Planar test
point × polygon

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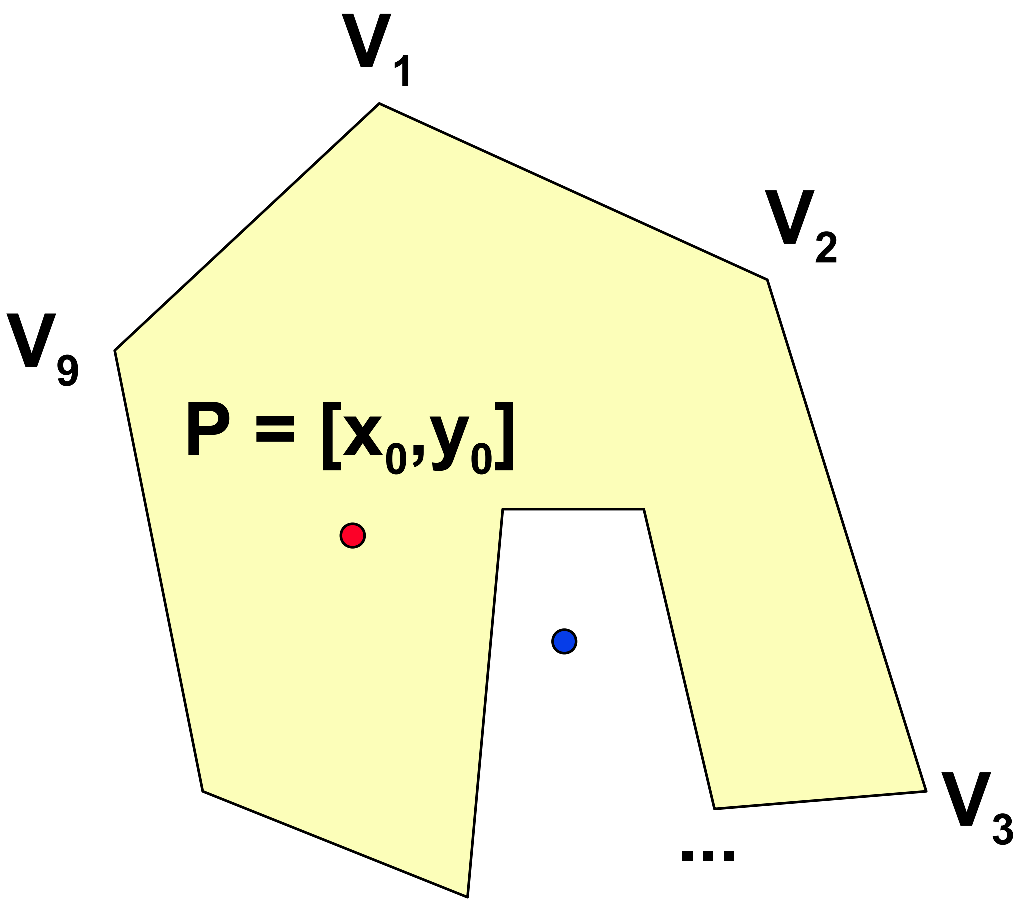
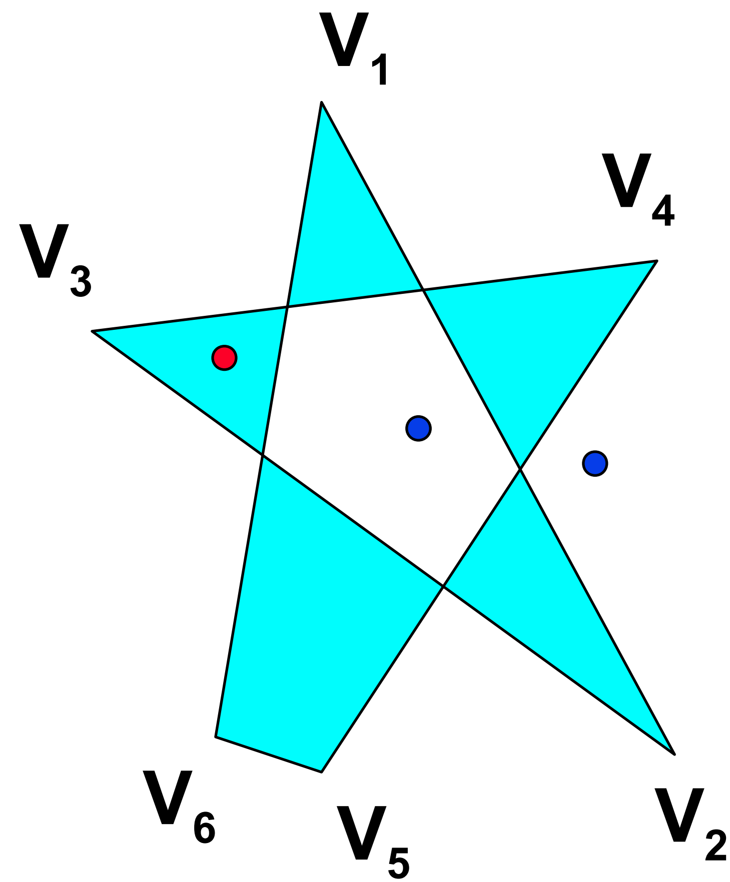
Polygon interior?





$V_i = [x_i, y_i]$

Different definitions

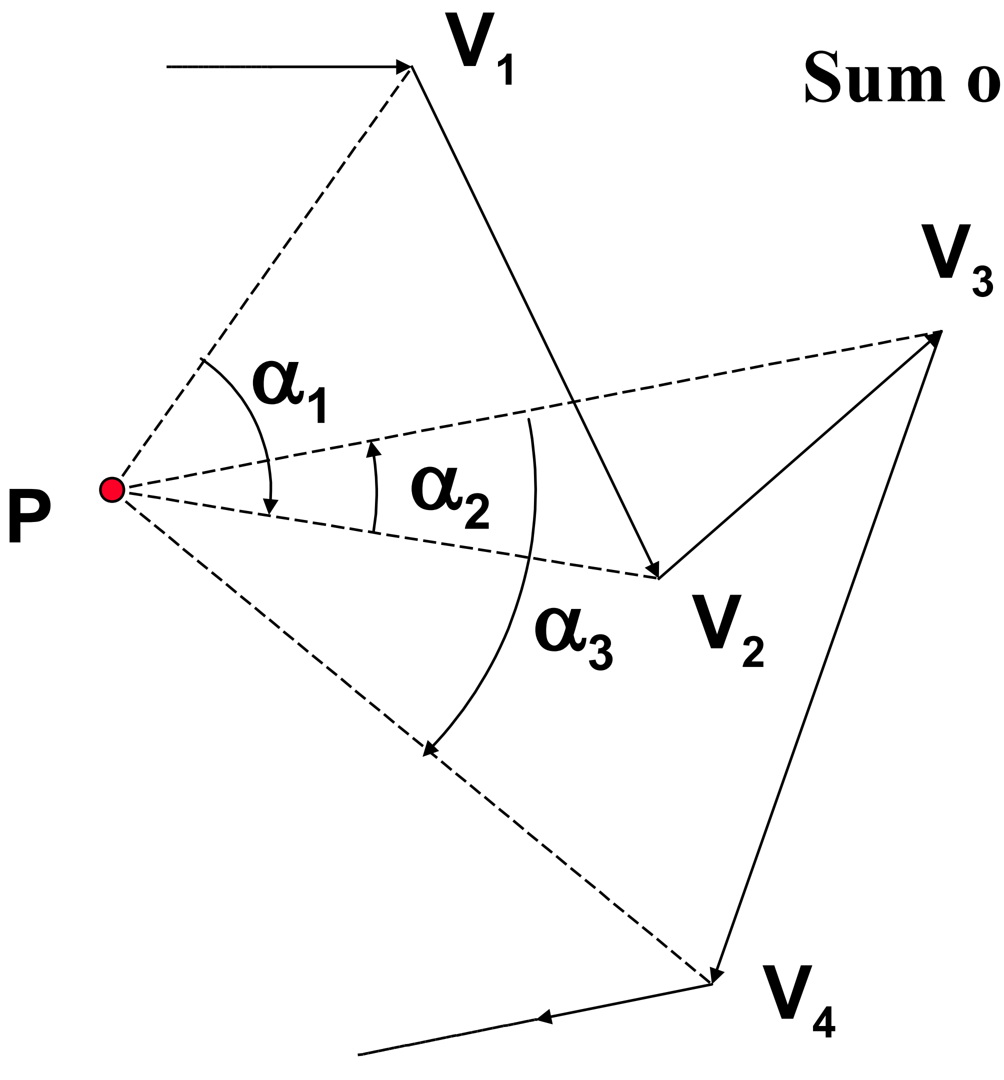
Point **P** lies inside the polygon **[V_1, .. V_M]**, if:

❶ it is separated from the outside (infinite component of the plane) by **odd number of borders** (“odd-even rule”, Jordan theorem)

❷ it is separated from the outside by **at least one border** (i.e. not element of the infinite component)

❸ its ”**winding number**” with respect to the polygon's outline **W** is **nonzero** (“thread loop + pin”)

Winding number computation



Sum of the oriented angles:

$$\Sigma_i \alpha_i = 2\pi \cdot W$$

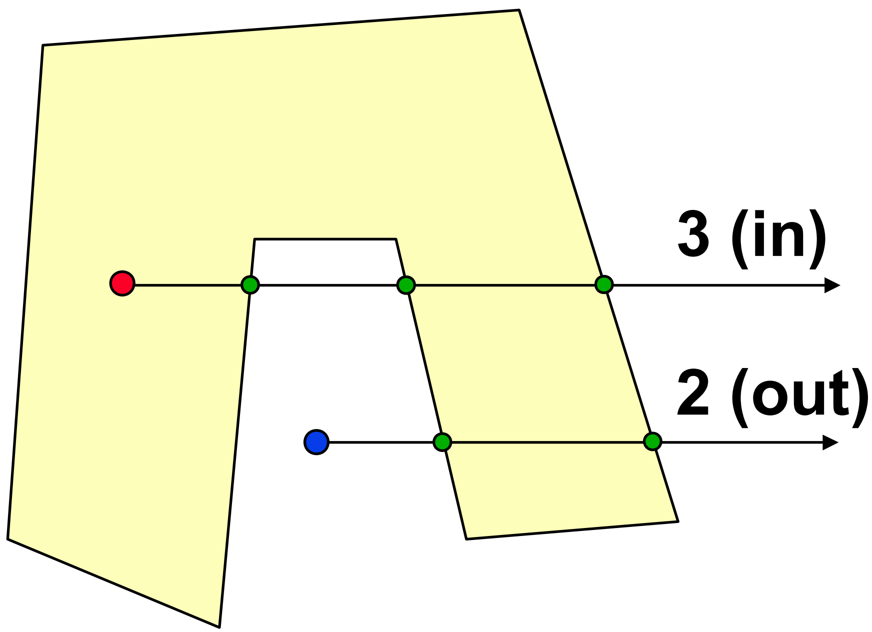
0°, ±360°, ±720°, ..

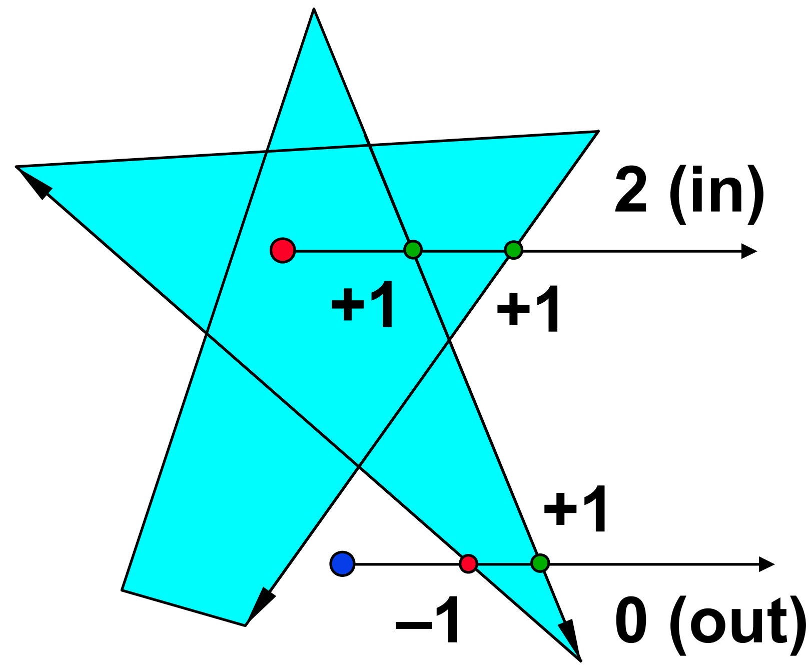
outside **inside**

For efficiency:

table of **arctg(y/x)**

Half line vs. polygon outline

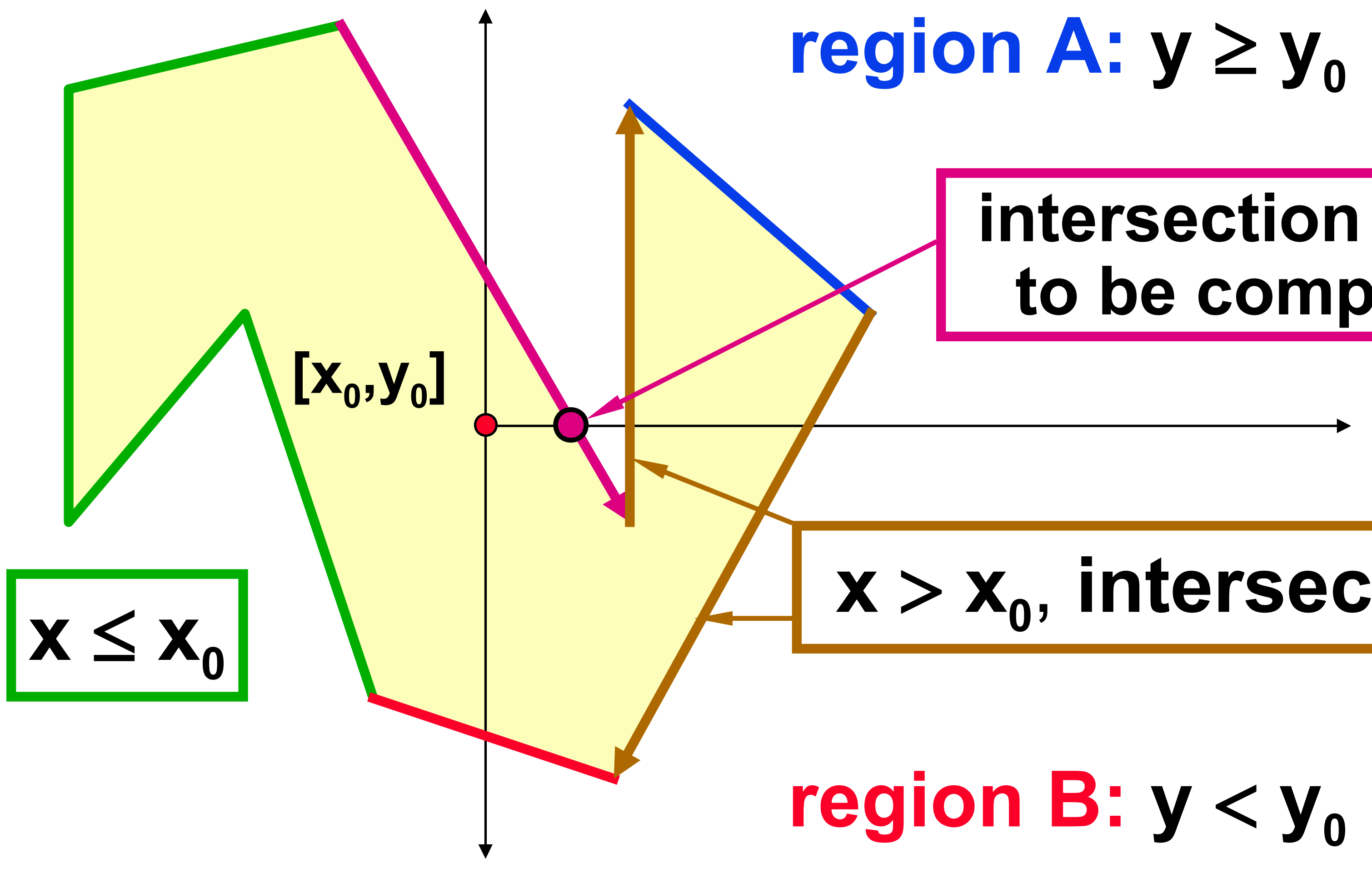




Definition ❶
(nonoriented edges)

Definition ❸
(oriented edges)

Implementation



Implementation

- sequential pass through edges V_1V_2, V_2V_3, .. V_MV_1
  - every vertex has the flags $x > x_0$, $y \geq y_0$
- **trivial negative edges:** both vertices have equal boolean value of either condition:
  - $x \leq x_0$, $y \geq y_0$ or $y < y_0$
- **trivial positive** edges (intersection exists):
  - for both vertices: $x > x_0$
  - for exactly one vertex: $y \geq y_0$

Implementation

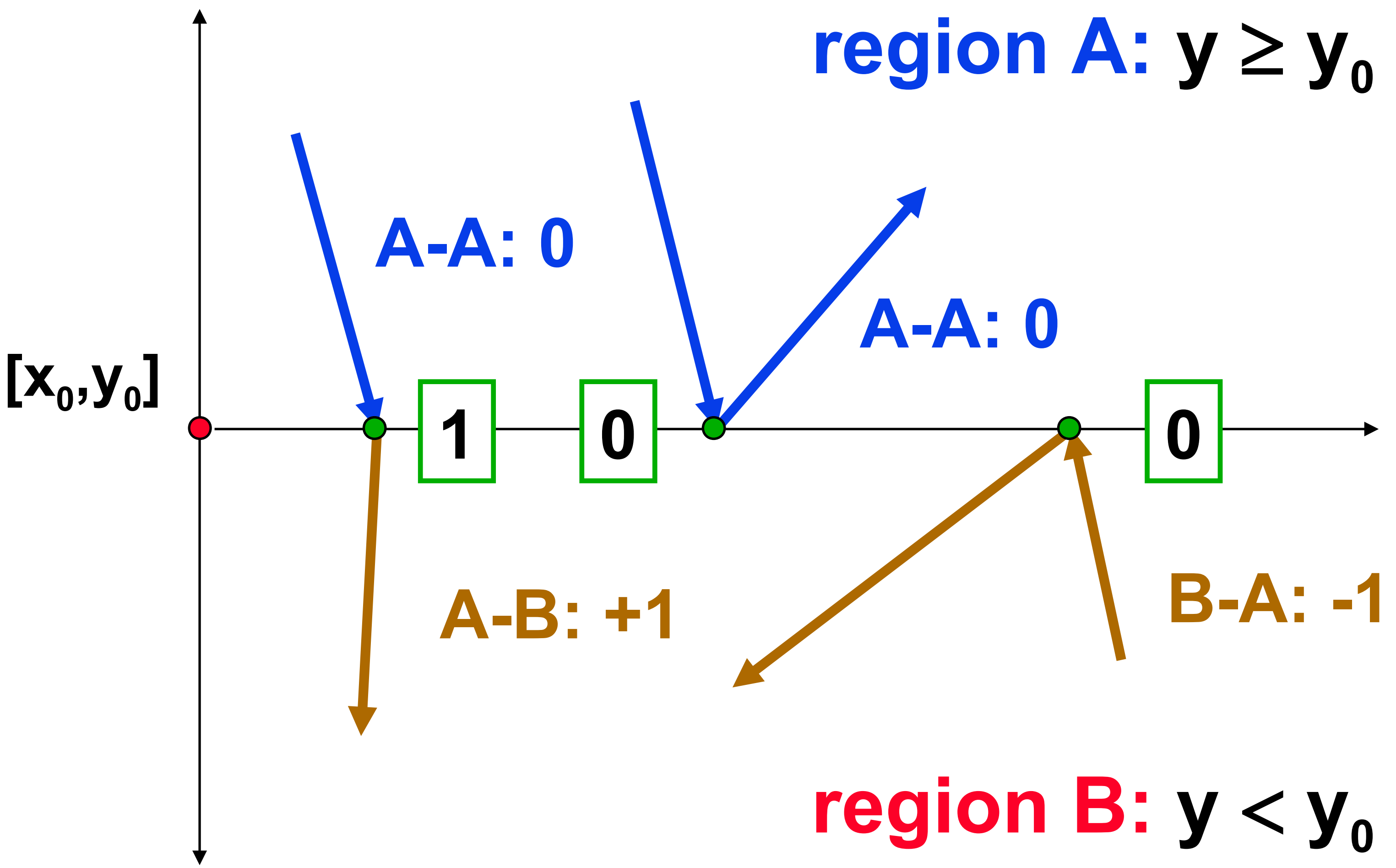
- the rest of the edges are **nontrivial**:
  - for exactly one vertex: $\mathbf{x > x_0}$
  - for exactly one vertex: $\mathbf{y \geq y_0}$
  - the **exact intersection** of an edge with the half line $\mathbf{y = y_0}$ has to be computed

- for any **positive edge** (intersection) the contribution is:

  +1 or **-1** according to edge orientation (definition ❸)

  +1 for an unoriented edge (definition ❶)

Special cases



References

- **A. Glassner**: ***An Introduction to Ray Tracing***, Academic Press, London 1989, 53-59
- **J. Foley, A. van Dam, S. Feiner, J. Hughes: *Computer Graphics, Principles and Practice*, 34**