A Zero-Variance-Based Sampling Scheme for Monte Carlo Subsurface Scattering Supplemental Document

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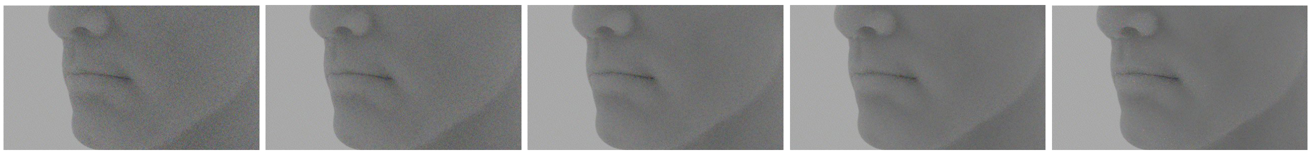

(a) Classical sampling (36s) (b) 25% Dwivedi (33s) (c) 50% Dwivedi (29s) (d) 75% Dwivedi (25s) (e) 100% Dwivedi (21s)

Figure 1: *Our Dwivedi-modified sampling is driven by adjoint importance solutions for semi-infinite, homogeneous medium with a flat boundary. Thus, its application to general curved geometry can lead to occasional high sample weights, or fireflies (see (e) around the nose) because the estimated importance of each path was not accurately predicted. We mitigate this problem by using MIS at each transition to combine classical sampling and the Dwivedi sampling. Note more fireflies near the nose and curved areas and their reduction as MIS mixes the classical and Dwivedi methods. Also note how the render time changes. While the use of the classical sampling is essential to avoid the fireflies for the curved regions, for flat regions like the cheek the pure Dwivedi walk is ideal and mixing in the classical sampling actually adds some noise. To get the best performance, the mixing weights could be driven by local curvature. (All images were rendered using 25 samples per pixel, uniform white illumination, a single-scattering albedo of 0.943 for all wavelengths, and index-matched smooth boundaries.)*

Additional Details

The figures in this supplemental document and their captions provide further details on our method. In addition to the results shown here, the method can also be easily applied for transmission through layers if the thickness and medium orientation are provided at each entry point. In fact, our numerical experiments have shown that the variance reduction achieved in the optically thick transmissive case is orders of magnitude greater relative to reflection from translucent media shown here.

Dwivedi's work [1982] was the first to propose a combination of exponential transition distance stretching and direction sampling based on the discrete eigenmode for the importance solution inside a homogeneous half-space. This was a significant improvement upon earlier heuristic applications of direction-dependent exponential transforms [Clark 1966] as a variance reduction technique for deep penetration problems, where the 'guiding' parameters were either proposed by empirical studies [Ponti 1971] or with multistage adaptive methods [Spanier 1971]. In addition, the importance function is driven from the rigorous diffusion portion of the exact solution for the homogeneous half-space problem, which becomes close to exact far from sources and boundaries, especially for low absorption levels [McCormick and Kuscer 1973]. In such cases, the Dwivedi method is a very good approximation of the idealized zero-variance scheme.

We choose the Dwivedi sampling scheme for our subsurface scattering application because it offers a good compromise between sampling complexity and variance reduction. Our ongoing investigation shows that significant further variance reduction can be achieved by approximating the zero-variance scheme [Hoogenboom 2008] more accurately. This, however, comes at the expense of more complex sampling routines.

Acknowledgments

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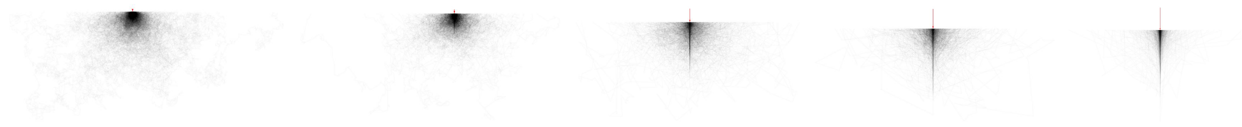
Additional figures on the following page.



(a) classical (any α) (b) Dwivedi $\alpha = 0.99$ (c) Dwivedi $\alpha = 0.9$ (d) Dwivedi $\alpha = 0.7$ (e) Dwivedi $\alpha = 0.3$

Figure 6: *In each subfigure we show 2000 randomly sampled paths created using either classical volumetric sampling (a) or the Dwivedi sampling scheme (b-e). The figures have differing scales—the red arrow is one mean-free-path long and indicates the illumination position and direction. All paths continue inside the semi-infinite medium with isotropic scattering until an escape is sampled. Each path is rasterized with the same opacity, regardless of sample weight. Irrespective of absorption level (the value of α), the classical scheme samples the wide distribution of paths shown in (a), even though many of these paths are heavily absorbed and contribute negligible energy to the final result. Russian roulette helps avoid this wasteful sampling, but increases variance of each sample as a consequence. The Dwivedi sampling scheme we use adapts to the absorption levels of the medium and creates shorter, important paths more often, while simultaneously decreasing the variance of each sample.*

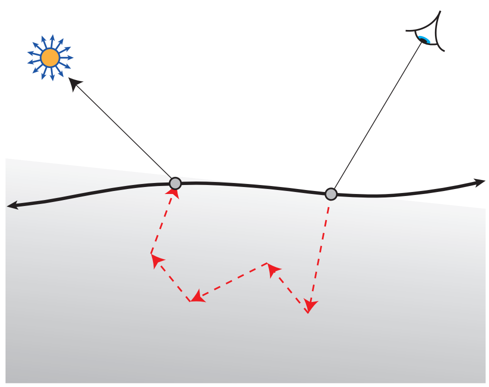


Figure 2: *Our unbiased subsurface scattering solution uses a zero-variance-based scheme to sample the subsurface subpaths—the portions of light transport paths that correspond to subsurface scattering (dashed red line). A semi-infinite half-space importance solution (illustrated as a gradient), that is used to guide the paths back to the boundary, is aligned to the surface normal at the point of entry. Multiple importance sampling is used to combine the new scheme with classical sampling, to robustly handle curved regions while still reducing variance and increasing performance overall.*

classical sampling our (same time)

Figure 3: *Despite the fact that the importance function driving the sampling assumes uniform hemispherical illumination (that is, all directions upon escaping from the medium have the same importance), the modified path sampling lowers variance even when the illumination is nonuniform. The images rendered with classical sampling use 100 samples/pixel while in our results we trace about 50% more samples/pixel in the same time. While the speedup is a profitable side-effect, most of the variance reduction is due to the sampling pdfs closely approximating the zero-variance sampling scheme. As in Fig. 1, the subsurface medium has a single-scattering albedo of 0.943 for all wavelengths and index-matched smooth boundaries. The individual rows show results for different environment maps. The first two rows show the same images as in the abstract.*

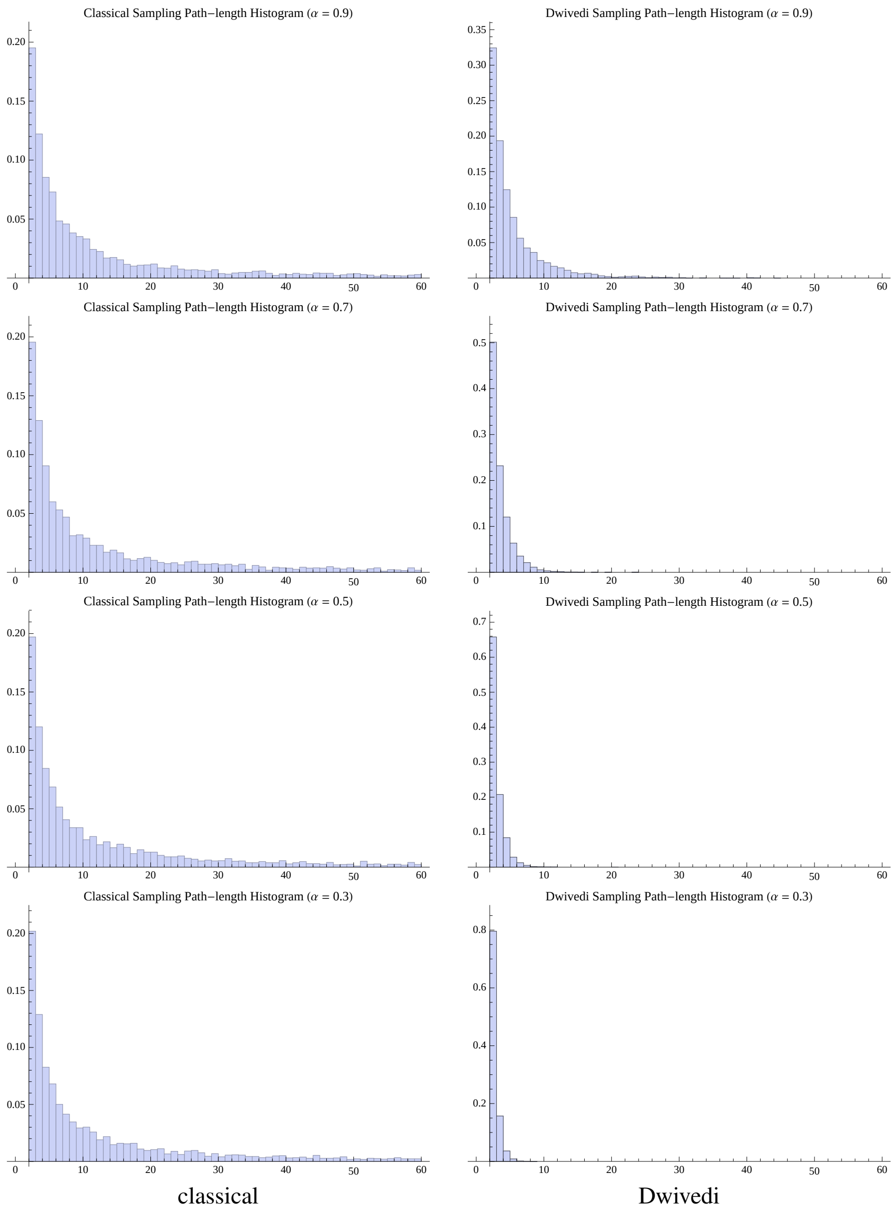


Figure 7: *Comparison of the distributions of path lengths (in terms of path segment count) generated by classical sampling (without Russian roulette) and our application of Dwivedi sampling for the problem of reflection of normally-incident illumination from an isotropically-scattering semi-infinite medium. The zero-variance-based Dwivedi sampling scheme generates much shorter paths on average whilst simultaneously* decreasing *variance (as opposed to Russian roulette). The method automatically adapts to the single-scattering albedo* α *of the medium.*