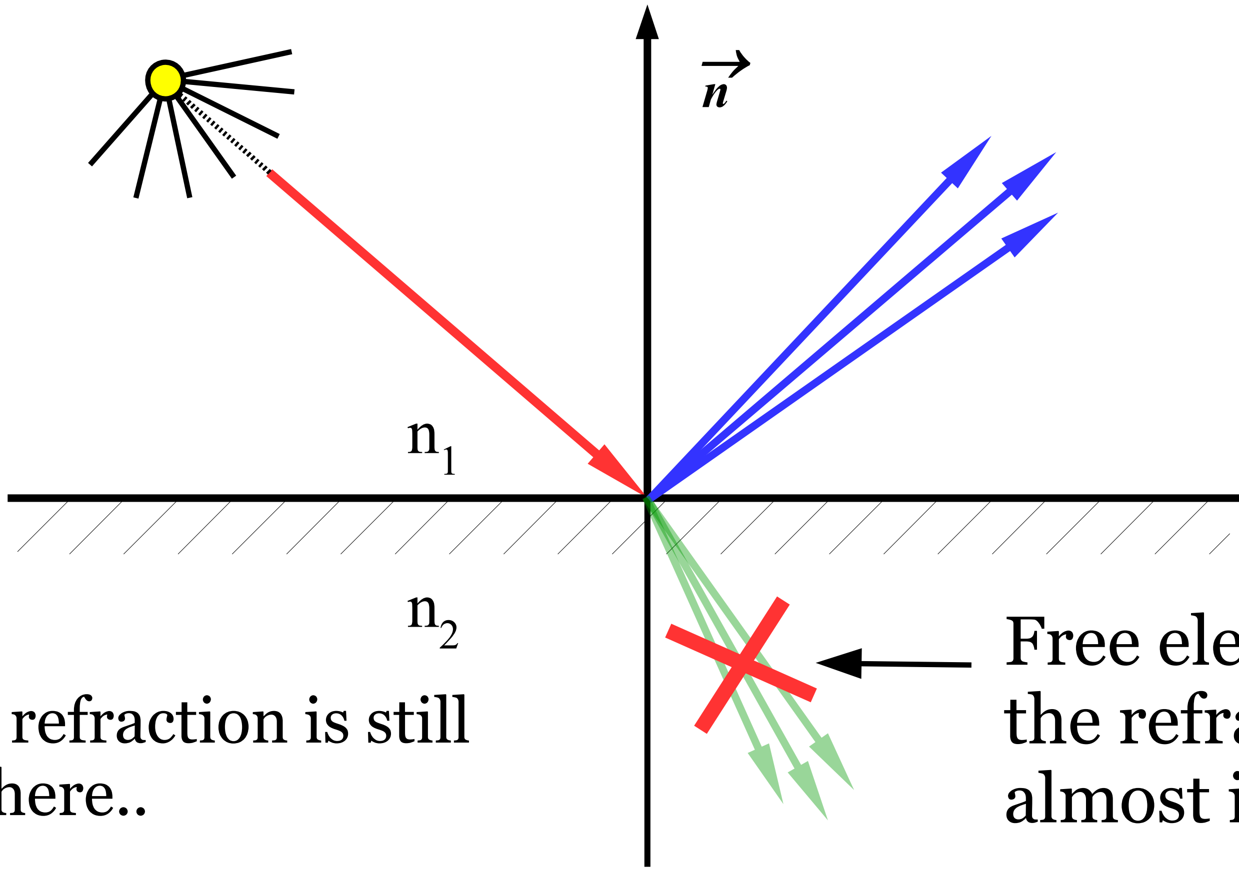
Real surface (microscopic view)



What we see from the distance..



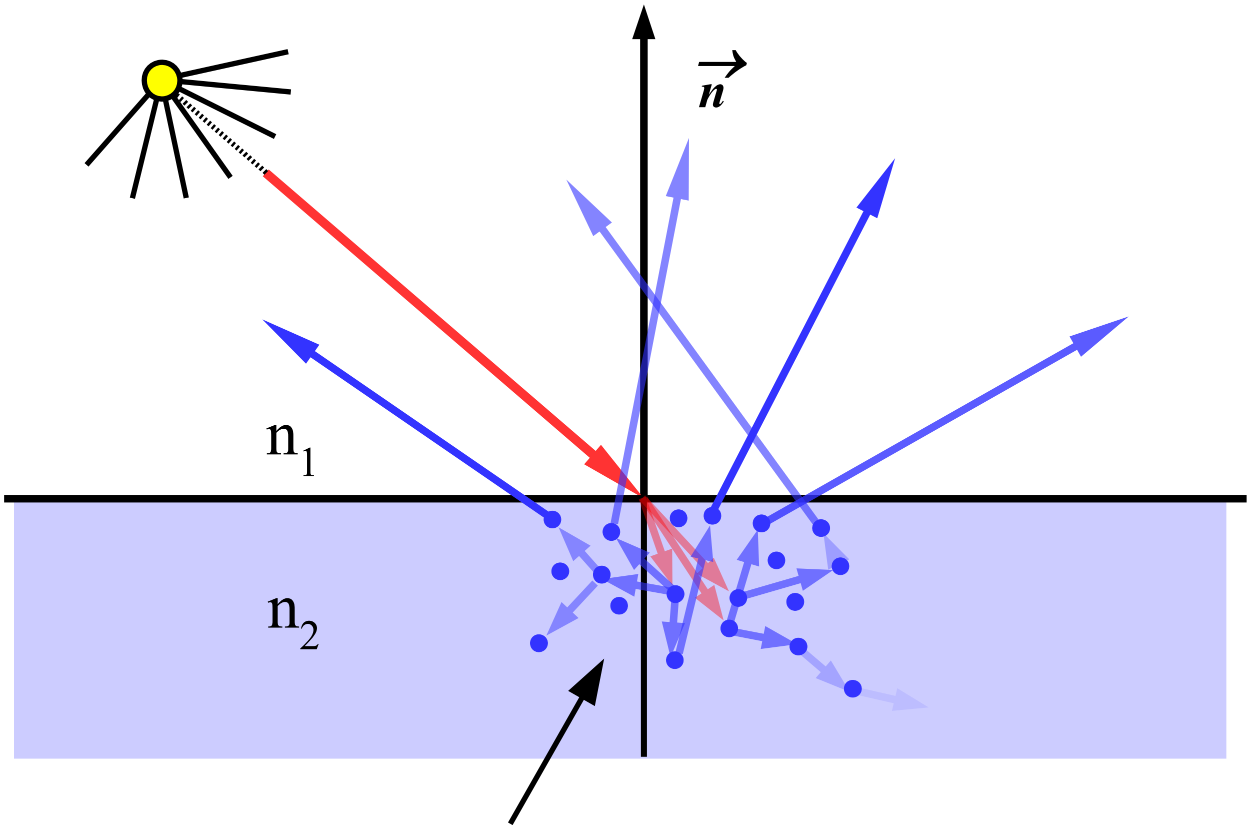
Metals (conductors)



Index of refraction is still defined here..

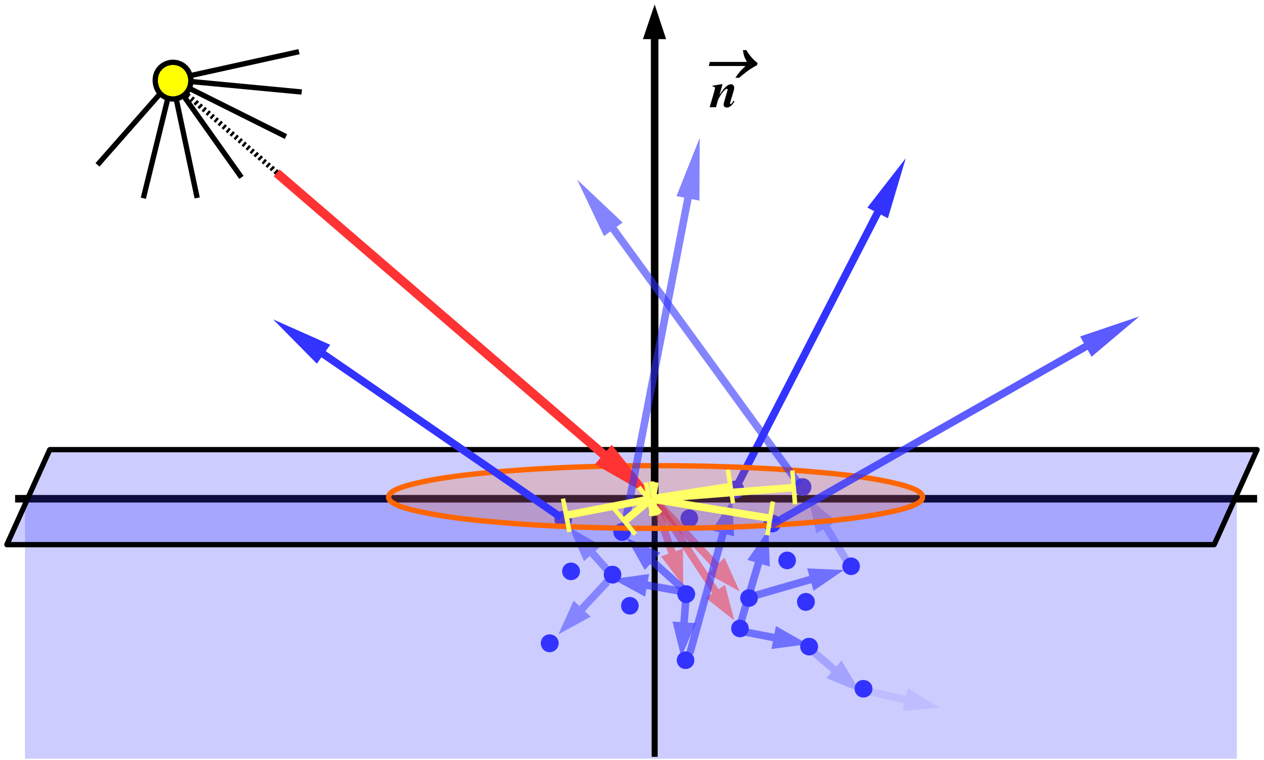
Free electrons **absorb** the refracted light almost immediately

Dielectrics (insulators)



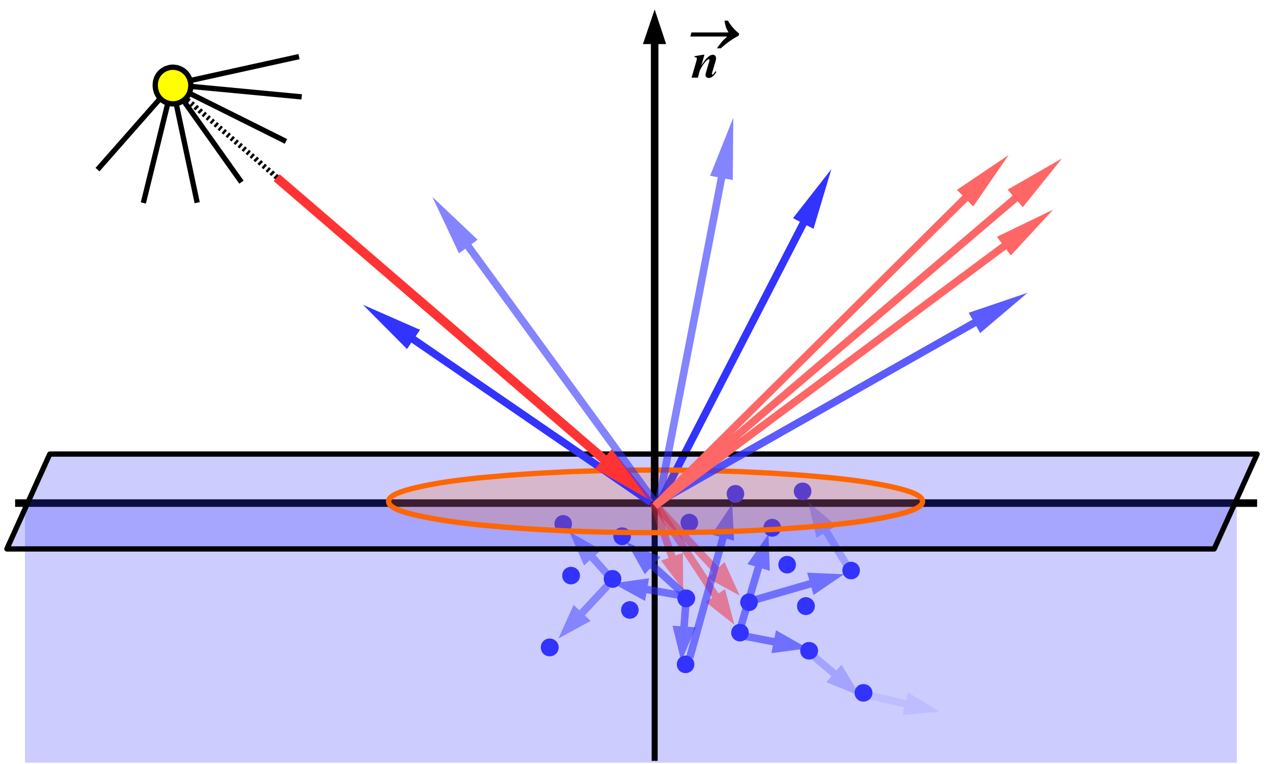
Scattering inside of the material

BSSRDF idea



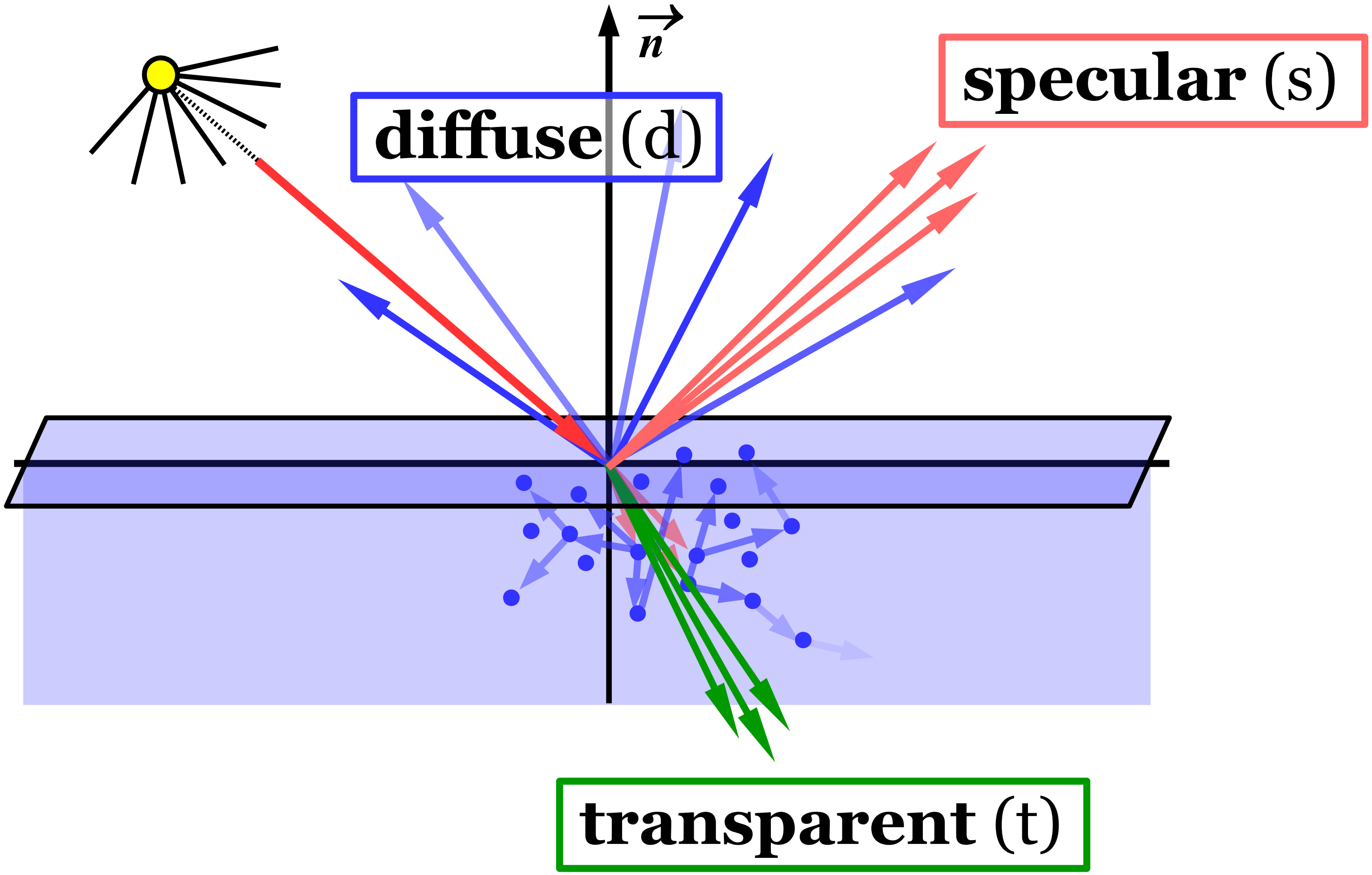
("Bi-directional Scattering-Surface
Reflectance Distribution Function")

Ignoring exit–to–entry distance



BRDF = “Bi-directional Reflectance Distribution Function”

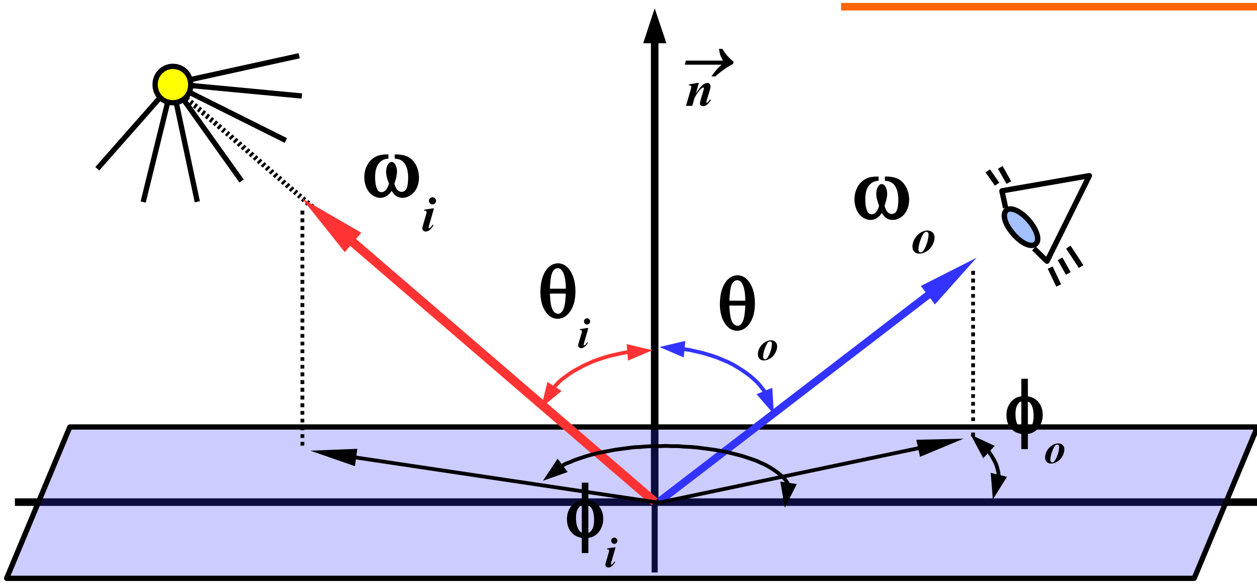
Shading terms (components)



BRDF formulation

BRDF function: $\mathbf{R}^5 \rightarrow \mathbf{R}$

$$f(\omega_i, \omega_o, \lambda)$$



$\cos\theta_i$

$$L_o(\omega_o) = \int_\Omega f(\omega_i, \omega_o) \cdot L_i(\omega_i)\,(n \cdot \omega_i)\, d\omega_i$$

BRDF plausibility

- non-negative $$f(\omega_i, \omega_o, \lambda) \geqslant 0$$
- reciprocal $$f(\omega_i, \omega_o, \lambda) = f(\omega_o, \omega_i, \lambda)$$
- energy-conserving
$$\int_{\Omega} f(\omega_i, \omega_o)\,(n \cdot \omega_i)\, d\omega_i \leqslant 1$$

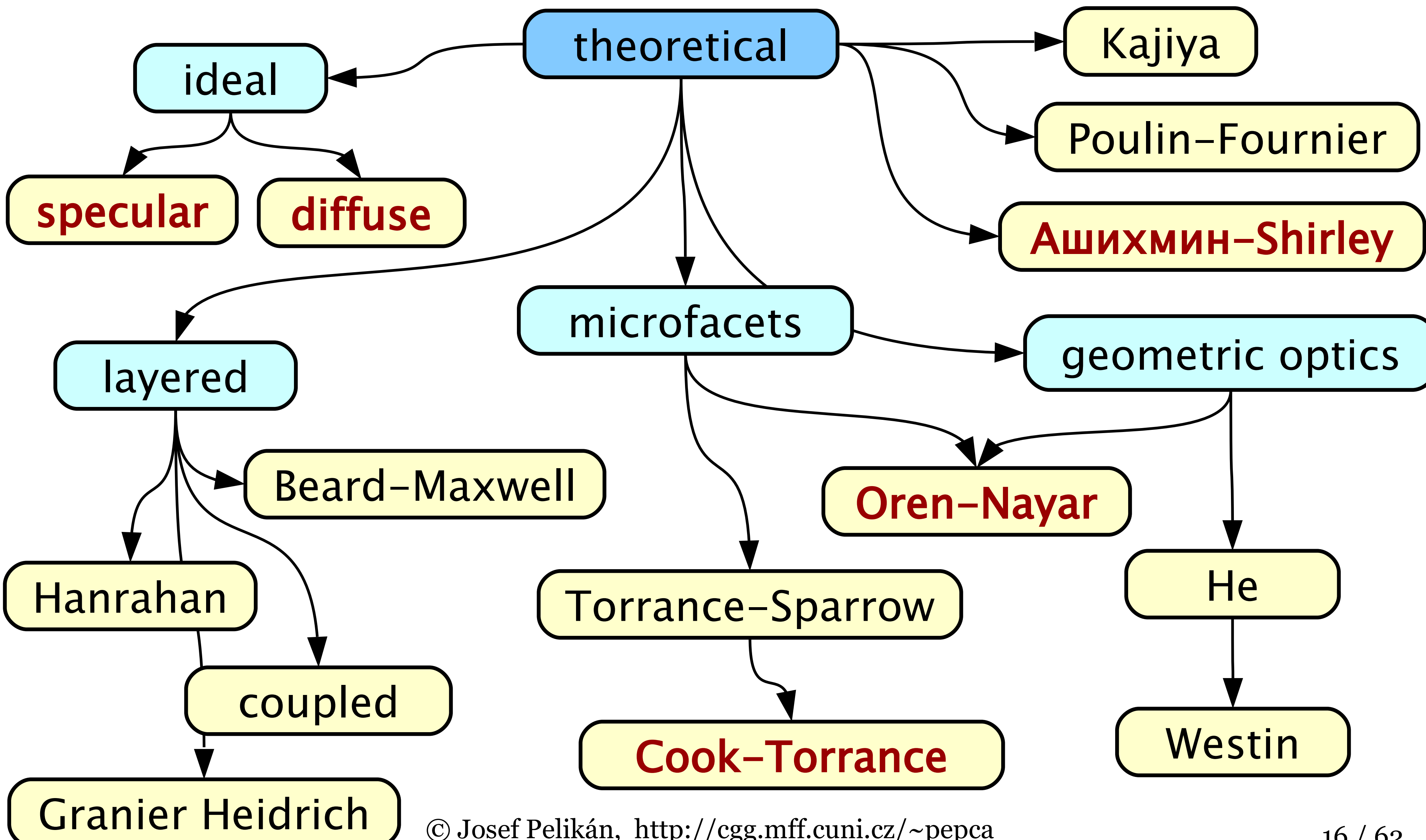
History (physical/empirical)

- **Beckmann, Spizzichino** (1963): electromagnetic wave reflection on rough surfaces (optics)
- **Torrance, Sparrow** (1967): off-specular reflections on rough surfaces (optics)
- **Phong** (1975): famous empirical model, used many decades
- **Blinn** (1977): first light reflection presentation at SIGGRAPH
- **Cook, Torrance** (1981): generalization, implementation, first physically based BRDF model in computer graphics

History (physical/empirical)

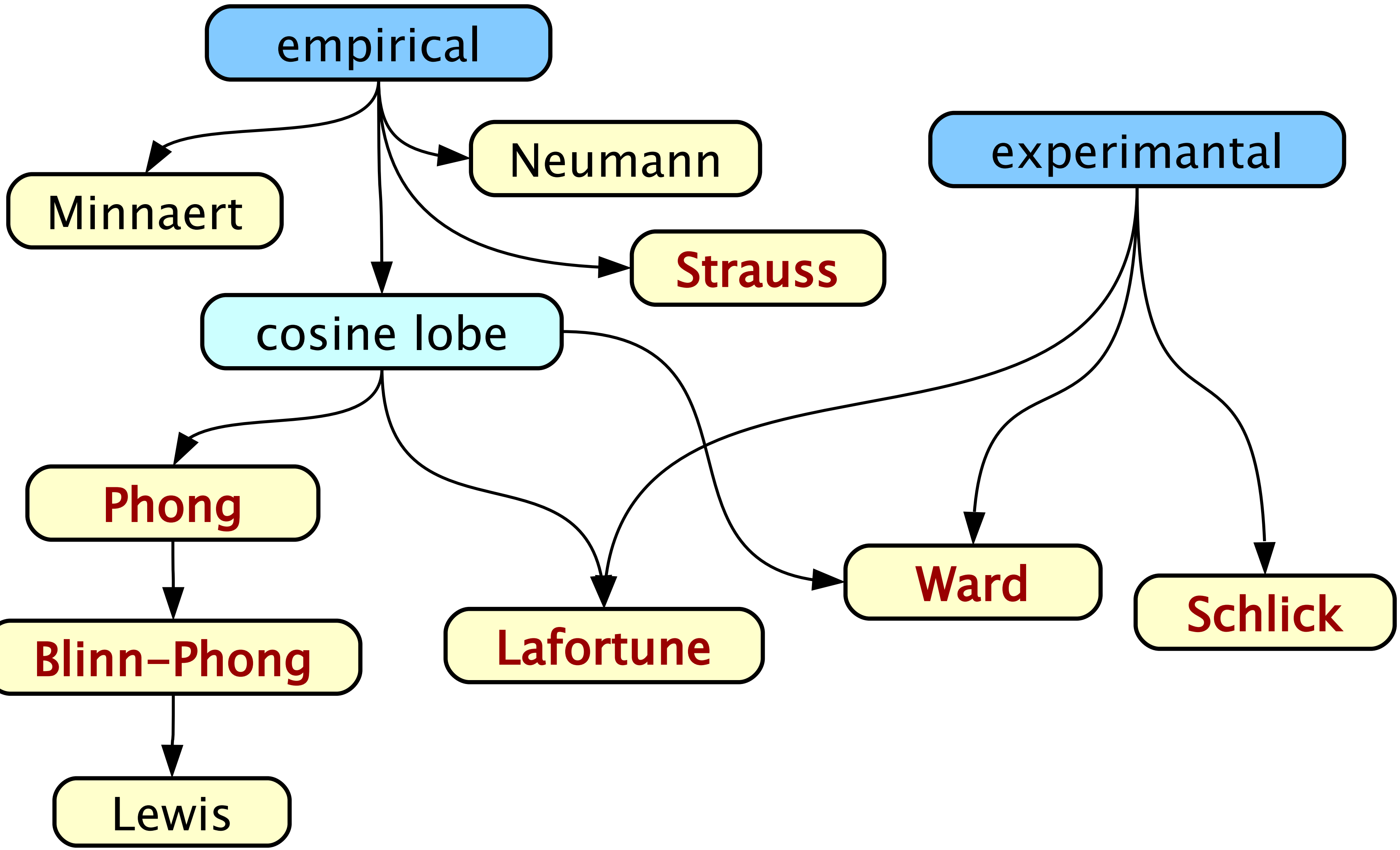
- **He** (1991): more complex wave optics, polarization, diffraction, interference..
- **Ward** (1992): anisotropic material, microfacets
- **Schlick** (1994): fast Fresnel formula approximation, two-layer reflectance model
- **Lafortune** (1997): multiple lobes, fitted to lab data
- **Ашихмин, Shirley** (2000): anisotropic Phong
- **Walter** (2007): microfacet refraction model (BSDF = Bidirectional Scattering Distribution Function)
- **Ашихмин, Bagher** (2007, 2012): models based on arbitrary microfacet distribution (measured..)

BRDF ZOO I

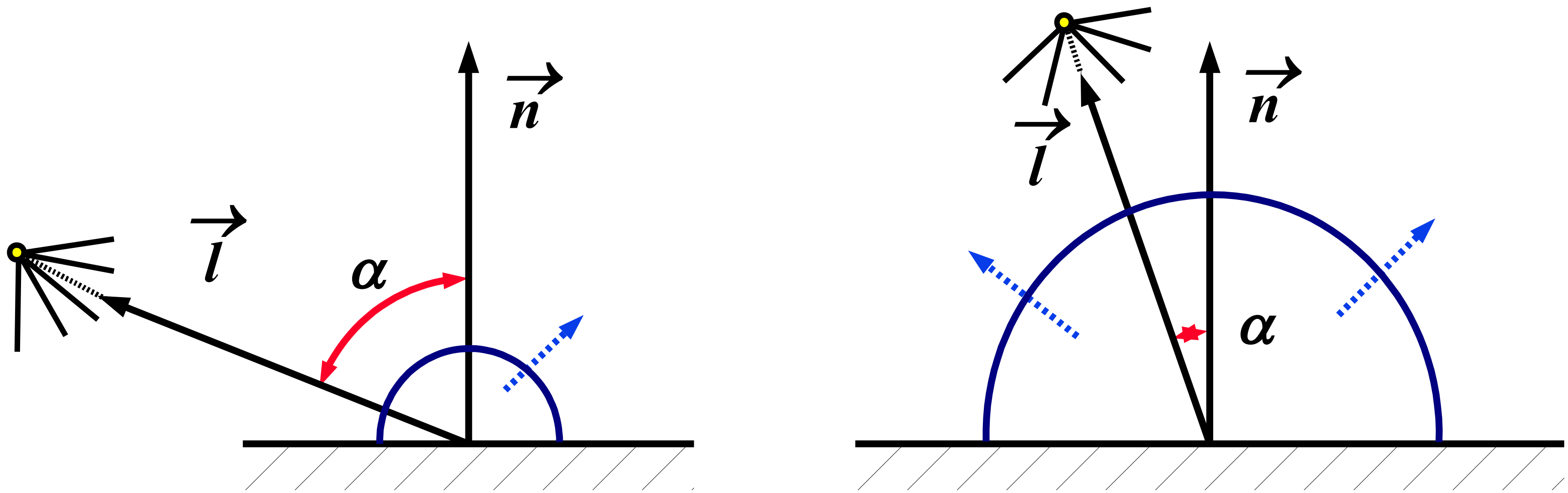


theoretical
ideal
specular
diffuse
Kajiya
Poulin–Fournier
Ашихмин–Shirley
microfacets
geometric optics
layered
Beard–Maxwell
Oren–Nayar
Hanrahan
Torrance–Sparrow
He
coupled
Cook–Torrance
Westin
Granier Heidrich

BRDF ZOO II

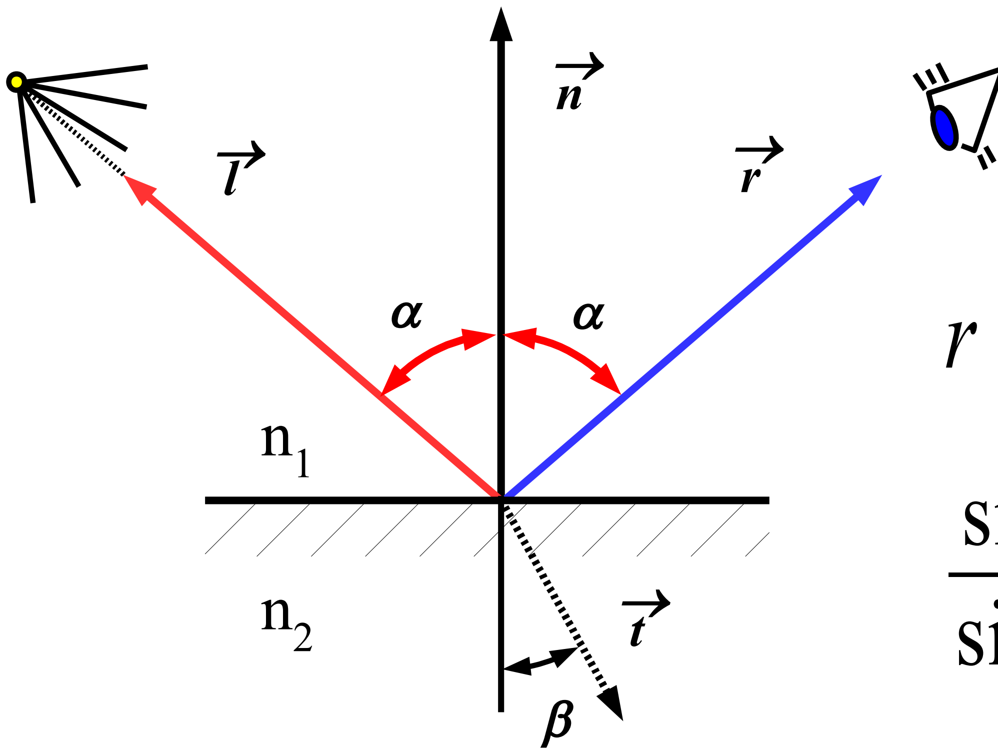

empirical
Minnaert
Neumann
Strauss
cosine lobe
experimantal
Phong
Blinn–Phong
Lewis
Lafortune
Ward
Schlick

Ideal diffusion



- **ideal diffuse material (Lambertian surface)**
  - reflection probability is constant
  - examples: furry surface, noisy microstructure w/o any pattern
- **Lambert law**: reflected intensity depends solely on **cos α**

Ideal (mirror) reflection

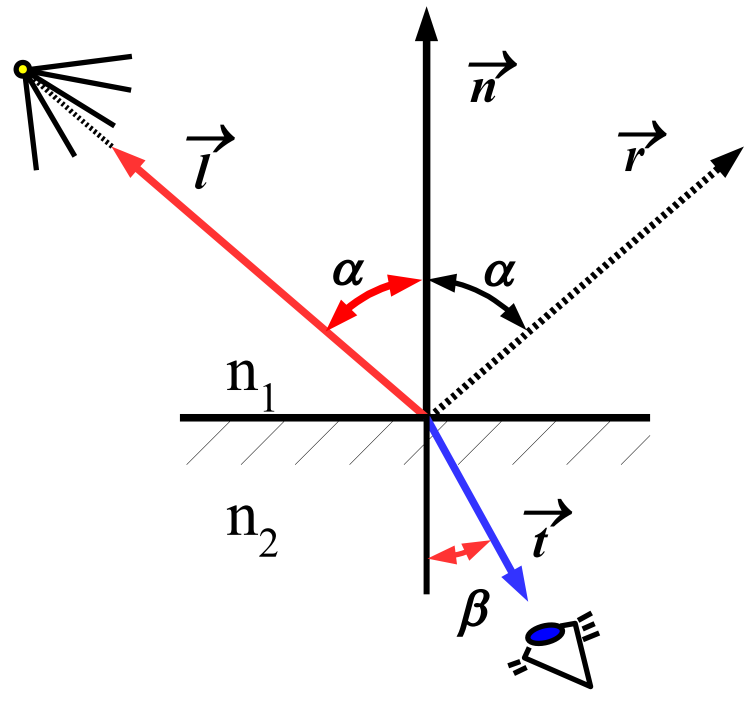


$$r = 2n\left(n \cdot l\right) - l$$

$$\frac{\sin\beta}{\sin\alpha} = \frac{n_1}{n_2} = n_{12}$$

- ratio of reflected and refracted light is determined by the **Fresnel equations** (19. century)

Refraction (Snell's law, Ibn Sahl, 984)



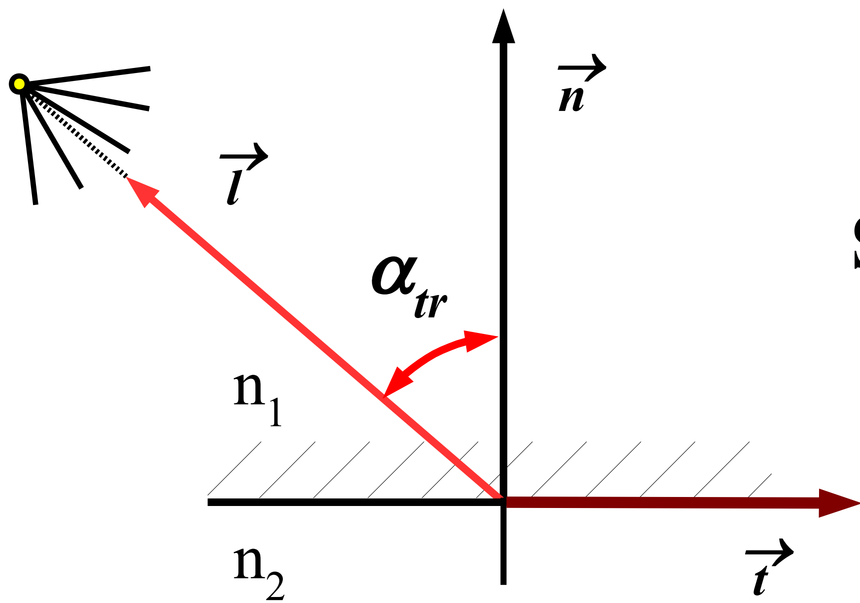
$$\frac{\sin\beta}{\sin\alpha} = \frac{n_1}{n_2} = n_{12}$$

$$\cos\beta = \sqrt{1 - n_{12}{}^2 \sin^2\alpha} = \sqrt{1 - n_{12}{}^2 \cdot (1 - (n \cdot l)^2)}$$

$$t = \left[n_{12}(n \cdot l) - \sqrt{1 - n_{12}{}^2 \cdot (1 - (n \cdot l)^2)} \right] \cdot n - n_{12} \cdot l$$

Total internal reflection

- going from more dense environment to less dense one ($n_1 > n_2$)
- for incident angles greater than **critical angle α_{tr}** there is no refraction at all!



$$\sin \alpha_{tr} = \frac{n_2}{n_1}$$

Fresnel equations (polarization)

- two **polarizations** (electric field perpendicular **„s“** /senkrecht/ or parallel **„p“** to the <u>incident plane</u>)
- reflectance “**R**“ and transmittance “**T**“ (power ratios):

$$R_s = \left[\frac{\sin(\beta - \alpha)}{\sin(\beta + \alpha)} \right]^2 \qquad T_s = 1 - R_s$$

$$R_p = \left[\frac{\tan(\beta - \alpha)}{\tan(\beta + \alpha)} \right]^2 \qquad T_p = 1 - R_p$$

Fresnel equations (alternative)

- no need to compute angles (cosines are easy):

$$R_s = \left[\frac{n_1 \cos\alpha - n_2 \cos\beta}{n_1 \cos\alpha + n_2 \cos\beta} \right]^2$$

$$R_p = \left[\frac{n_1 \cos\beta - n_2 \cos\alpha}{n_1 \cos\beta + n_2 \cos\alpha} \right]^2$$

Unpolarized light

- averaging values R_s a R_p:

$$R = \frac{1}{2}\frac{(a-u)^2+b^2}{(a+u)^2+b^2}\left[\frac{(a+u-1/u)^2+b^2}{(a-u+1/u)^2+b^2}+1\right]$$

$$a^2 = \frac{1}{2}\left(\sqrt{(n_\lambda^2-k_\lambda^2+u^2-1)^2+4\mathrm{n}_\lambda^2 k_\lambda^2}+n_\lambda^2-k_\lambda^2+u^2-1\right)$$

$$b^2 = \frac{1}{2}\left(\sqrt{(n_\lambda^2-k_\lambda^2+u^2-1)^2+4\mathrm{n}_\lambda^2 k_\lambda^2}-n_\lambda^2+k_\lambda^2-u^2+1\right)$$

$$u = \cos\alpha = n\cdot l \qquad n = n_\lambda - i\,k_\lambda$$ (for dielectric $k_\lambda=0$)

Dielectric (insulator) materials

- $\mathbf{k}_\lambda = \mathbf{0} \quad \Rightarrow$

$$a^2 = n_\lambda^2 + u^2 - 1 \qquad\qquad b = 0$$

$$R = \frac{1}{2}\frac{(a-u)^2}{(a+u)^2}\left(\frac{[u(a+u)-1]^2}{[u(a-u)+1]^2} + 1\right)$$

Remarks (Fresnel)

- if $\boldsymbol{\alpha} = \boldsymbol{\pi/2}$ (i.e. $\mathbf{u} = \mathbf{0}$), then reflectance $\mathbf{R_\lambda(90)= 1}$ regardless of the wavelength λ
- for perpendicular ray ($\boldsymbol{\alpha} = \mathbf{0}$):

$$R_0 = R_s = R_p = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

$$T_0 = T_s = T_p = 1 - R_0 = \frac{4\, n_1 n_2}{(n_2 + n_1)^2}$$

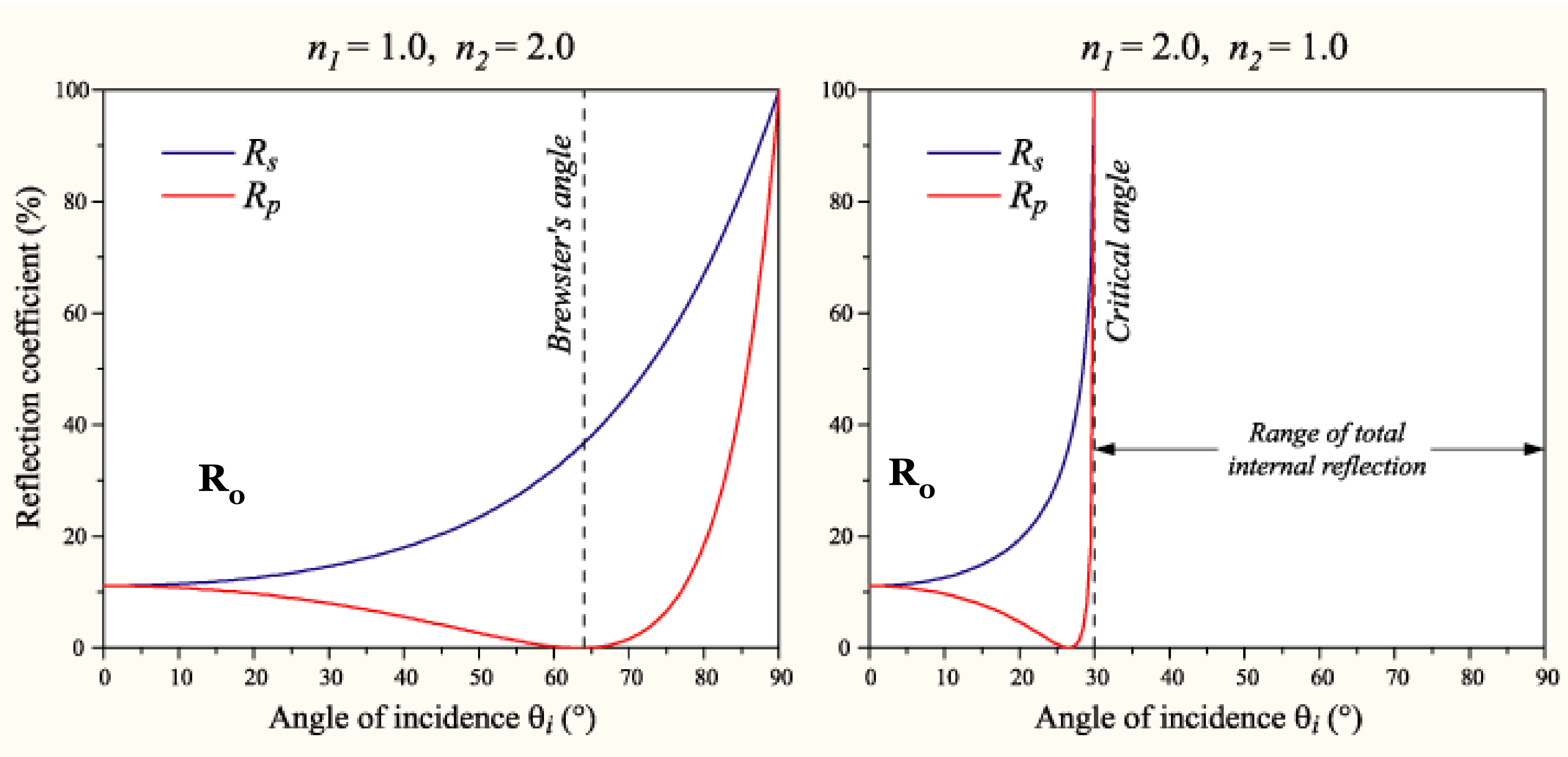
Wavelength λ

For **l** and **v** perpendicular to the surface (i.e. $\alpha = \mathbf{0}$):

$$F(\lambda,0) = \left(\frac{n_\lambda - 1}{n_\lambda + 1}\right)^2 \quad \text{a} \quad n_\lambda = \frac{1 + \sqrt{F(\lambda,0)}}{1 - \sqrt{F(\lambda,0)}}$$

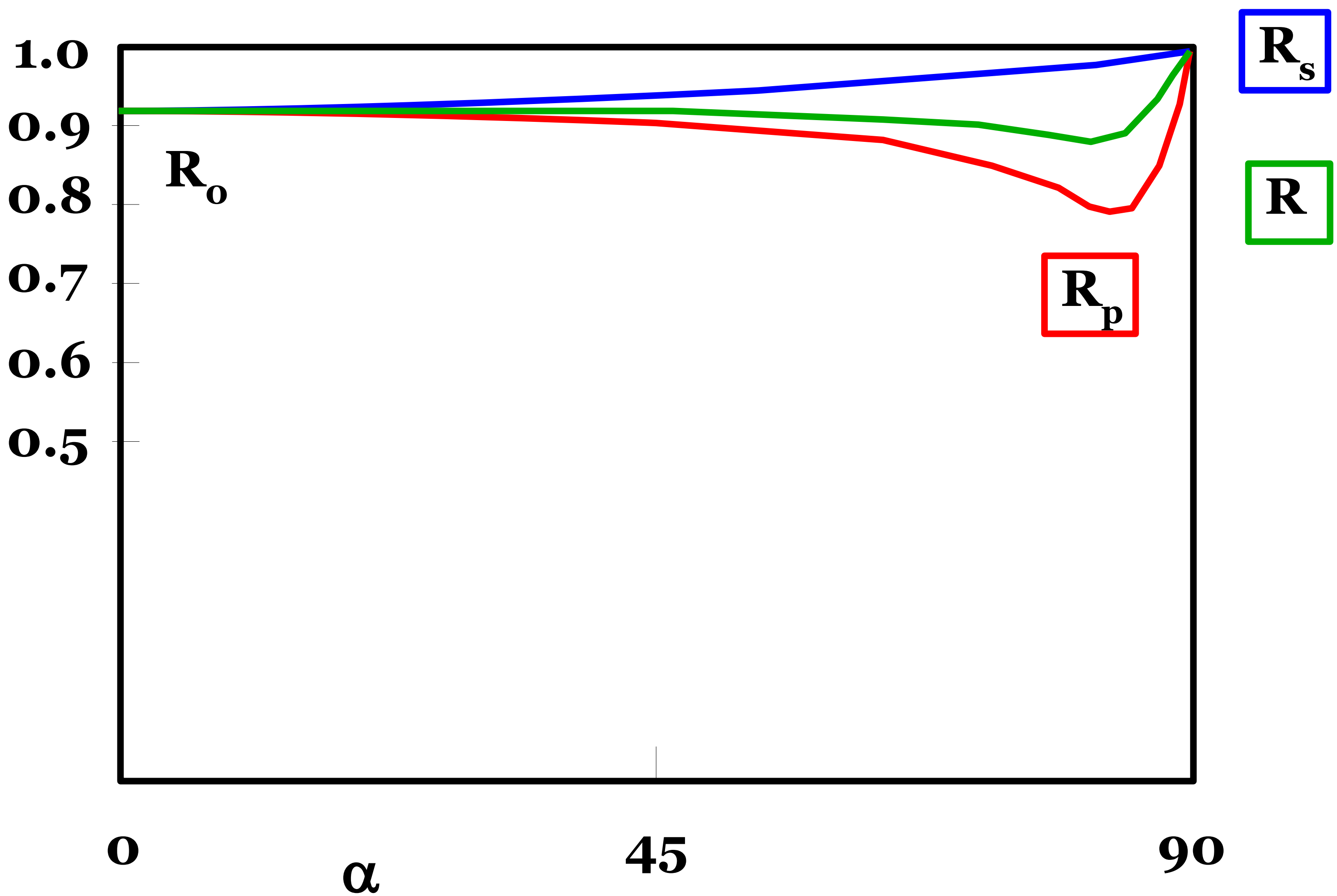
- quantities $\mathbf{F_\lambda(0)}$ were measured in labs for many real materials (both conductors and insulators)
  - so we know the $\mathbf{n_\lambda}$ indices
- specular reflection **depends on λ** (except for $\alpha = \pi/2$)

Reflectance – dielectric material



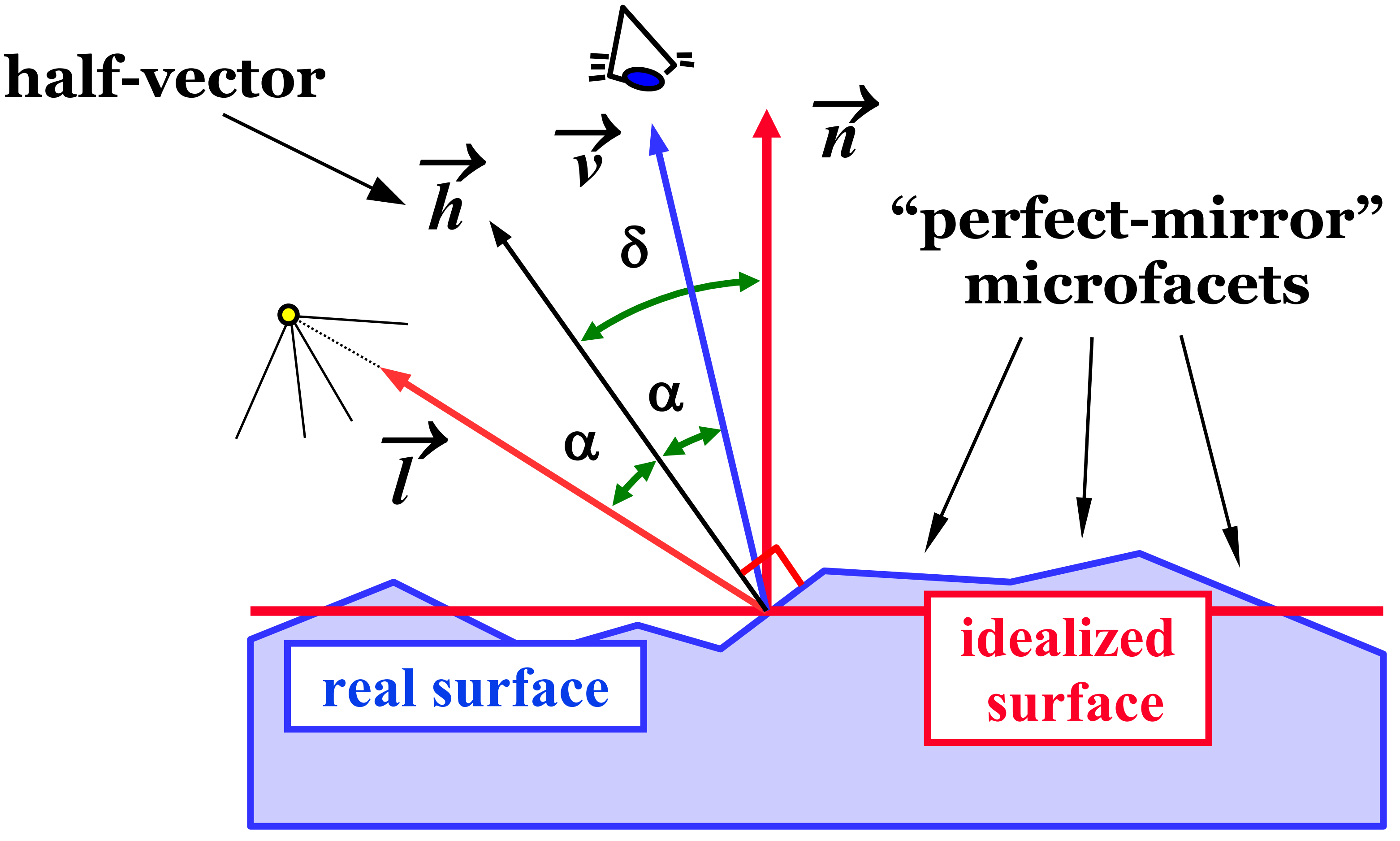
(cc) Ulflund, Wiki

Reflectance – metal



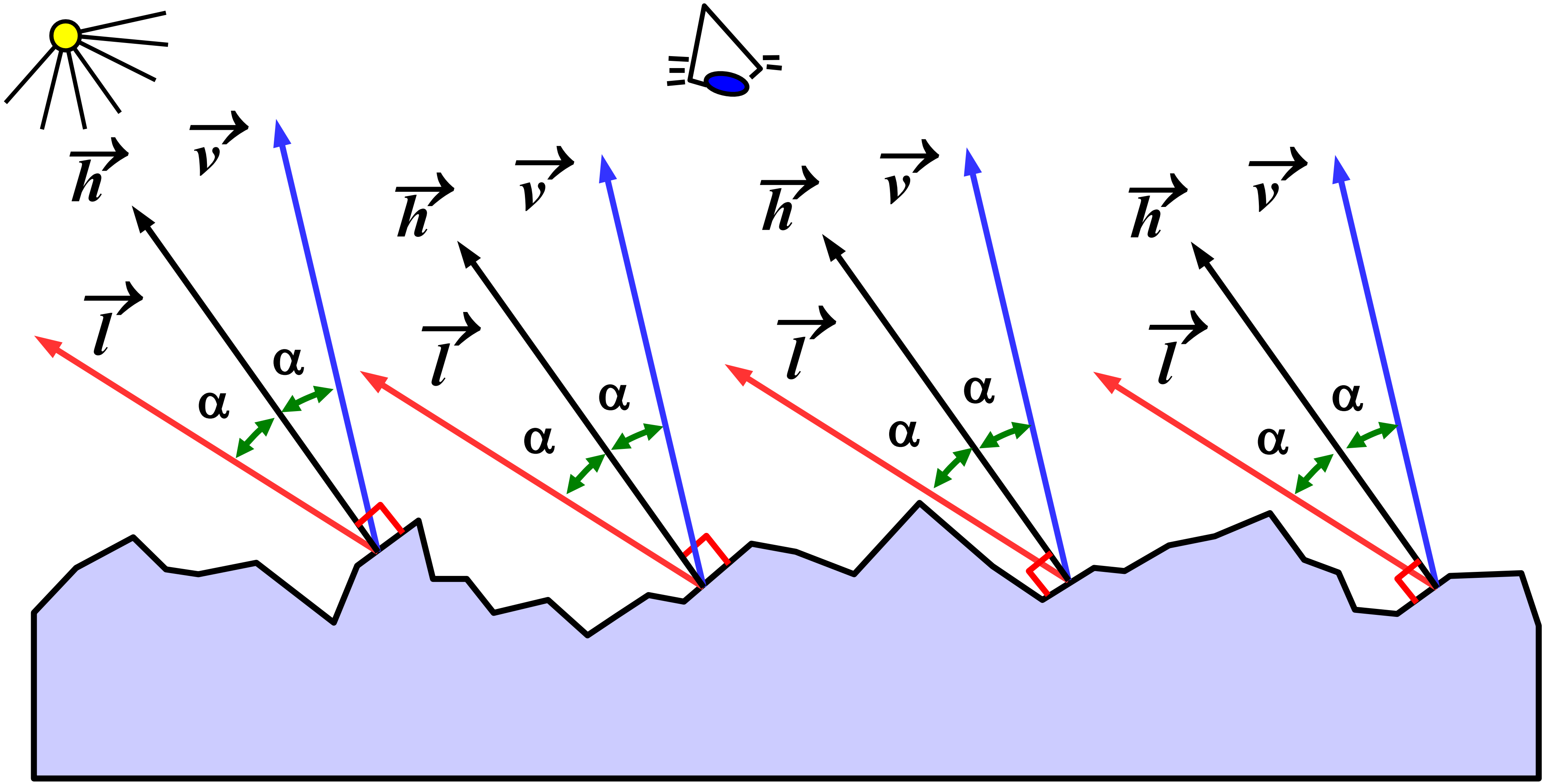
Microfacet theory

- Beckmann, Spizzichino (63), Torrance, Sparrow (67)

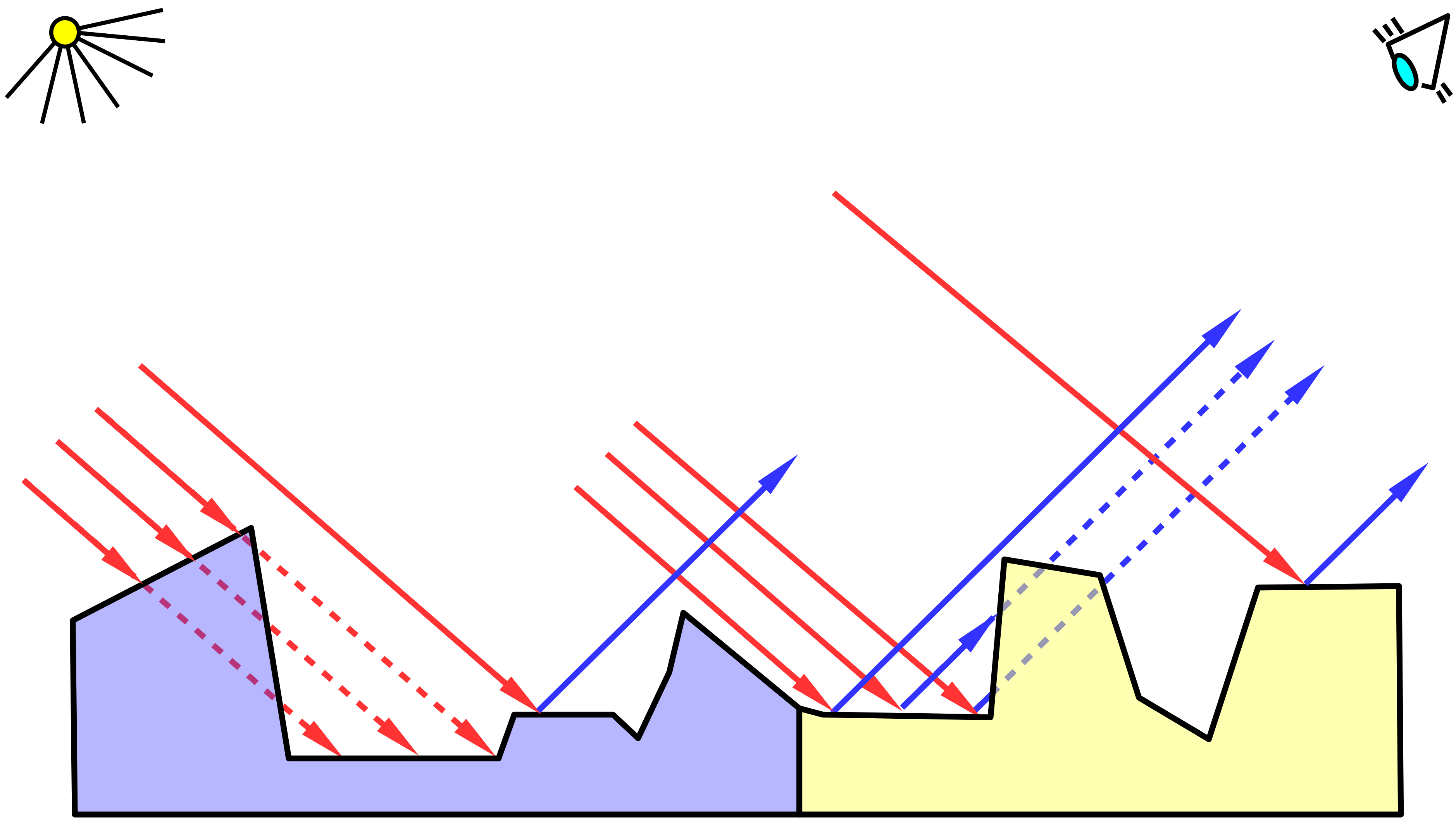


Perfect reflection for half-angle

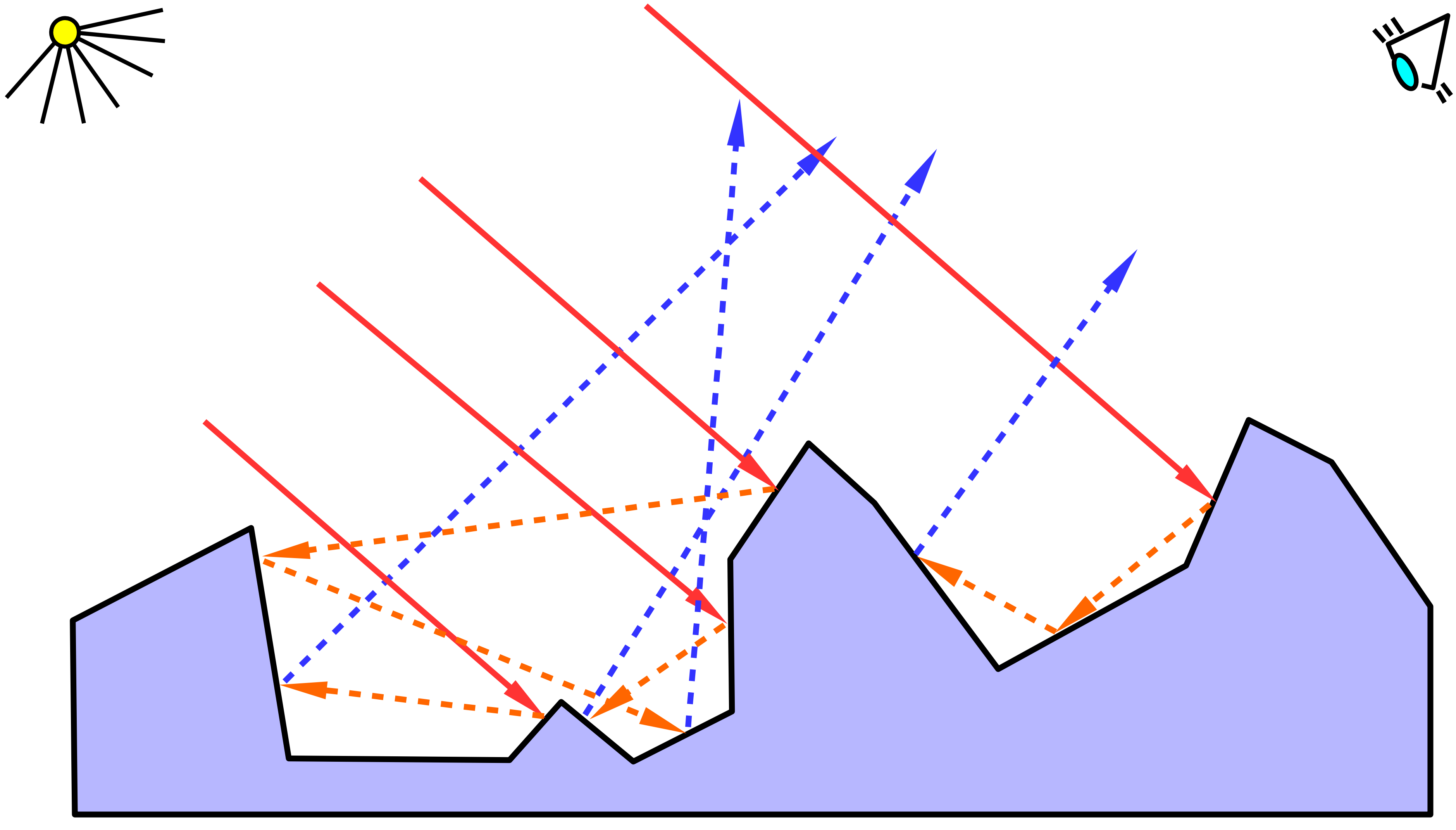
Only ideal **half-angle microfacets** can contribute !



Shadowing and masking



Multiple bounces are lost



Microfacet specular BRDF

$$R_\lambda(h) = \frac{F_\lambda(\alpha)}{4} \cdot \frac{D(h) \cdot G(l,v,h)}{(n \cdot l)(n \cdot v)}$$

- $\boldsymbol{R_\lambda(h)}$ ... specular reflectance for wavelength $\boldsymbol{\lambda}$
- $\boldsymbol{F_\lambda(\alpha)}$ ... Fresnel ideal reflectance for wavelength $\boldsymbol{\lambda}$ and incident angle $\boldsymbol{\alpha}$
- $\boldsymbol{D(h)}$ ... microfacet PDF ("how many microfacets" have $\boldsymbol{h}$ as a normal vector)
- $\boldsymbol{G(l,v,h)}$ ... geometric factor (shadowing & masking)

Fresnel term F

Fresnel term for unpolarized light

$$F(\lambda,\beta) = \frac{1}{2} \cdot \frac{(g-c)^2}{(g+c)^2} \left\{ 1 + \frac{[c(g+c)-1]^2}{[c(g-c)-1]^2} \right\}$$

for $c = \cos\beta = (V \cdot H)$,

$$g^2 = n_\lambda{}^2 + c^2 - 1$$

- $\mathbf{n}_\lambda$ … index of refraction for wavelength $\boldsymbol{\lambda}$
  - for conductors $\mathbf{n}_\lambda' = \mathbf{n}_\lambda - \boldsymbol{i}\,\boldsymbol{\kappa}_\lambda$ ($\boldsymbol{\kappa}_\lambda$ … absorbtion coeff.)

Fresnel – base specular color

Metal	F(0) [linear]	F(0) [sRGB]
Titanium	0.542, 0.497, 0.449	194, 187, 179
Chromium	0.549, 0.556, 0.554	196, 197, 196
Iron	0.562, 0.565, 0.578	198, 198, 200
Nickel	0.660, 0.609, 0.526	212, 205, 192
Platinum	0.673, 0.637, 0.585	214, 209, 201
Copper	0.955, 0.638, 0.538	250, 209, 194
Palladium	0.733, 0.697, 0.652	222, 217, 211
Zinc	0.664, 0.824, 0.850	213, 234, 237
Gold	1.022, 0.782, 0.344	255, 229, 158
Aluminum	0.913, 0.922, 0.924	245, 246, 246
Silver	0.972, 0.960, 0.915	252, 250, 245

Schlick's approximation

- Fresnel term for other angles, based on $\mathbf{F_\lambda(0) = c}$

$$F_{schlick}(c, l, h) = c + (1 - c)\,(1 - (l \cdot h))^5$$

Any angle, any λ (R. Hall)

- let's assume we have a **base material color $F_\lambda(0)$** and an **angle-function** for some (standard) λ_0
  - set of wavelengths can be limited (3÷6 components)

$$F_\lambda(\alpha) \approx F_\lambda(0) + \left(1 - F_\lambda(0)\right)\frac{max\left(0,\ F_{\lambda_0}(\alpha) - F_{\lambda_0}(0)\right)}{1 - F_{\lambda_0}(0)}$$

Normal distribution function

Fast and simple formula – Gaussian distribution:

$$\underline{D(h, m)} = \chi(n \cdot h)\,(n \cdot h) \cdot e^{-\left(\frac{\delta}{m}\right)^2}$$

- $\chi(a) = a > 0\ ?\ 1 : 0$
- $\cos\delta = \mathbf{n} \cdot \mathbf{h}$
- ***m*** … "surface roughness" (standard deviation of the surface slope)
  - **< 0.1** … very smooth
  - **> 0.8** … rough (almost diffuse)

Beckmann's distribution (~normalised)

$$D_{be}(h,m) = \frac{\chi(n\cdot h)}{\pi m^2 (n\cdot h)^4}\; e^{-\left(\frac{\tan\delta}{m}\right)^2}$$

$$= \frac{\chi(n\cdot h)}{\pi m^2 (n\cdot h)^4}\; e^{\frac{(n\cdot h)^2-1}{m^2\,(n\cdot h)^2}}$$

Blinn–Phong (normalised)

$$D_{bp}(h, m) = \chi(n \cdot h) \; \frac{m+2}{2\pi} \; (n \cdot h)^m$$

Trowbridge & Reitz

$$D_{tr}(h, m) = \frac{\chi(n \cdot h)\, m^2}{\pi\left((n \cdot h)^2 (m^2 - 1) + 1\right)^2}$$

- $\boldsymbol{m}$ can be greater than 1

GGX (Walter et al. 2007)

$$\underline{D_{GGX}(h, m)} = \frac{\chi(n \cdot h)\, m^2}{\pi\, (n \cdot h)^4 \left(m^2 + \tan^2 \delta\right)^2}$$

Isotropic Ward (1992)

$$D_{wiso}(h, m) = \frac{\chi(n \cdot h)}{\pi m^2} e^{-\frac{\tan^2 \delta}{m^2}}$$

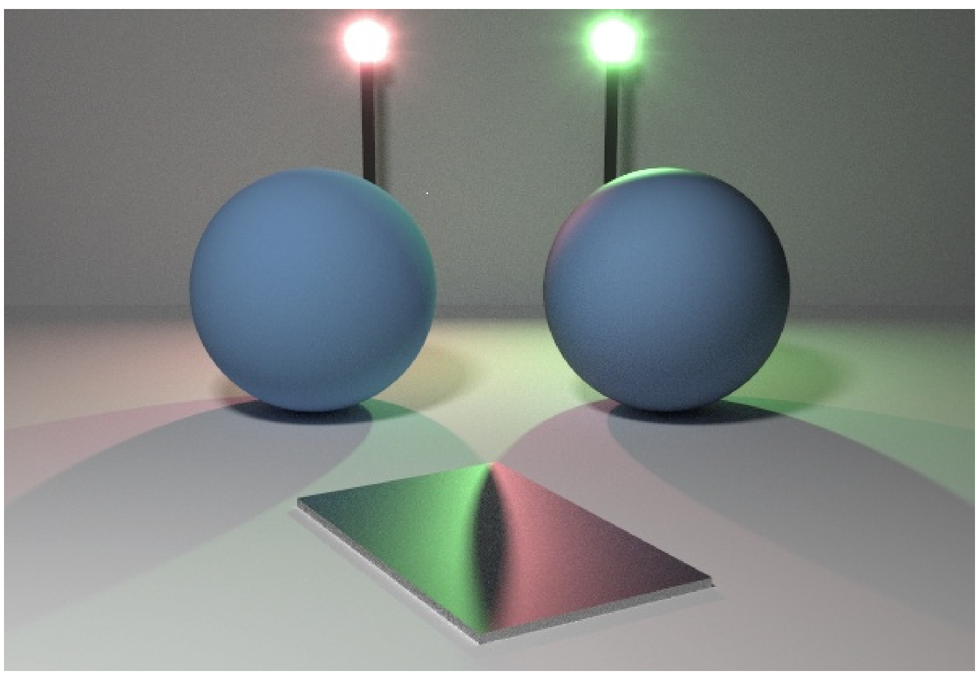
Anisotropic Ward (1992)

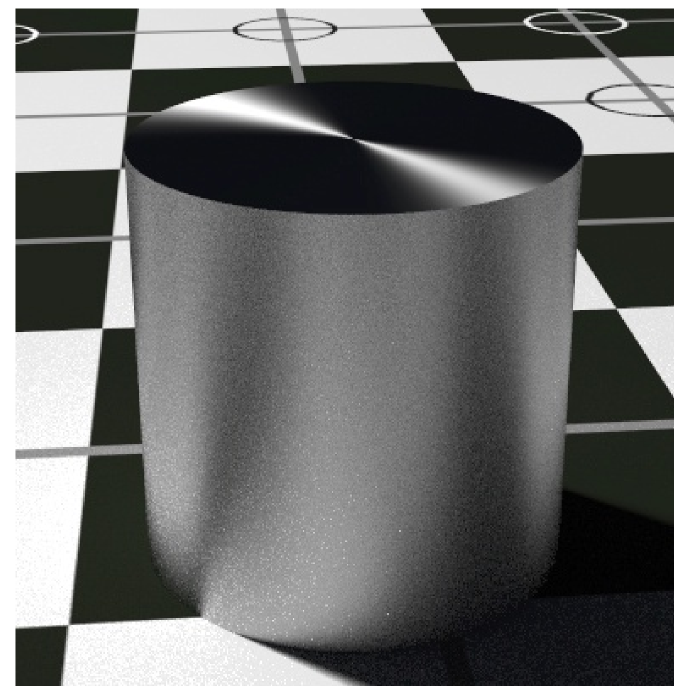
$$D_w(h, m_x, m_y) = \frac{\chi(n \cdot h)}{\pi\, m_x m_y}\, e^{-\tan^2\delta \left(\frac{\cos^2\phi_h}{m_x^2} + \frac{\sin^2\phi_h}{m_y^2} \right)}$$

- ϕ_h … azimuth angle of the half-vector

Ашихмин–Shirley anisotropic (2000)

$$D_{as}(h, e_x, e_y) = \sqrt{(e_x+1)(e_y+1)}\,(h \cdot n)^{e_x \cos^2\phi_h + e_y \sin^2\phi_h}$$





(c) Ашихмин, 2000

Material blends

Idea of **blending several materials together** makes sense: $\mathbf{m_1 \ldots m_k}$

$$D(\alpha) = \sum_{i=1}^{k} w_i \cdot D(m_i, \alpha)$$

- $\mathbf{w_i}$ … weight coefficients

$$\mathbf{\Sigma\, w_i = 1}$$

Geometric term G (Cook–Torrance)

Compensation for masking and shadowing

$$G_{ct}(l, v, h) = min\left(1, \frac{2(n \cdot h)(n \cdot v)}{(v \cdot h)}, \frac{2(n \cdot h)(n \cdot l)}{(v \cdot h)}\right)$$

Cook and Torrance assumed infinitely long
“V”-shaped grooves (not exactly true)

- optimized: **Kelemen**, more accurate: **Smith**

G alternative (GGX)

$$G_{GGX}(l, v, h) = \chi\left(\frac{v \cdot h}{v \cdot n}\right) \frac{2}{1 + \sqrt{1 + m^2 \tan^2 \theta_v}}$$

- $\boldsymbol{m}$ ... roughness from the GGX distribution

Cheap G (Kelemen–Szirmay–Kalos)

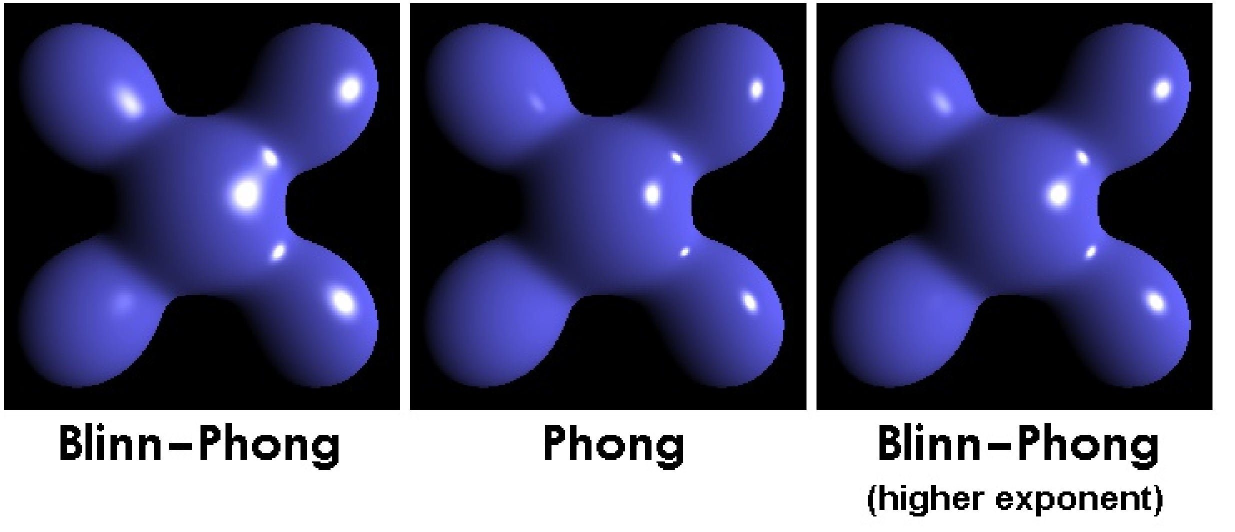
$$\frac{G_{KSK}(l, v, h)}{(n \cdot l)(n \cdot v)} \approx \frac{1}{(l \cdot h)^2}$$

Blinn's contribution

- Blinn-Phong model (here $\beta = \angle v,r$)
  - light source and viewer in infinity:

$$\cos^h \beta \approx \cos^{4h} \beta/2$$

$$(R_i \cdot V)^h \approx (H_i \cdot N)^{4h}$$



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Schlick's contribution

- **Christophe Schlick** (1994) was experimenting with approximate substitution in Fresnel term
- fraction instead of power function

$$S^h \approx S / (h - hS + S)$$

  - slightly less sharp highlight compared to Blinn-Phong
- Fresnel term substitute (32× faster, error <1%)

$$R_{schlick}(c, l, n) = c + (1 - c)\,(1 - (l \cdot n))^5$$

Lafortune model (1997)

- **Generalized cosine lobe** model
  - derived using Householder matrix (3×3)

1. classical specular term (Phong) ..

$$f_r(l, v) = \rho_s C_s \cos^h \beta$$

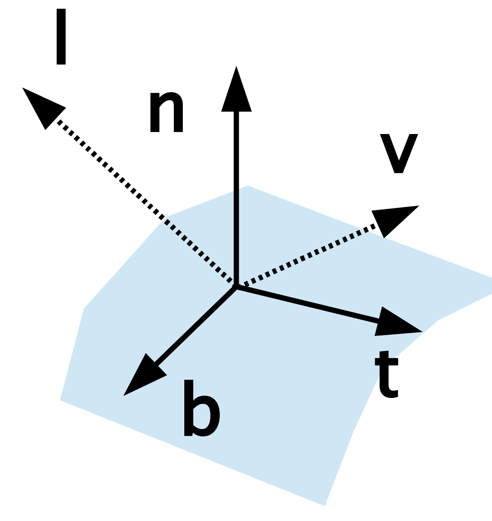
2. .. rewritten using Householder matrix notation

$$f_r(l, v) = \rho_s C_s (r \cdot v)^h$$
$$= \rho_s C_s \left[l^T (2 n n^T - I) v \right]^h$$

General plausible cosine lobe

- Householder matrix **M** (3×3)
  - for reciprocity it must be **symmetrical**

$$f_r(l,v) = \rho_s \left[l^T M v \right]^h$$



- SVD decomposition of matrix ***M***:

$$f_r(l,v) = \rho_s \left[l^T Q^T D Q v \right]^h$$

- ***Q*** ... coordinate transform, ***D*** ... diagonal matrix

$$f_r(l,v) = \rho_s \left[C_b l_b v_b + C_t l_t v_t + C_n l_n v_n \right]^h$$

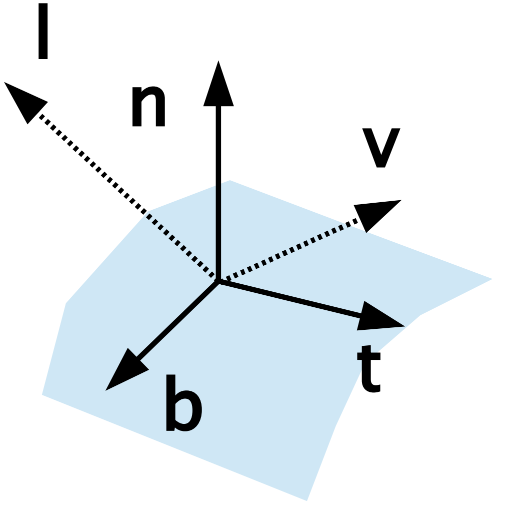
Cosine lobe options

- Phong lobe: $-C_b = -C_t = C_n = \sqrt{C_s}$
- general isotropic reflection: $C_b = C_t$
- anisotropy: $C_b \neq C_t$
- isotropic diffuse term: $C_b = C_t = 0,\ C_s = (h+2)/2\pi$
- off-specular reflection: $C_n < -C_b = -C_t$
- retro-reflections: $C_b, C_t, C_n > 0$

Compound model

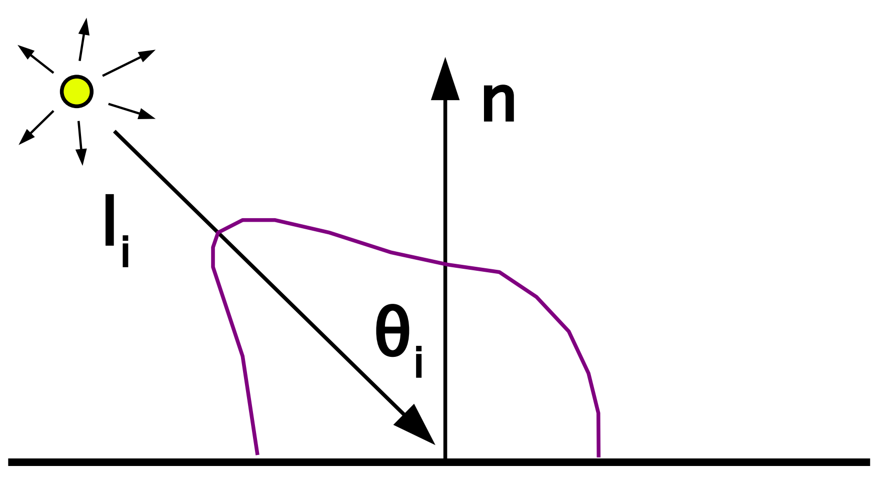
- Superposition of several lobes
  - each one is defined by: $\mathbf{C_{b,i}}$, $\mathbf{C_{t,i}}$, $\mathbf{C_{n,i}}$, $\mathbf{h_i}$

$$f_r(l,v) = \sum_i \left[C_{b,i} l_b v_b + C_{t,i} l_t v_t + C_{n,i} l_n v_n \right]^{h_i}$$



Lambert law is not perfect..

- pure "cosine" surface is not as common in the nature
  - rough, grainy surfaces (sandpaper, sand, etc.)
  - **full moon** – contours should be darker but actually **they are not !**
  - "back-scattering" effect, reflecting (passive) taillights

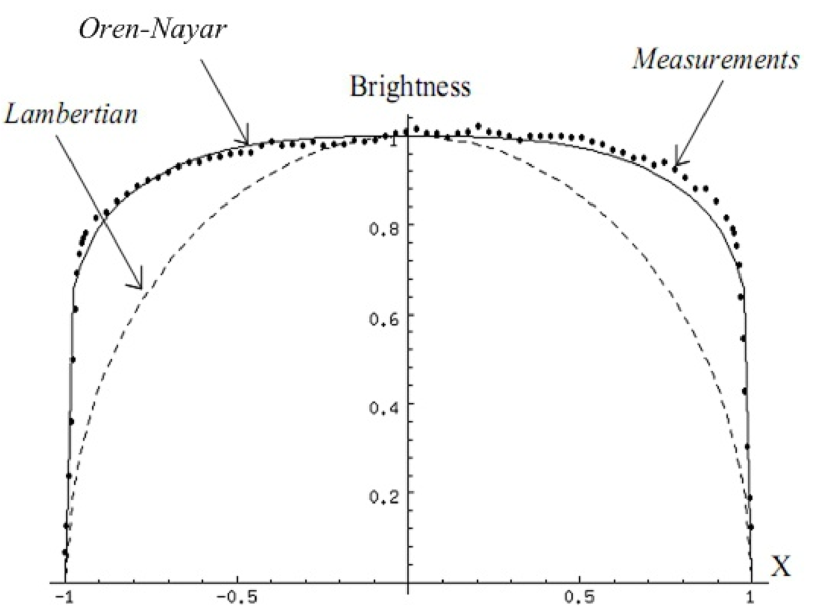


Oren–Nayar model

- based on **microfacet idea**
  - diffuse reflection on microfacets
  - simplified formulas – only most important ones

$$E_d = \frac{\rho}{\pi} \cdot E_0 \cdot \cos(\theta_i) \cdot (A + B \cdot max(0, \cos(\phi_o - \phi_i)) \cdot \sin(\alpha) \cdot \tan(\beta))$$

θ_i	incoming angle ($\angle$ l_i, n)
θ_o	outgoing angle ($\angle$ v, n)
Φ_i	incoming azimuth of ω_i
Φ_o	outgoing azimuth of ω_o
α	max(θ_i, θ_o)
β	min(θ_i, θ_o)



Oren–Nayar, final formula

$$E_d = (C_L \circ C_D) \cdot \cos(\theta_i) \cdot (A + B \cdot max(0, \cos(\phi_o - \phi_i)) \cdot \sin(\alpha) \cdot \tan(\beta))$$

$$A = 1 - 0.5 \cdot \frac{\sigma^2}{\sigma^2 + 0.33}$$ (value in denominator – up to 0.57)

$$B = 0.45 \cdot \frac{\sigma^2}{\sigma^2 + 0.09}$$

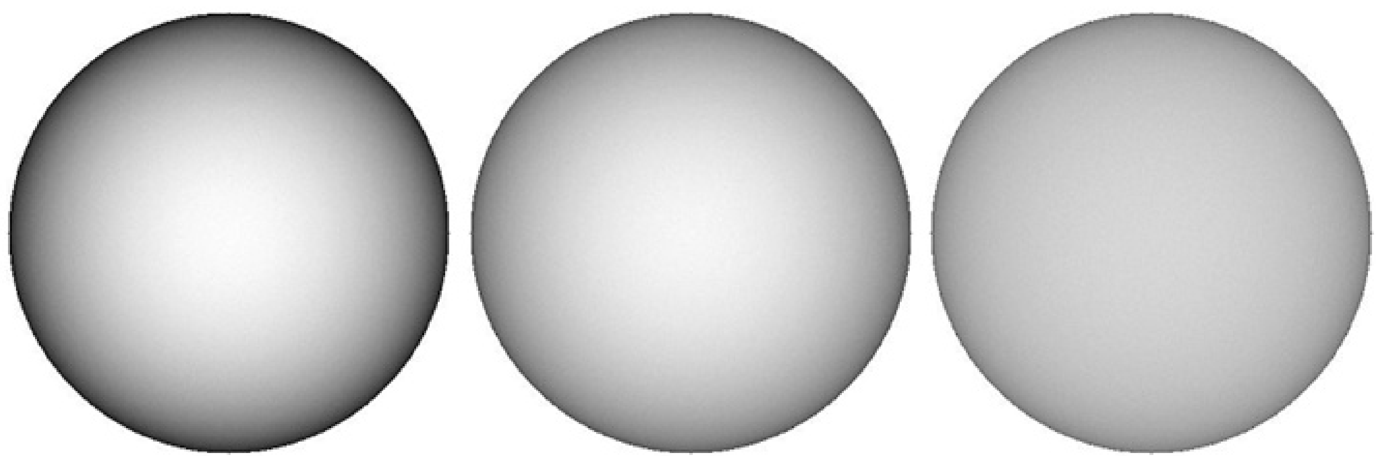
σ **roughness**: mean value of h (see Cook-Torrance)

C_L light source color

C_D material color

Oren–Nayar – samples

Real Image   Lambertian Model   Oren-Nayar Model



$\sigma = 0$ $\sigma = 0.1$ $\sigma = 0.3$

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