matfyz

Computer Graphics Charles University

Radiometry and Radiosity

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Global illumination, radiosity

Based on **physics**

- energy transport (light transport) in simulated environment
- first usage of radiosity in image synthesis: Cindy Goral (SIGGRAPH 1984)

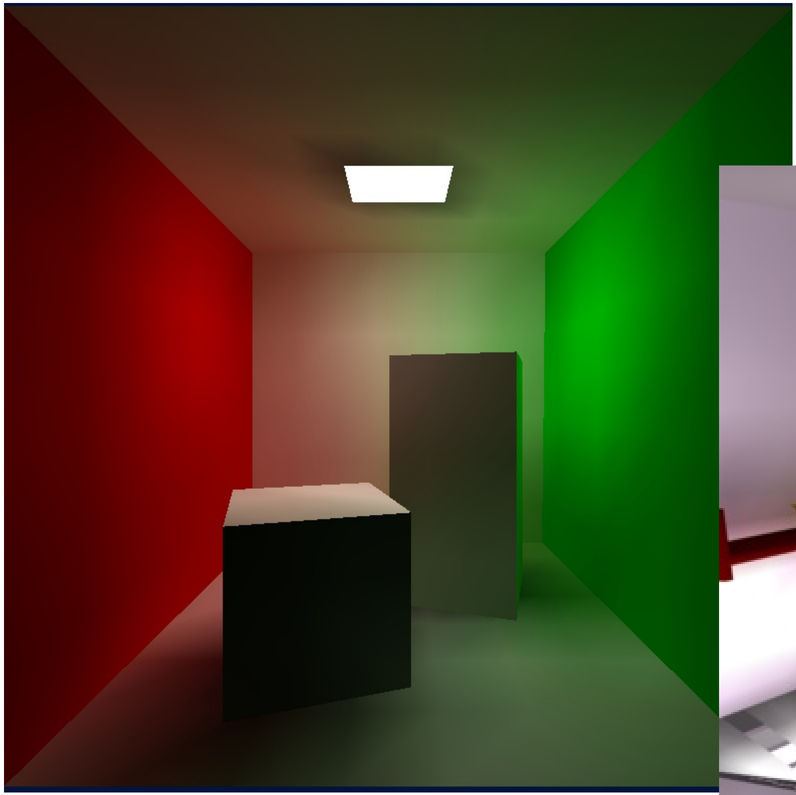
Radiosity is able to compute **diffuse light**, secondary lighting…

Basic radiosity cannot do sharp reflections, mirrors…

Time consuming computation

- Radiosity: light propagation only, R-T or GPU: rendering

Radiosity – examples



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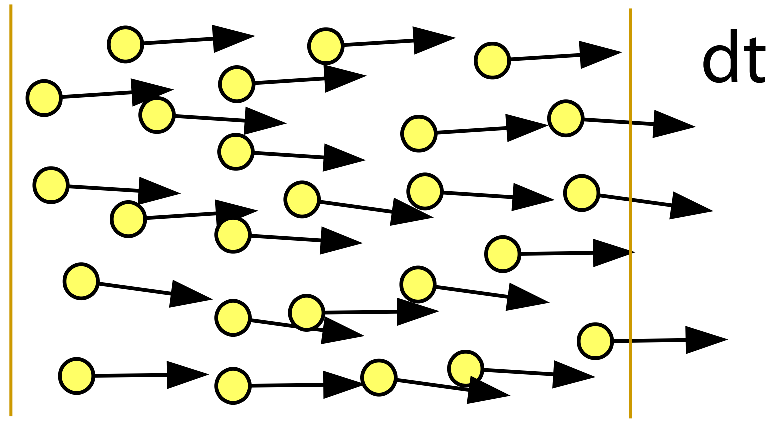
Basic radiometry I

Radiant flux, **Radiant power**

$$\Phi = \frac{dQ}{dt} \qquad [\,\mathrm{W}\,]$$

Number of photons (converted to energy) per time unit
(100W bulb: ~10^{19} photons/s, eye pupil from a monitor: 10^{12} photons/s)

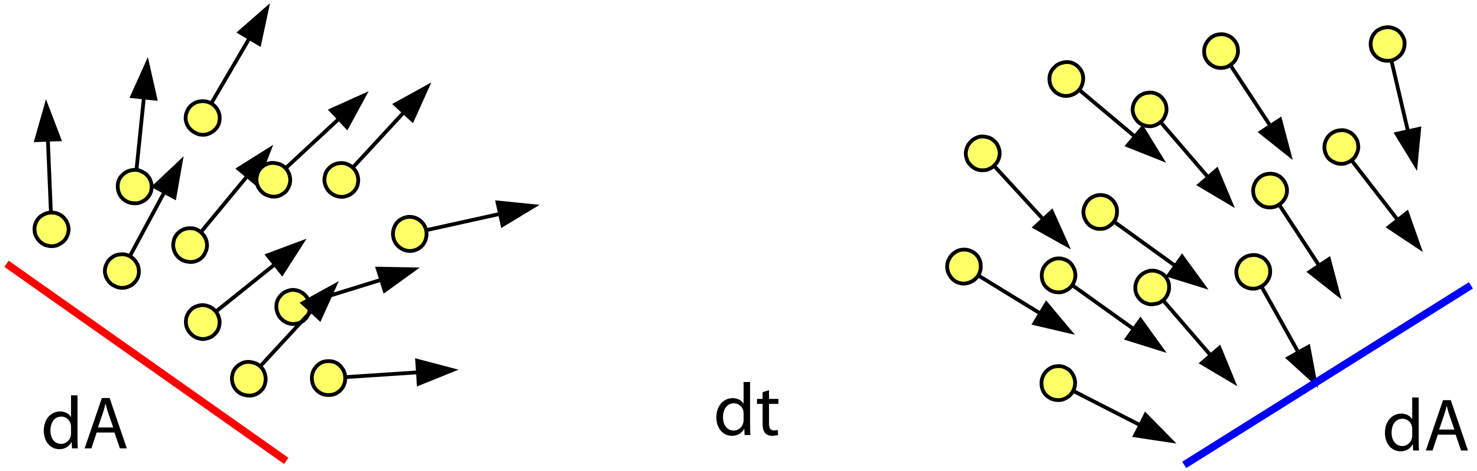


Basic radiometry II

Irradiance, Radiant exitance, Radiosity

$$E(x) = \frac{d\,\Phi(x)}{d\,A(x)} \qquad [\,\mathrm{W/m^2}\,]$$

Photon areal density (converted to energy) incident or radiated per time unit



Basic radiometry III

Radiance

$$L(x,\omega) = \frac{d^2\Phi(x,\omega)}{d\,A_\omega^\perp(x)\,d\,\sigma(\omega)} \qquad [\,W/m^2/sr\,]$$

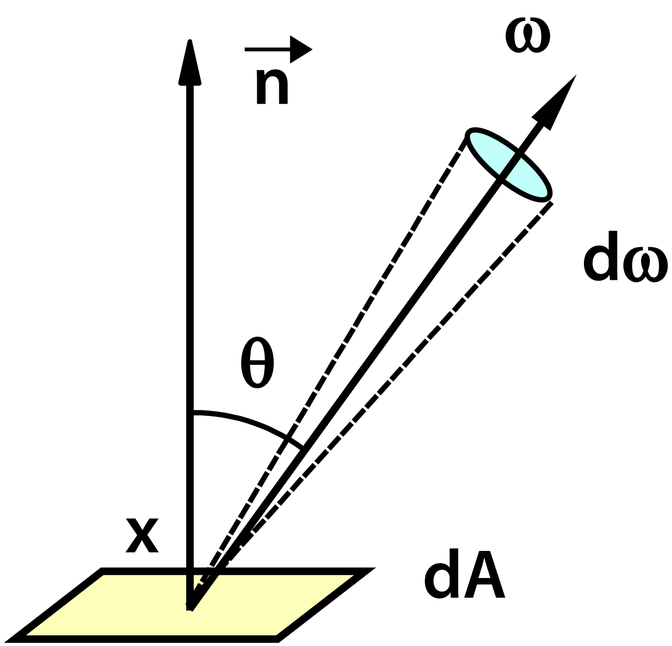
Number of photons (converted to energy) per time unit passing through a small area perpendicular to the direction ω. Radiation is directed to a small cone around the direction ω.

Radiance is a quantity defined as a **density** with respect to $\mathbf{dA}^\perp$ and with respect to solid angle $\mathbf{d\sigma(\omega)}$.

Radiance I

Received/emitted radiance in direction ω

– $\mathbf{L_{in}}(\omega)$ ($\mathbf{L_e}(\omega)$, $\mathbf{L_{out}}(\omega)$)   [W/(m²· sr)]

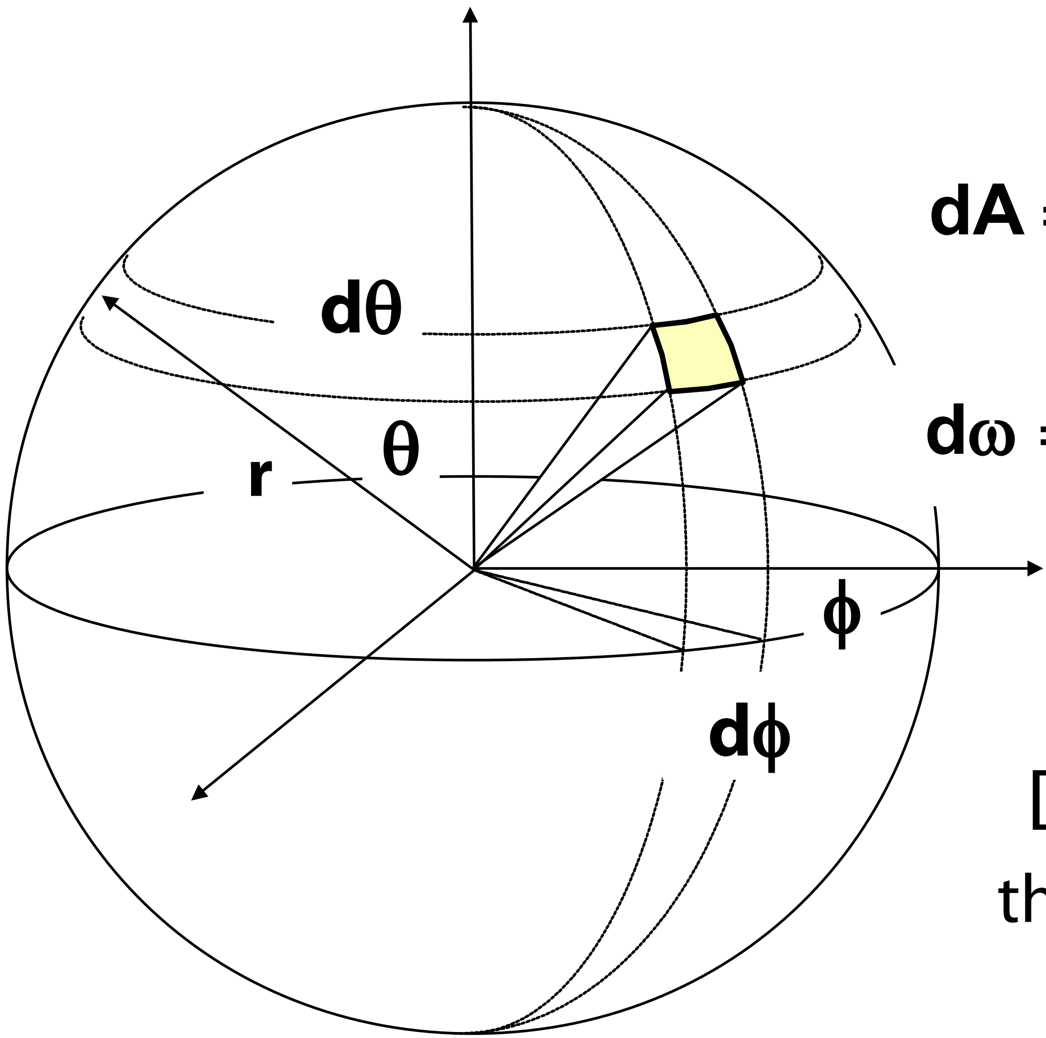


$$L_{out}(x,\omega) = \frac{d^2\Phi}{dA\, d\omega\, \cos\theta}$$

$$= \frac{dB_{out}}{d\omega\, \cos\theta}$$

$$= \frac{dI}{dA\, \cos\theta}$$

Solid angles



$$dA = r^2 \sin\theta \, d\theta \, d\phi$$

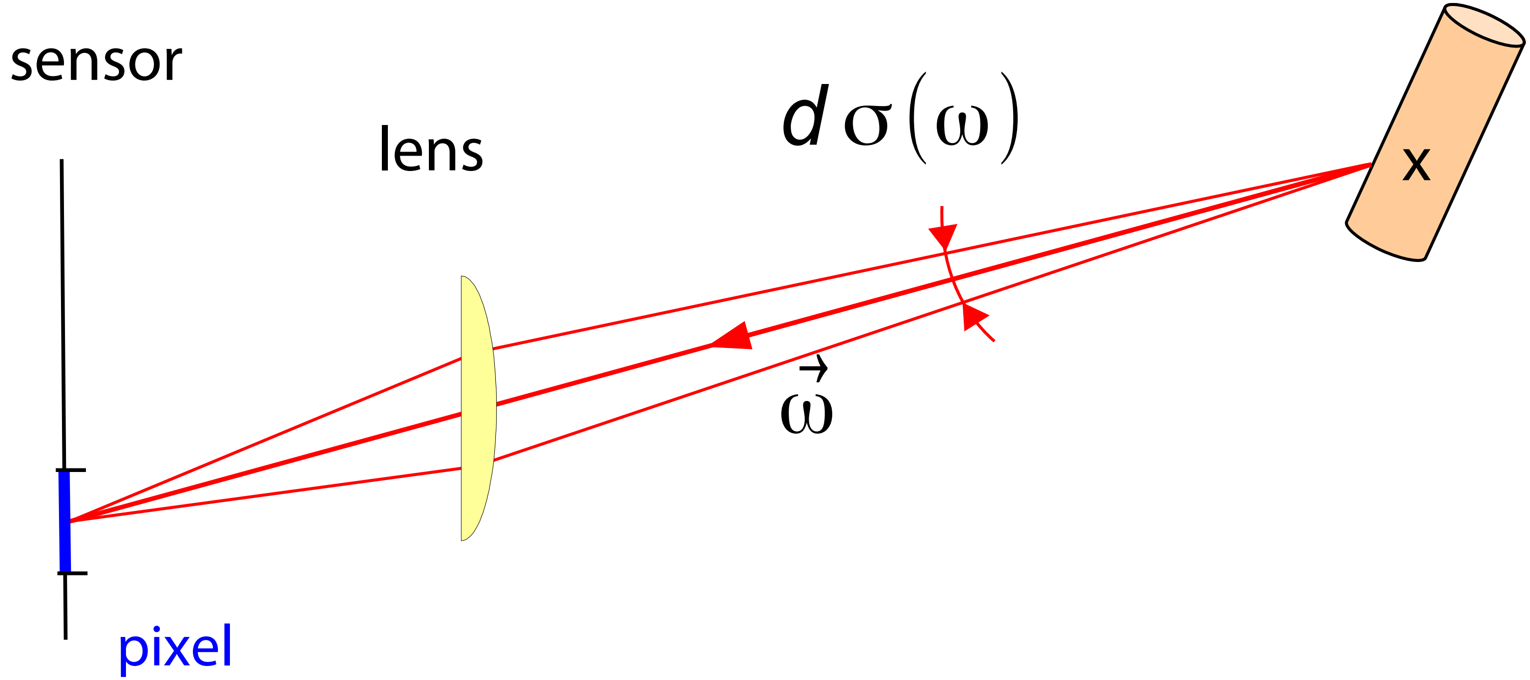
$$d\omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$$

[ω] ... **steradian (sr)**

the whole sphere ... **4π sr**

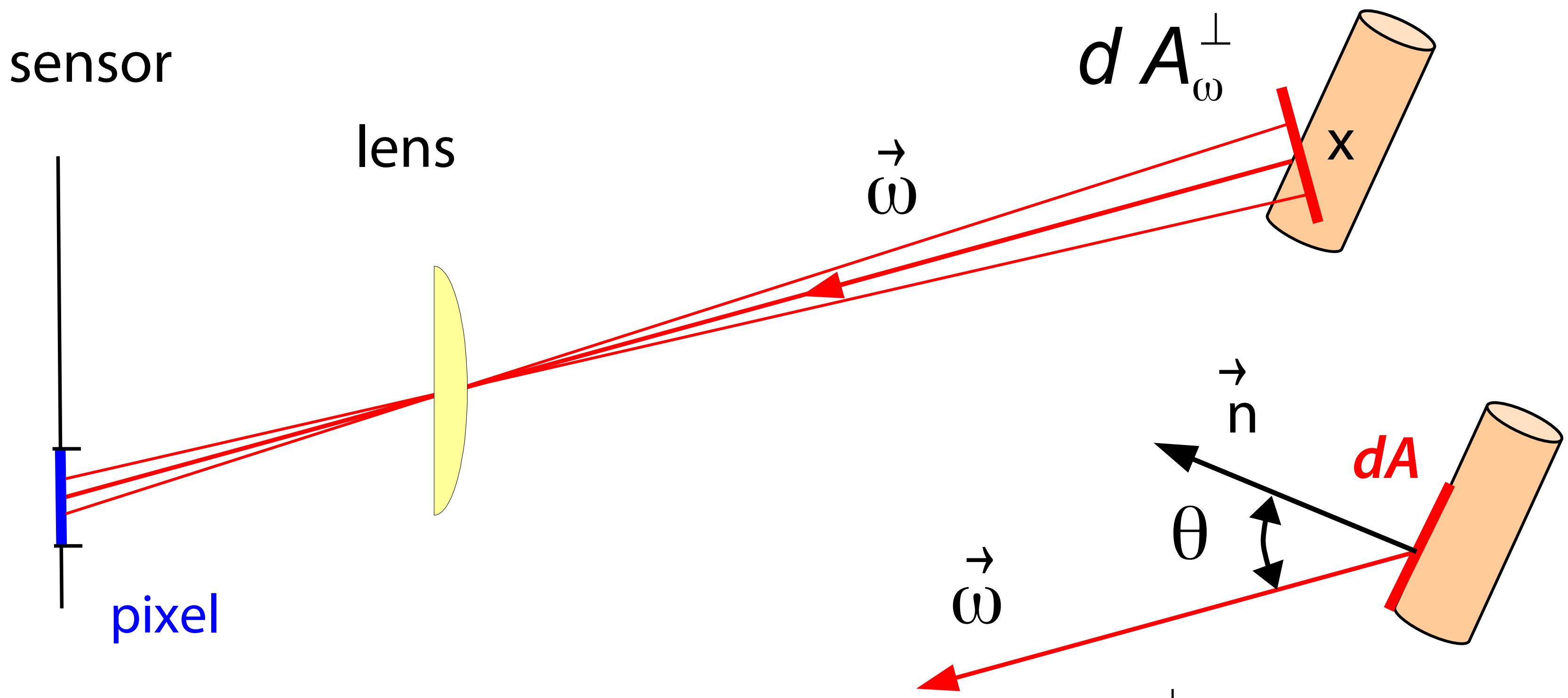
Radiance II

$$\Phi(x,\omega) \propto d\sigma(\omega)$$



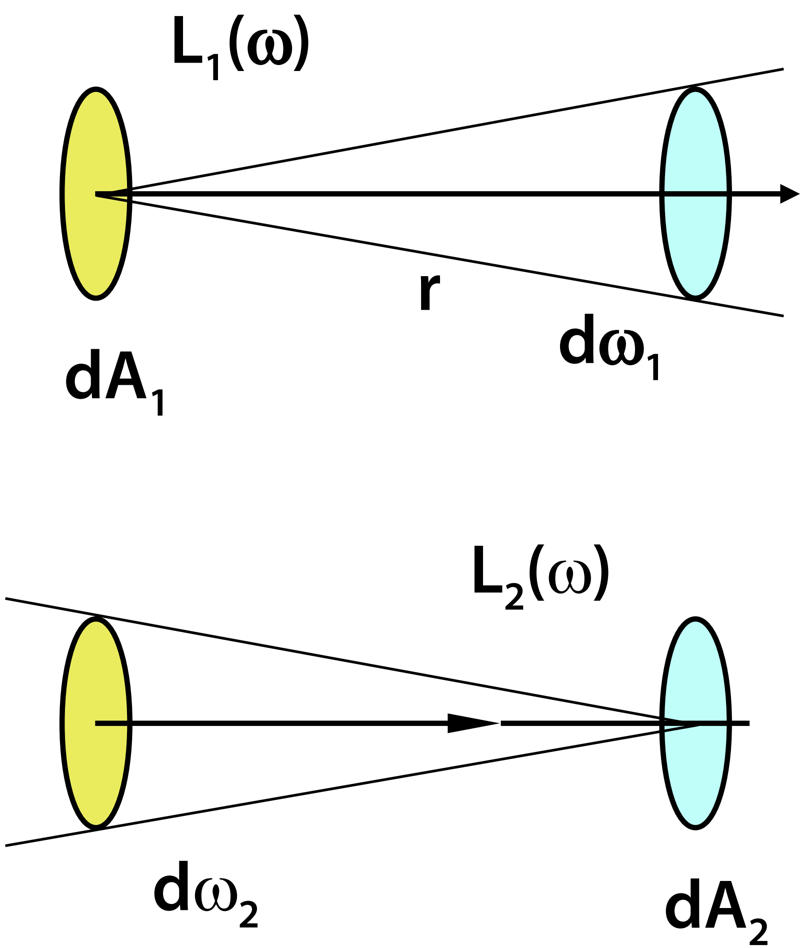
Radiance III

$$\Phi(x,\omega) \propto dA_{\omega}^{\perp}(x)$$



$$dA_{\omega}^{\perp} = dA \cdot \cos\theta$$

Energy preservation law (ray / fiber)

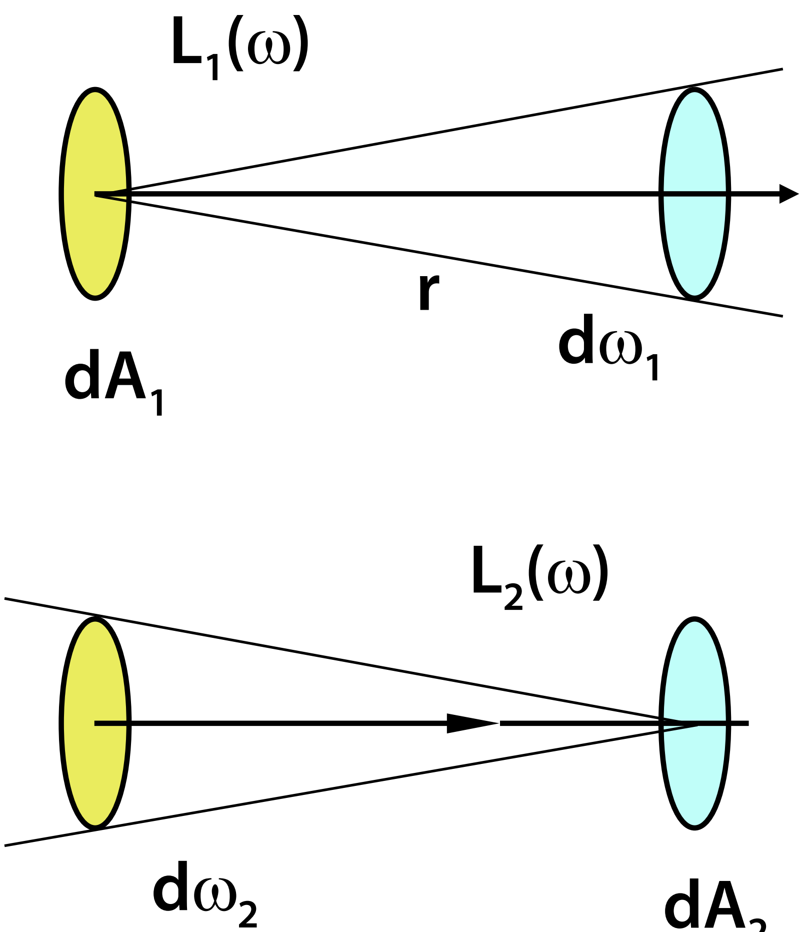


$$L_1 \, d\omega_1 \, dA_1 = L_2 \, d\omega_2 \, dA_2$$

emitted power

received power

Energy preservation law (ray / fiber)



$$L_1 \, d\omega_1 \, dA_1 = L_2 \, d\omega_2 \, dA_2$$

$$T = d\omega_1 \, dA_1 = d\omega_2 \, dA_2 = $$

$$= \frac{dA_1 \, dA_2}{r^2}$$

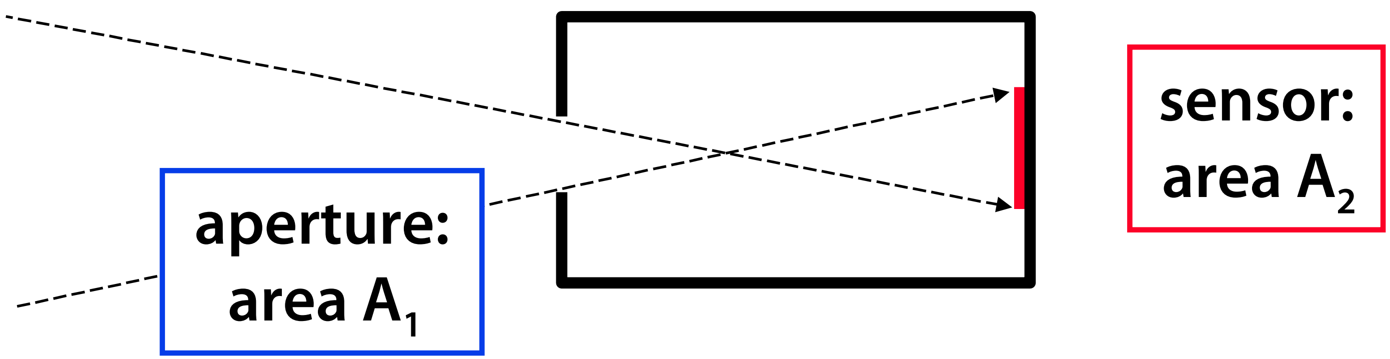
ray capacity

$$\boxed{L_1 = L_2}$$

ray ... radiance L

Light measurement

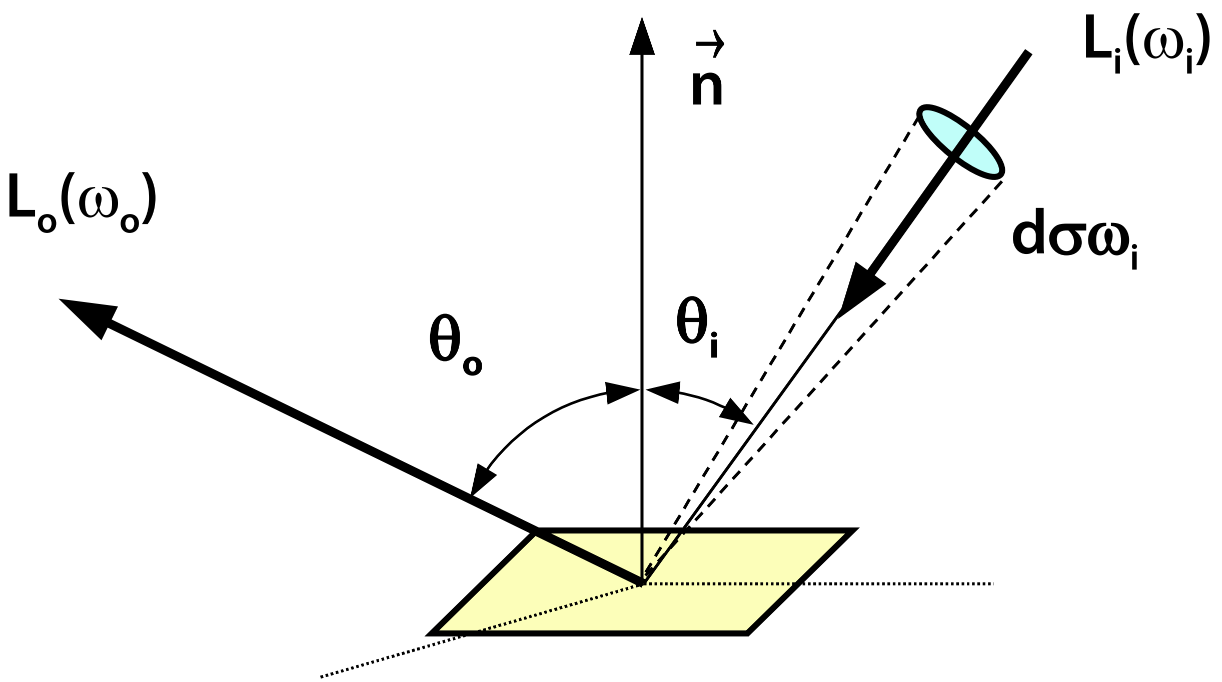
Measured quantity is proportional to **radiance** from visible scene



$$\underline{R} = \int_{A_2} \int_{\Omega} L_{in}(A, \omega) \cdot \cos\theta \, d\omega \, dA = \underline{L_{in} \cdot T}$$

BSDF (Local transfer function)

(„Bidirectional Scattering Distribution Function“, older term: BRDF)



$$f_s(\omega_i \to \omega_o) = \frac{dL_o(\omega_o)}{dE(\omega_i)} = \frac{dL_o(\omega_o)}{L_i(\omega_i)\,\cos\theta_i\,d\sigma^{\perp}(\omega_i)}$$

Helmholtz law (reciprocity)

For **real surfaces** (physically plausible)

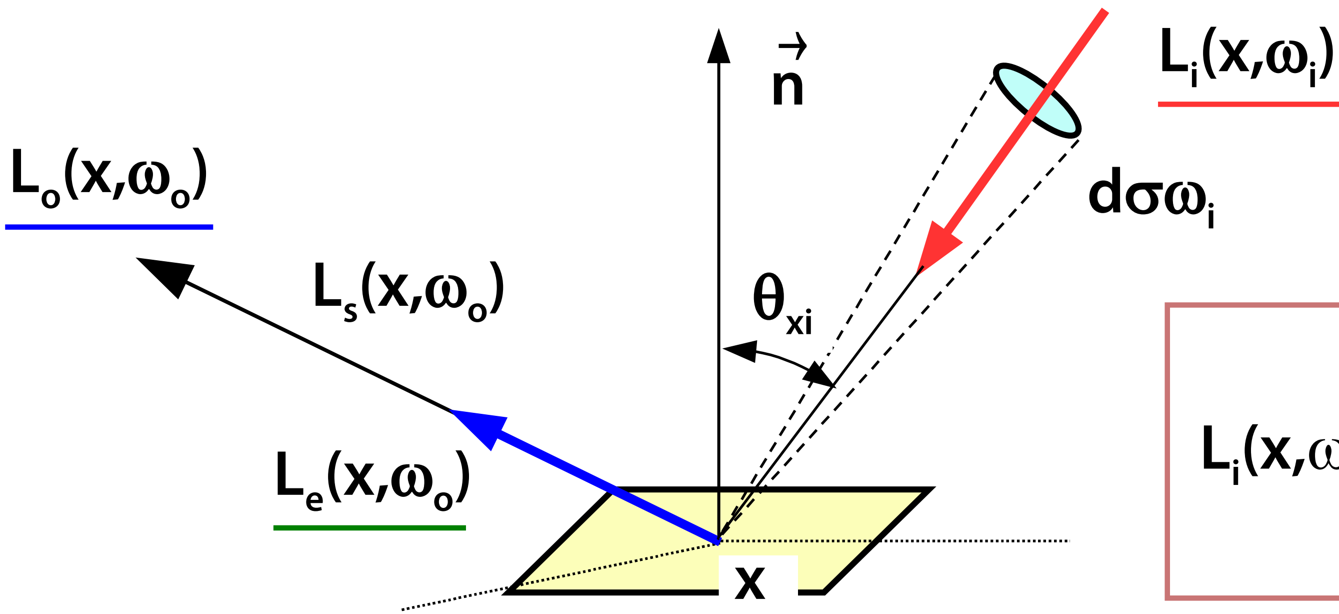
$$\mathbf{f}(\omega_{in} \to \omega_{out}) = \mathbf{f}(\omega_{out} \to \omega_{in})$$

General **BSDF needs not be isotropic** (invariant to rotation around the surface normal)

- metal surfaces polished in one direction…

$$\mathbf{f}(\theta_{in}, \phi_{in}, \theta_{out}, \phi_{out}) \neq \mathbf{f}(\theta_{in}, \phi_{in} + \phi, \theta_{out}, \phi_{out} + \phi)$$

Local rendering equation



ray-cast

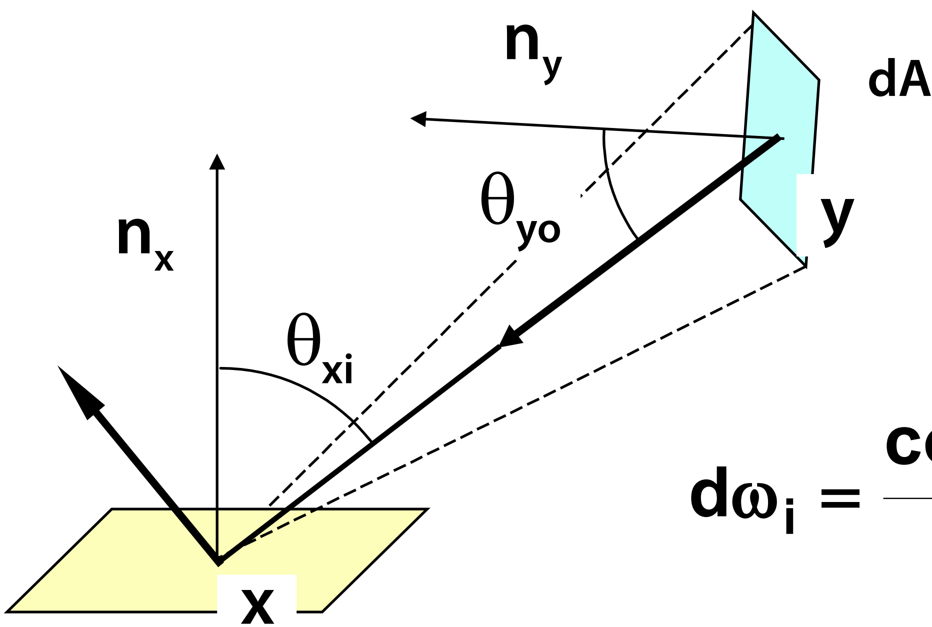
vacuum

$$L_i(x,\omega) = L_o(x_M(x,w),-\omega)$$
$$= L_o(y,-\omega)$$

own emission at **x**

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int L_o(y,-\omega_i)\cdot f_s(x,\omega_i\to\omega_o)\cdot d\sigma_x^{\perp}(\omega_i)$$

Radiance received from a surface



$$d\omega_i = \frac{\cos\theta_{yo}\; dA}{\|x - y\|^2}$$

Geometric term: $$G(y, x) = \frac{\cos\theta_{yo}\cos\theta_{xi}}{\|x - y\|^2}$$

Radiance received from a surface

$$L_o(x,\omega_o) =$$

integral over all incoming directions

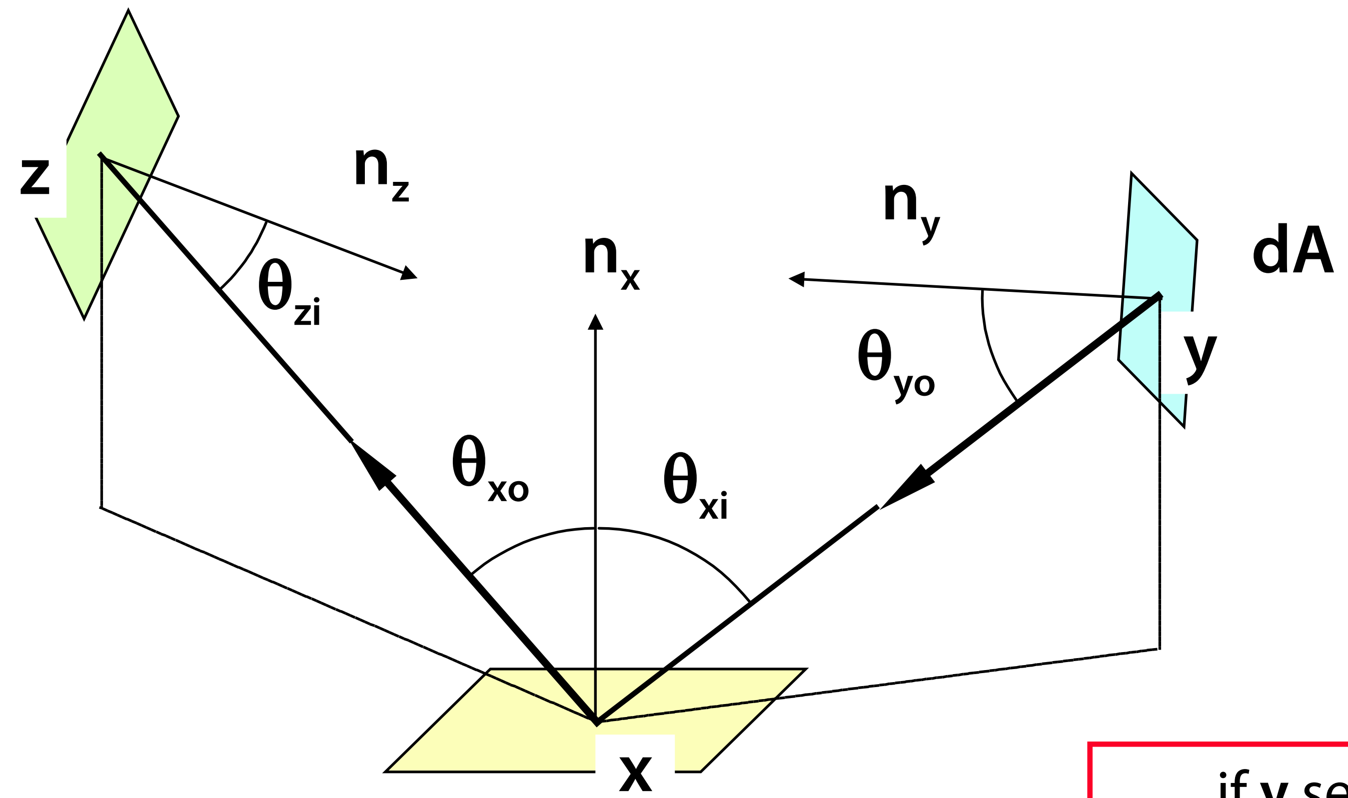
$$= L_e(x,\omega_o) + \int_{\Omega} f(x,\omega_i \to \omega_o) \cdot L_i(x,\omega_i) \cdot \cos\theta_{xi} \, d\omega_i =$$

$$= L_e(x,\omega_o) + \int_{S} f(x,\omega_i \to \omega_o) \cdot L_o(y,-\omega_i) \cdot G(y,x) \, dA$$

integral over an emitting surface

(assumption: the whole surface **S** is visible from **x**)

Reflected light



if **y** sees **x**

(terminology only) $$\mathbf{L}(\mathbf{y},\mathbf{x}) = \mathbf{L_o}(\mathbf{y},\mathbf{x}-\mathbf{y}) = \mathbf{L_i}(\mathbf{x},\mathbf{y}-\mathbf{x})$$

$$\mathbf{f}(\mathbf{y},\mathbf{x},\mathbf{z}) = \mathbf{f}(\mathbf{x},(\mathbf{y}-\mathbf{x}) \rightarrow (\mathbf{z}-\mathbf{x}))$$

Indirect radiance equation

$$V(y,x) = \begin{cases} 1 & \text{if } y \text{ sees } x \\ 0 & \text{else} \end{cases}$$

$$\underline{L(x,z)} = \underline{L_e(x,z)} + \int_S \underline{f(y,x,z)} \cdot L(y,x) \cdot \underline{G(y,x) \cdot V(y,x)} \, dA$$

$L_e(x,z)$: **own** (emitted) **radiant exitance**

$f(y,x,z)$: **BRDF**

$G(y,x) \cdot V(y,x)$: **geometric terms**

Radiosity equation

Assumption – **ideal diffuse** (Lambertian) surface

- **BRDF** is not dependent on incoming/outgoing angles
- outgoing radiance **L(y,ω)** is independent on direction ω

$$L(x,z) = L_e(x,z) + f(x) \cdot \int_S L(y,x) \cdot G(y,x) \cdot V(y,x)\, dA$$

$$L(x,z) = B(x)/\pi, \quad L_e(x,z) = E(x)/\pi, \quad f(x) = \rho(x)/\pi$$

$$B(x) = E(x) + \rho(x) \cdot \int_S B(y) \cdot \frac{G(y,x) \cdot V(y,x)}{\pi}\, dA$$

Discrete solution

$$\mathbf{B}(\mathbf{x}) = \mathbf{E}(\mathbf{x}) + \rho(\mathbf{x}) \cdot \int_S \mathbf{B}(\mathbf{y}) \cdot \mathbf{g}(\mathbf{y},\mathbf{x})\, \mathbf{dA}$$

where $$\mathbf{g}(\mathbf{y},\mathbf{x}) = \frac{\mathbf{G}(\mathbf{y},\mathbf{x}) \cdot \mathbf{V}(\mathbf{y},\mathbf{x})}{\pi}$$

Solution **B** is infinit-dimensional

Discretization of the task

- **Monte-Carlo** ray-tracing (dependent on camera)
- classical **radosity** (finite/boundary elements FEM)

General radiosity method

❶ Object surfaces divided into set of **elements**

❷ Definition of **knot points** on elements

– **radiosity** will be computed there

❸ Choice of an **approximation method** and error metric

– basis functions for convex blend from knot points

❹ **Coefficients** of linear equation system

– “form-factors”

General radiosity method

❺ Solution of **a linear equation system**
- result: radiosities at knot points

❻ Reconstruction of values on **whole surfaces**
- linear blends using basis functions and knot point radiosities

❼ **Rendering** of results (arbitrary view)
- light is proportional to radiosity

Remarks

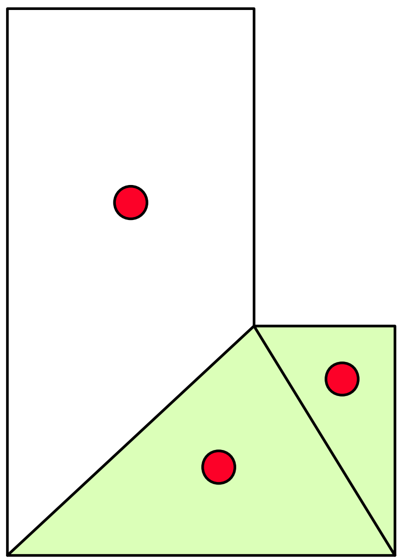
Step ❸ is performed in the **algorithm design** phase

– does not appear in the implementation

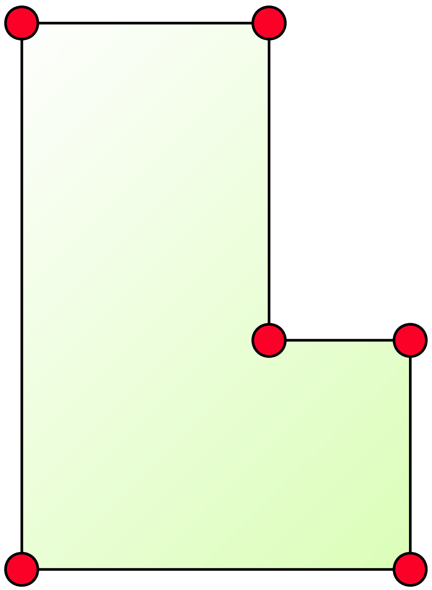
Some **advanced methods** do not strictly follow the sequence ❶ to ❼

– sometimes a computation flow goes back to some previous phase, some phases could be iterated…

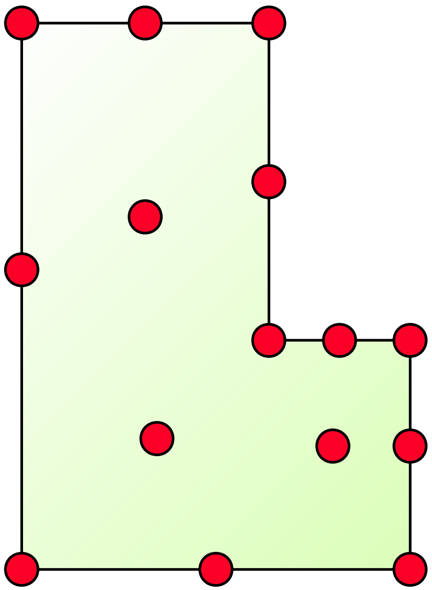
Radiosity approximation



constant
(knots in
centers)



bilinear
(knots in
vertices)



quadratic
(more inside
knots…)

Constant elements

On every element A_i **constant reflectivity** is assumed ρ, radiosity B = average of **B(x)**

- terminology: ρ_i, B_i for $i = 1 \dots N$

$$B(x) = E(x) + \rho(x) \cdot \int_S B(y) \cdot g(y, x)\, dA$$

average over area A_i

$$B_i = E_i + \rho_i \cdot \frac{1}{A_i} \int_{A_i} \left[\sum_{j=1}^{N} B_j \int_{A_j} g(y, x)\, dA_j \right] dA_i$$

radiosity received in point **x** (lying on A_i)

Basic radiosity equation

Swapping sum and integral

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^{N} B_j \cdot \frac{1}{A_i} \int_{A_i} \int_{A_j} g(y, x)\ dA_j\ dA_i$$

Geometric term – **form factor F_{ij}**

(part of energy irradiated from $\mathbf{A_i}$ received directly by $\mathbf{A_j}$)

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^{N} B_j\, F_{ij} \quad \left[\frac{W}{m^2}\right]$$

Intuitive derivation

$$B_i A_i = E_i A_i + \rho_i \cdot \sum_{j=1}^{N} B_j A_j F_{ji} \quad [\mathrm{W}]$$

Emitted power = own power + reflected power

„Reciprocal rule“ $A_j F_{ji} = A_i F_{ij}$

$$B_i A_i = E_i A_i + \rho_i \cdot \sum_{j=1}^{N} B_j F_{ij} A_i \quad \Big| \cdot A_i^{-1}$$

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^{N} B_j F_{ij} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

System of linear equations

$$B_i - \rho_i \cdot \sum_{j=1}^{N} B_j F_{ij} = E_i \qquad i = 1..N$$

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & -\rho_1 F_{1,2} & .. & -\rho_1 F_{1,N} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & .. & -\rho_2 F_{2,N} \\ .. & .. & .. & .. \\ -\rho_N F_{N,1} & -\rho_N F_{N,2} & .. & 1-\rho_N F_{N,N} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ .. \\ B_N \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ .. \\ E_N \end{bmatrix}$$

Vector of unknown vars $[B_i]$

System of linear equations

For **planar (convex) surfaces**: $\mathbf{F_{ii} = 0}$

– the diagonal contains only unit values

Nondiagonal items are usually very small (abs value)

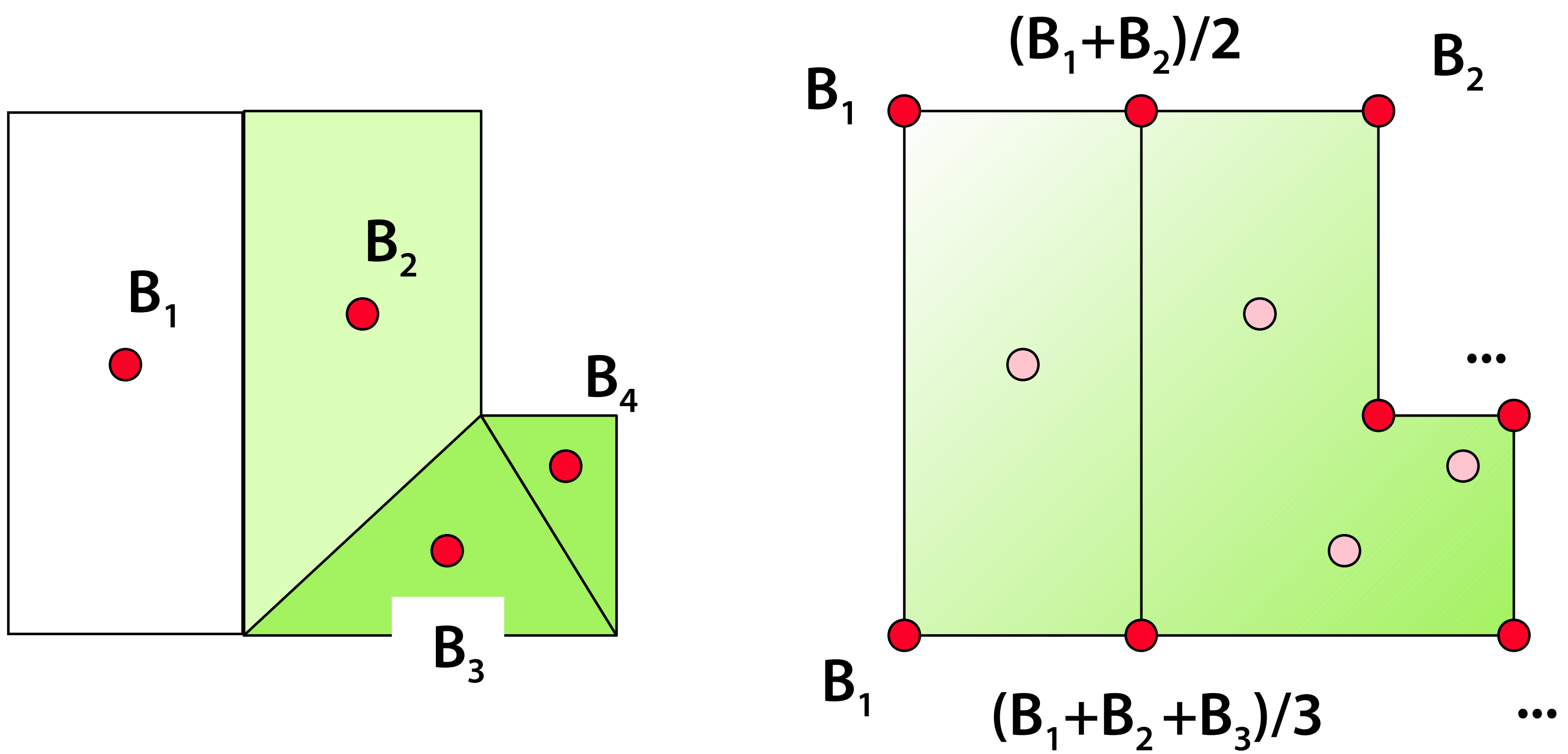
– matrix is "diagonally dominant"

$\Rightarrow$ system is stable and can be solved by **iterative methods** (Jacobi, Gauss-Seidel)

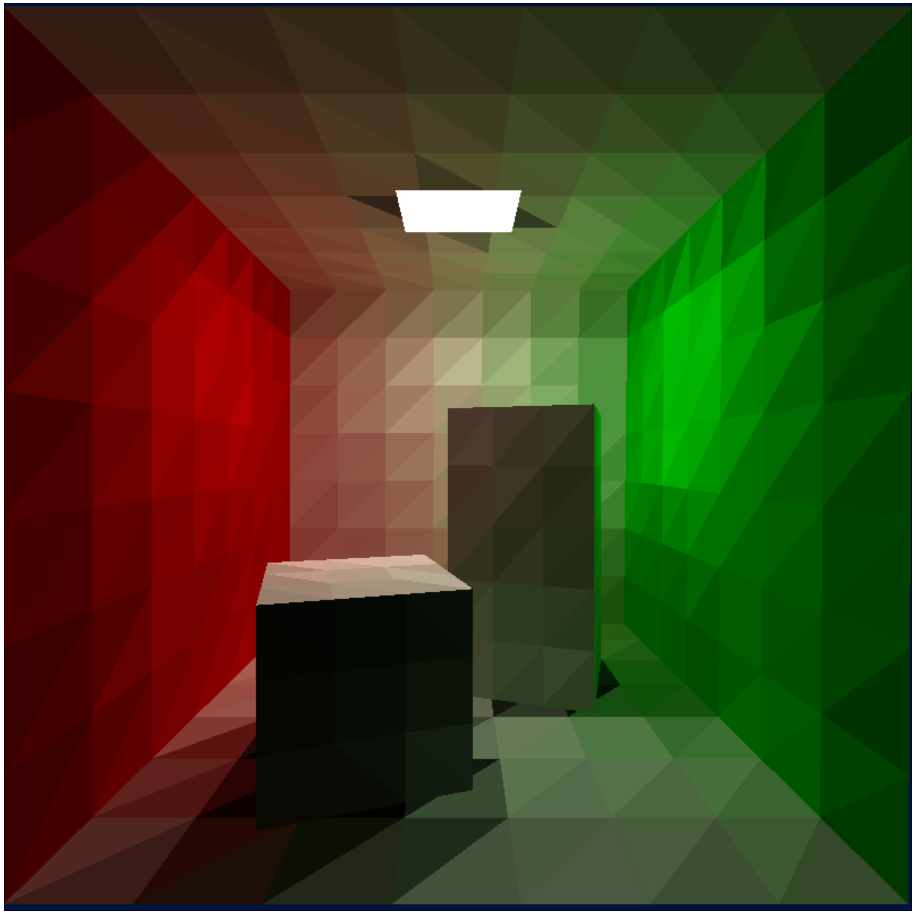
For a **change of light (light sources) [$\mathbf{E_i}$]** system needs not to be fully re-computed, only the reverse phase could be done

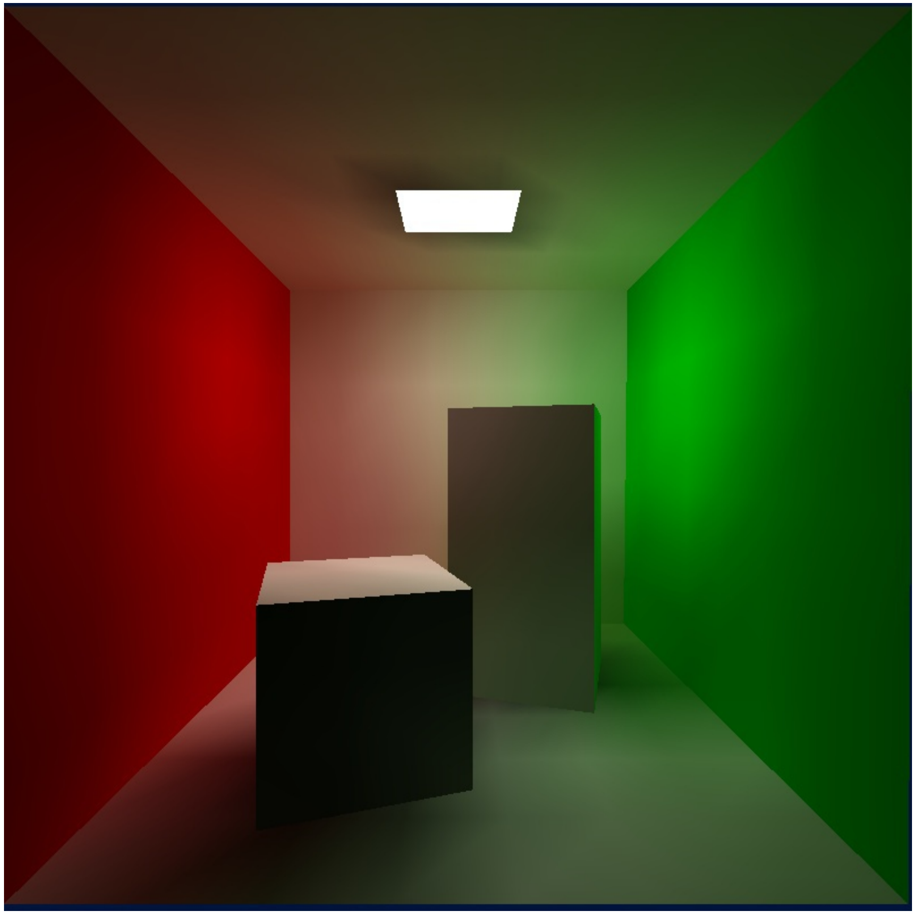
Radiosity transfer to vertices

Even in constant element approach usage of some color interpolation method is recommended (**Gouraud**)



Linear color interpolation





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