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Procedural Modeling of Urban Road Networks Supplementary Material

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1. Historical Context

Below, Historical Context values for all results are given. Note that $\boldsymbol{P}(t,\alpha)$ is *fully defined* by means of $\boldsymbol{S}_c(t)$, $\boldsymbol{S}_w(t)$, $\boldsymbol{S}_p(t)$, as mentioned in the caption of Figure 7, and only these coefficients are given in the table below. We use a step of our algorithm as the basic unit of time. All results were generated over the course of 300 steps.

Historical Context parameters for Figures 11a, 11b, 11d, 11e, 11f, 12.

Time	Step Center	Step Width	Step Height	Recommended Road Capacity	Major Road Coefficient	Minor Road Coefficient	Unbuilt Harbour Coefficient	Built Harbour Coefficient	Bridge Coefficient	Extendible Road Coefficient	Street Pattern
t	$\boldsymbol{S}_c(t)$	$\boldsymbol{S}_w(t)$	$\boldsymbol{S}_p(t)$	$\boldsymbol{A}(t)$	$\boldsymbol{C}_1(t)$	$\boldsymbol{C}_2(t)$	$\boldsymbol{C}_3(t)$	$\boldsymbol{C}_4(t)$	$\boldsymbol{C}_5(t)$	$\boldsymbol{C}_6(t)$	$\boldsymbol{K}$
1–8	2.3	0.5	100	300	0.85	0.4	50000	16666	50000	0.3	Medieval
9-34	2.3	0.5	100	800	0.25	6.3	5000	1666	50000	0.3	Grid
35-95	2.3	0.5	100	1100	0.25	6.3	5000	1666	50000	0.3	Grid
96-300	2.3	0.5	100	1100	0.25	6.3	5000	1666	50000	0.3	Erratic Grid

Historical Context parameters for Figure 11c.

Time	Step Center	Step Width	Step Height	Recommended Road Capacity	Major Road Coefficient	Minor Road Coefficient	Unbuilt Harbour Coefficient	Built Harbour Coefficient	Bridge Coefficient	Extendible Road Coefficient	Street Pattern
t	$\boldsymbol{S}_c(t)$	$\boldsymbol{S}_w(t)$	$\boldsymbol{S}_p(t)$	$\boldsymbol{A}(t)$	$\boldsymbol{C}_1(t)$	$\boldsymbol{C}_2(t)$	$\boldsymbol{C}_3(t)$	$\boldsymbol{C}_4(t)$	$\boldsymbol{C}_5(t)$	$\boldsymbol{C}_6(t)$	$\boldsymbol{K}$
1–43	2.3	0.5	100	300	0.85	0.4	50000	16666	50000	0.3	Medieval
44–60	1.9	1.5	40	300	0.125	0.1	5000	1666	50000	0.3	Medieval
61–300	2.3	0.5	100	1100	0.25	6.3	5000	1666	50000	0.3	Grid

Values of the time independent $\boldsymbol{O}_1,\ldots,\boldsymbol{O}_4$ scalar constants.

$\boldsymbol{O}_1$	$\boldsymbol{O}_2$	$\boldsymbol{O}_3$	$\boldsymbol{O}_4$
$0.3\cdot(180/\pi)$	50	0.5	0.5

2. Per-City Parameter Values and Timings

Per-City Parameter Values.

Name	$\boldsymbol{b}$	$\boldsymbol{u}$
Figure 11a	0.084	0.55
Figure 11b	0.111	0.55
Figure 11c	0.14	0.7
Figure 11d	0.09	0.45
Figure 11e	0.12	0.55
Figure 11f	0.2	0.35
Figure 12	0.108	0.55

Average running time over 5 runs in seconds.

Figure 11a	Figure 11b	Figure 11c	Figure 11d	Figure 11e	Figure 11f	Figure 12
272.01s	150.53s	166.58s	157.256s	209.505s	149.753s	237.68s

3. Notation

The symbols are split into three groups:

1. All functions and coefficients related to the *historical context*, that is those reused for a set of generated cities, are in upper-case bold, for example $\boldsymbol{G}(t), \boldsymbol{K}(t), \boldsymbol{O}_1$.
2. Values that are supplied or *generated for each city* are in lower case bold, for example $\boldsymbol{t}(t)$ and $\boldsymbol{m}$.
3. Other, *non-input symbols*, mostly used in the description of the algorithm itself, are in a normal, non-bold font, for example t, G, α, and $\textsc{Cost}(P)$.

In many cases, e.g. s and S, a lower-case letter will denote members of set denoted by the same upper-case letter. All of the pathfinding related symbols are in calligraphic letters. Functions used in pseudo-code are in in small-caps. Where possible, the first letter of the described term is used as a symbol, for example B for the *b*uilt-up area.

Historical Context (upper-case bold).

	$\boldsymbol{G}(t)$	Growth per simulation step, expressed in number of nodes $j \in \boldsymbol{J}$ that will spawn new streets.
	$\boldsymbol{K}(t)$	Sequence of minor road pattern definitions.
	$\boldsymbol{R}(t)$	Major road construction parameters; an umbrella function, see below for a complete list.
$\boldsymbol{R}(t) =$	$\boldsymbol{P}(t,\alpha)$	Turn angle penalty function, expressed using $\boldsymbol{S}_c(t)$, $\boldsymbol{S}_w(t)$, $\boldsymbol{S}_p(t)$.
	$\boldsymbol{S}_c(t)$	Center of the step function.
	$\boldsymbol{S}_w(t)$	Width of the step function.
	$\boldsymbol{S}_p(t)$	Height of the step function.
	$\boldsymbol{A}(t)$	Recommended major road capacity.
	$\boldsymbol{C}_1(t)$	Major road edge cost coefficient for $\mathcal{G}'$.
	$\boldsymbol{C}_2(t)$	Minor road edge cost coefficient for $\mathcal{G}'$.
	$\boldsymbol{C}_3(t)$	Unbuilt harbor edge cost coefficient for $\mathcal{G}'$.
	$\boldsymbol{C}_4(t)$	Built harbor edge cost coefficient for $\mathcal{G}'$.
	$\boldsymbol{C}_5(t)$	(Unbuilt) bridge cost coefficient for $\mathcal{G}'$.
	$\boldsymbol{C}_6(t)$	Extendable road edge cost coefficient for $\mathcal{G}'$.
	$\boldsymbol{O}_1$	Edge slope coefficient.
	$\boldsymbol{O}_2$	Segment traffic capacity per unit $\textsc{Width}$.
	$\boldsymbol{O}_3$	Trade route length coefficient.
	$\boldsymbol{O}_4$	Trade route cost coefficient.

Per-City Input (lower-case bold).

$\boldsymbol{t}(t)$	Traffic for any city pair at time t.
$\boldsymbol{m}$	Terrain elevation map.
$\boldsymbol{nc}$	List of neighboring cities.
$\boldsymbol{y}$	List of all possible neighboring city pairs.
$\boldsymbol{y}$'	An ordered list that is a subset of $\boldsymbol{y}$.
$\boldsymbol{u}$	Reuse coefficient for paths/roads during initial road network construction.
$\boldsymbol{c}$	Number of neighboring cities, $\boldsymbol{c} = \lvert\boldsymbol{nc}\rvert$.
$\boldsymbol{b}$	Coefficient that relates traffic $\boldsymbol{t}(t)$ and growth $\boldsymbol{G}(t)$ by $\boldsymbol{t}(t) = \boldsymbol{b} \cdot \boldsymbol{G}(t)$

Non-input symbols.	
t	Time, in steps (iterations) of the algorithm.
$T(\dots)$	Traffic over a segment or a trade route.
$T_A(\dots)$	Available traffic capacity for a segment or a trade route.
G	Planar graph representing the road network, $G=(J,S)$.
j	Node, an individual vertex of G.
J	Set of all nodes, that is the vertices of G.
s	Segment, an individual edge of G.
S	Set of all segments, that is the edges of G.
$\mathcal{G}$	Pathfinding graph, $\mathcal{G}=(\mathcal{L},\mathcal{E})$.
$\mathcal{L}$	Regular lattice, i.e. the set of vertices of $\mathcal{G}$. Spaced at ℓ units (metres). Subscripts L, W, F denote lattice vertices over land, water, and forbidden areas respectively.
$\mathcal{E}$	Set of edges of $\mathcal{G}$. Subscripts L, W, H, B denote edges over land, water, harbor edges, and bridge edges respectively.
e	Edge in $\mathcal{G}$.
ℓ	Spacing of the regular lattice $\mathcal{L}$, in metres.
$\mathcal{G}'$	Temporary copy of $\mathcal{G}$ that allows for pathfinding over both the road network and terrain.
$\mathcal{P}$	Path (sequence of edges) found in $\mathcal{G}$ or $\mathcal{G}'$.
r	"Square" radius up to which the vertices in $\mathcal{L}$ are connected by edges.
p_i	Pair of (distinct) cities in $\boldsymbol{nc}$.
C	Neighbouring city $C\in\boldsymbol{nc}$.
P	Trade route, represented as a sequence of segments s, that is edges in G.
P_i	Set of all trade routes that connect the cities of the neighbouring city pair p_i.
$P_{i,j}$	The j-th trade route that connect the cities of the neighbouring city pair p_i.
P_A	Trade route over existing roads proposal, running exclusively over segments, that is edges, in G.
P_B	Trade route over both existing roads and free terrain, that is segments in G and edges in the pathfinding graph $\mathcal{G}$, after they are copied to $\mathcal{G}'$.
P_W	The better of P_A, P_B.
α	Angle between two consecutive segments, ranging from 0 (turn back) to π (no turn).
$\mathcal{S}$	City shell, the outer face of the planar graph G.
B	Built-up area of the city.
$B_{B,i}$	Built-up block.
$B_{T,i}$	Terrain block.
FILL	Fill operator that returns the set of points on the inside of a block or the city shell.
NB	Neighborhood operator that returns the set of points in the neighbourhood of a block or the city shell.
LENGTH(e)	Length of and edge $e\in\mathcal{E}$.
COST(P)	Cost of a trade route.
WEIGHT(e)	Weight of an edge $e\in\mathcal{E}$, that is the difficulty of getting from one vertex of the edge to another.
WIDTH(s)	Width of a road/segment $s\in S$, in metres.
EUCLIDEANDIST(a,b)	Euclidean distance between the positions of two objects a and b.
RANK(z)	Ranking function of nodes z that can grow new minor roads.
SMOOTHSTEP(x)	Smooth step function $3x^2-2x^3$.
Z	Set of nodes from which new minor roads may be grown.
Z'	List of nodes in Z ordered by RANK(z).
z	Nodes in either Z or Z'.
q	Nucleus of city growth.
q_N	Nuclei nearest to a given node z.
q_C	Nuclei on the initial road network that is the closest to geometrical center of the map X.
Q	Set of city growth nuclei.
STRENGTH(q)	Strength of nucleus, STRENGTH$(q)\in\langle 0,1\rangle$, with 0 for no influence at all and 1 for maximum influence.
μ	Expected value of the number of cities $\boldsymbol{c}$.
ϕ	Global angular offset of neighbouring cities. $\phi\in\langle 0,2\pi)$.
θ_i	Per-ray perturbation of the i-th neighbouring city. $\theta_i\in\langle -\frac{\pi}{2c},\frac{\pi}{2c}\rangle$.
$E_{L,\max}$	Maximum edge slope for an edge $e\in\mathcal{E}$.
$E_{L,\text{cut}}$	Maximum allowed edge slope for any edge $e\in\mathcal{E}$.
X	Geometrical center of the map.

Method	Focus	Time progressions	Minor Roads	Major Roads	Land Use	Contributions to road network modelling
Our method	Automatic	Yes	Rule based pattern expansion	Inter-city traffic simulation	None	Influence of neighbouring cities, formation of urban landscape, changing minor road patterns with time, effects of water transportation.
Aliaga et al. [AVB08]	Automatic User-aided	No	Random walk over set of intersections		None	Synthesizes road networks from example fragments, introduces random walk road network syntehsis.
Chen et al. [CEW*08]	User-aided	No	Tensor field tracing	Tensor field tracing	None	Tensor field driven minor andmajor road generation, easily steerable road network generation.
Emilien et al. [EBP*12]	Automatic	Yes	Shortest path	None	None	Scattered settlements, meaningful initial settlement positioning, roads connecting to neighbouring cities.
Galin et al. [GPGB11]	Automatic User-aided	No	None	Graph skeletons	None	First to generate large-scale major road networks, graph pruning based road generation.
Krecklau et al. [KMK12]	User-aided	Yes	Building/destroying roads between key-frames	Building/destroying roads between key-frames	Input map interpolation	Interpolation of historical map "key-frames" by construction/destruction events, uses a scheduling algorithm.
Lechner et al. [LWW03]	Automatic	Yes	Agent Based	None	Agent based	Extender and connector agents generate minor roads, one kind of developer agent for each land use generates parcels.
Muller et al. [PM01]	Automatic	No	Rule based pattern expansion	Follow population gradient	None	First to generate urban road networks, introduced rule based pattern expansion for generating minor roads.
Vanegas et al. [VABW09]	User-aided	No	Enhanced rule based pattern expansion	Sketched by user, possibly expansion	Behavioral modelling	Agent based behavioral modelling, integration with geometric modelling, road generation adapts to land use. Highly interactive.
Vanegas et al. [VGDA*12]	User-aided	No	Rule based pattern expansion	Rule based pattern expansion	User-defined	Ability to model using target indicators, coupling of existing forward procedural modelling methods and inverse modelling to generate desired cities.
Weber et al. [WMWG09]	Automatic	Yes	Rule based pattern expansion	Pattern expansion	Dynamic disequilibrium	First to generate cities that grow over time, highly configurable land-use simulation.
Yang et al. [YWVW13]	Automatic User-aided	No	Templates and/or streamline tracing	Templates and/or streamline tracing	none	Generates optimized urban layouts suitable for urban planning, hierarchically constructs layouts using either templates or streamline tracing.
Yu et al. [YS12]	Automatic	Yes	Pattern matching	Pattern matching	No	Propose a measure for finding best intersection/outgoing roads match in example data.

Table 1: Basic comparison of methods that support road network synthesis.

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