

Radiometry and radiosity

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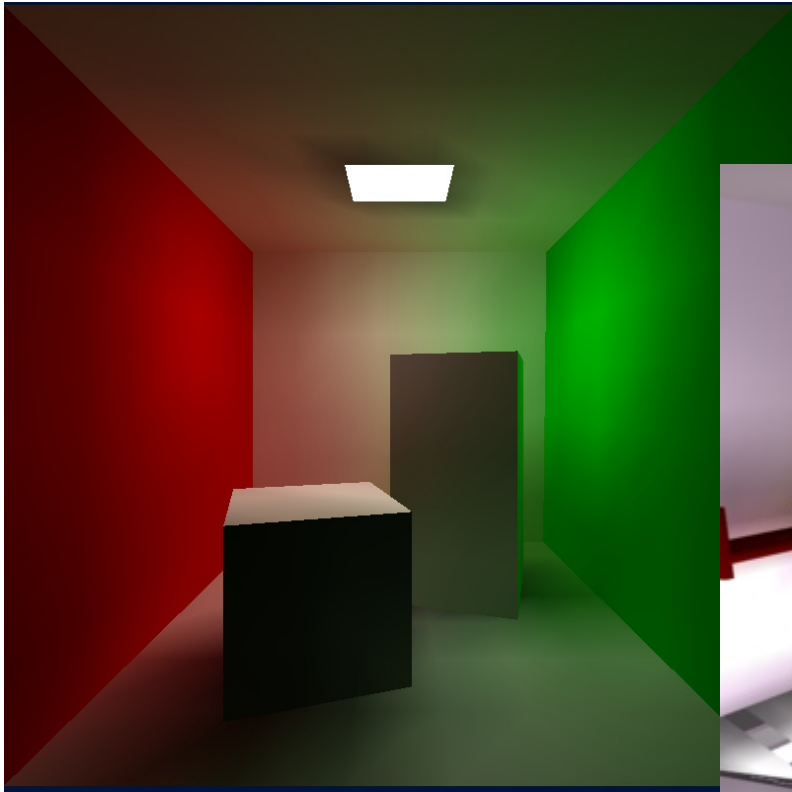


Global illumination, radiosity

- ◆ based on **physics**
 - energy transport (light transport) in simulated environment
 - first usage of radiosity in image synthesis: Cindy Goral (SIGGRAPH 1984)
- ➔ radiosity is able to compute **diffuse light**, secondary lighting, ..
- ➔ basic **radiosity** cannot do sharp reflections, mirrors, ..
- ◆ time consuming computation
 - Radiosity: light propagation only, RT: rendering



Radiosity – examples



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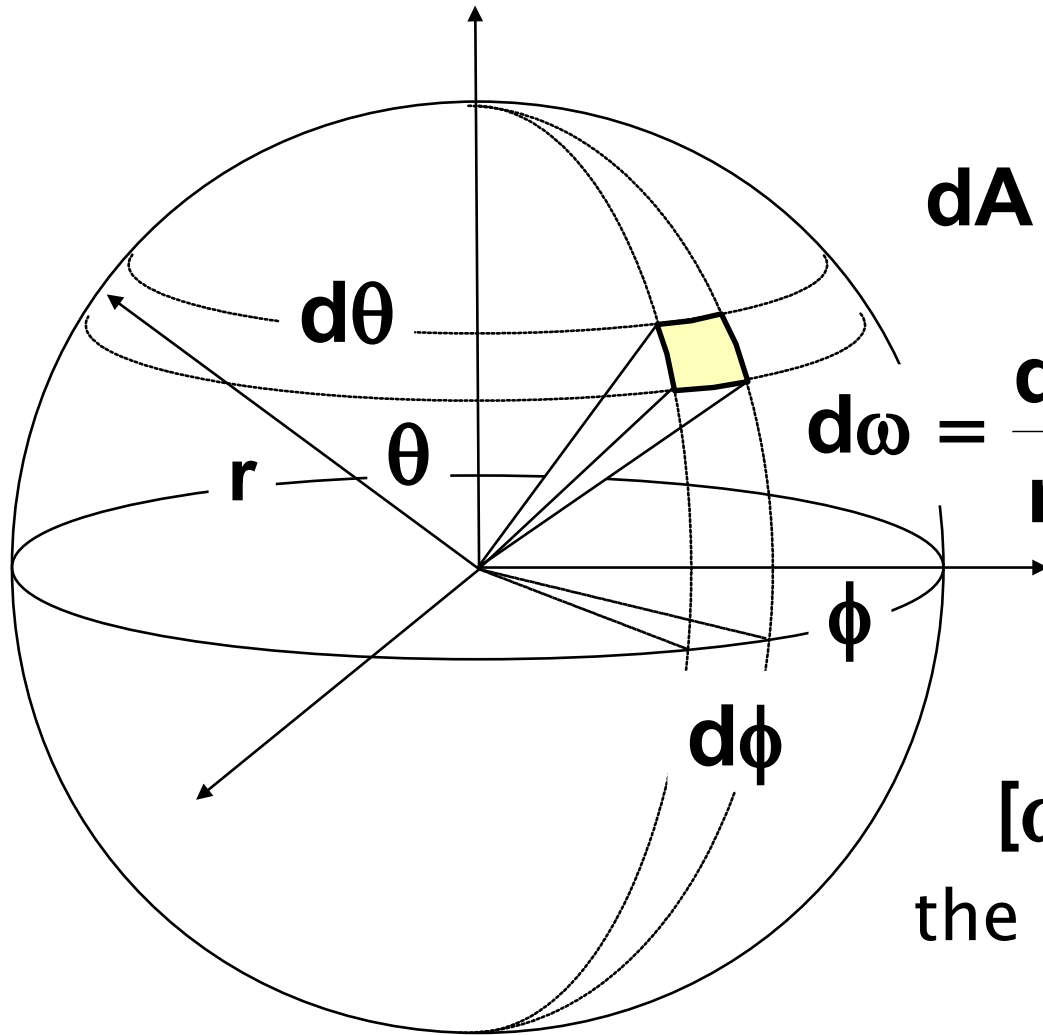


Basic radiometry

- **energy** received/emitted by some area:
 - Q_{in} (Q_{out}) [Joul]
- **power** (radiant flux) received/emitted by some area:
 - Φ_{in} (Φ_{out}) [Joul/sec = Watt]
- **radiosity / irradiance** (\leftarrow) / **radiant exitance** (\rightarrow)
(power/radiant flux density):
 - B_{in} (E , B_{out}) [W/m²]
- **intensity** (power density in a solid angle ω):
 - $I = d\Phi/d\omega$ [W/sr]



Solid angles



$$dA = r^2 \sin\theta \, d\theta \, d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$$

$[\omega]$.. steradian (sr)

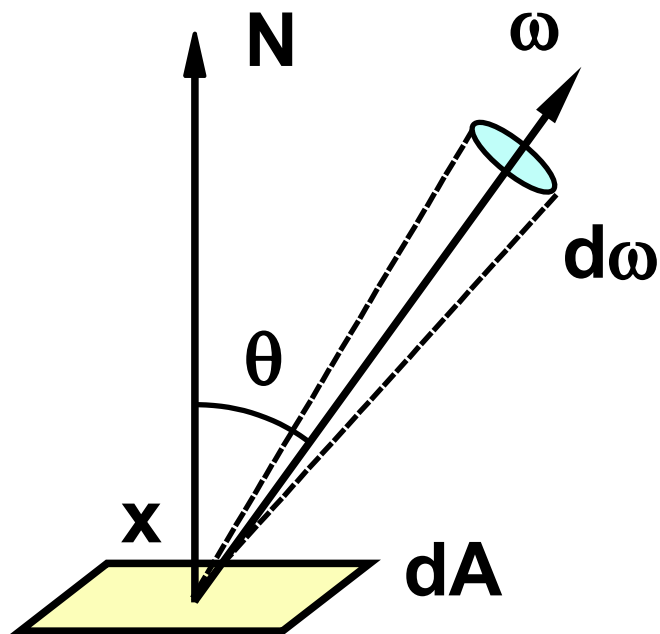
the whole sphere .. 4π sr



Radiance

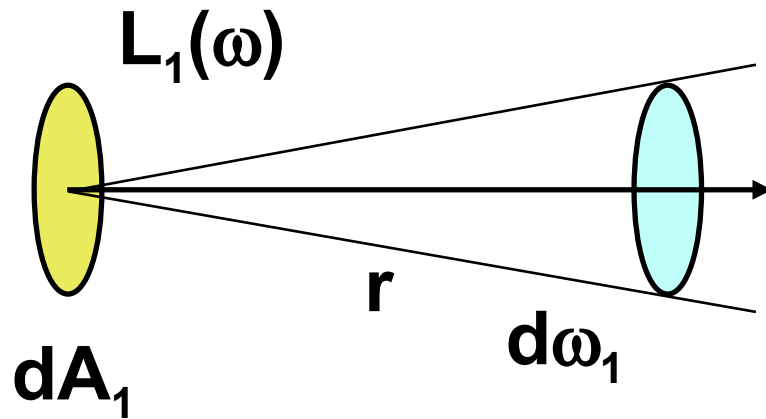
→ received/emitted **radiance** in direction ω :

– $L_{\text{in}}(\omega)$ ($L_e(\omega)$, $L_{\text{out}}(\omega)$) [W/(m² · sr)]



$$\begin{aligned} L_{\text{out}}(\mathbf{x}, \omega) &= \frac{d^2\Phi}{dA d\omega \cos\theta} \\ &= \frac{dB_{\text{out}}}{d\omega \cos\theta} \\ &= \frac{dl}{dA \cos\theta} \end{aligned}$$

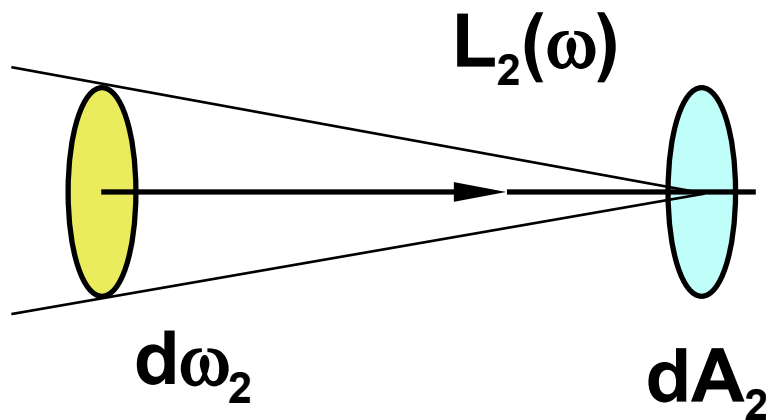
Energy preservation law (ray / fiber)



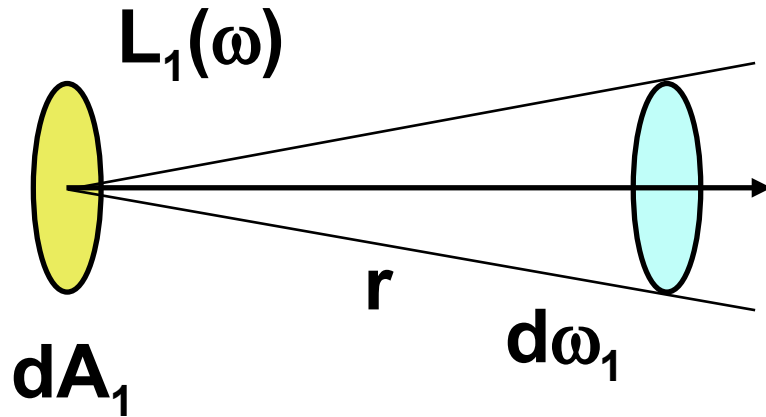
$$L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2$$

emitted
power

received
power



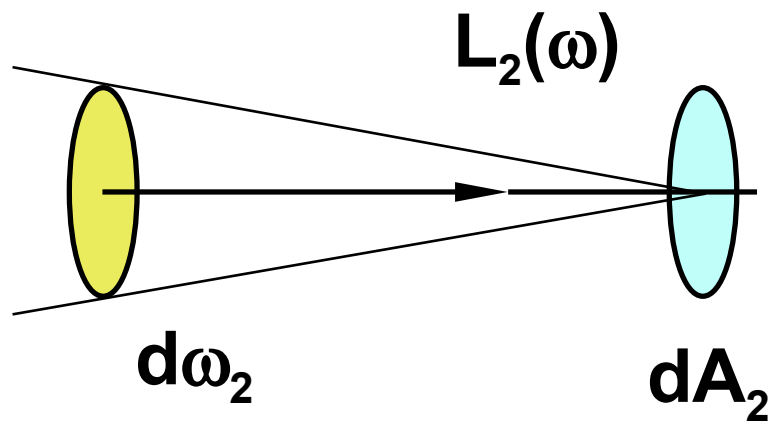
Energy preservation law (ray / fiber)



$$L_1 \, d\omega_1 \, dA_1 = L_2 \, d\omega_2 \, dA_2$$

$$\begin{aligned} \underline{T} &= d\omega_1 \, dA_1 = d\omega_2 \, dA_2 = \\ &= \frac{dA_1 \, dA_2}{r^2} \end{aligned}$$

ray capacity



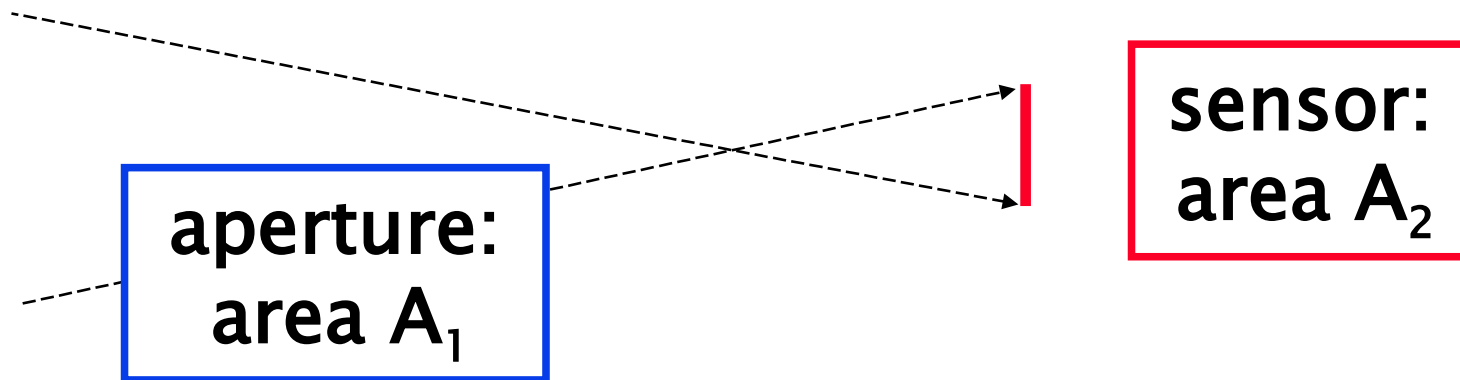
$$L_1 = L_2$$

ray ... radiance L



Light measurement

- **measured quantity** is proportional to **radiance** from visible scene

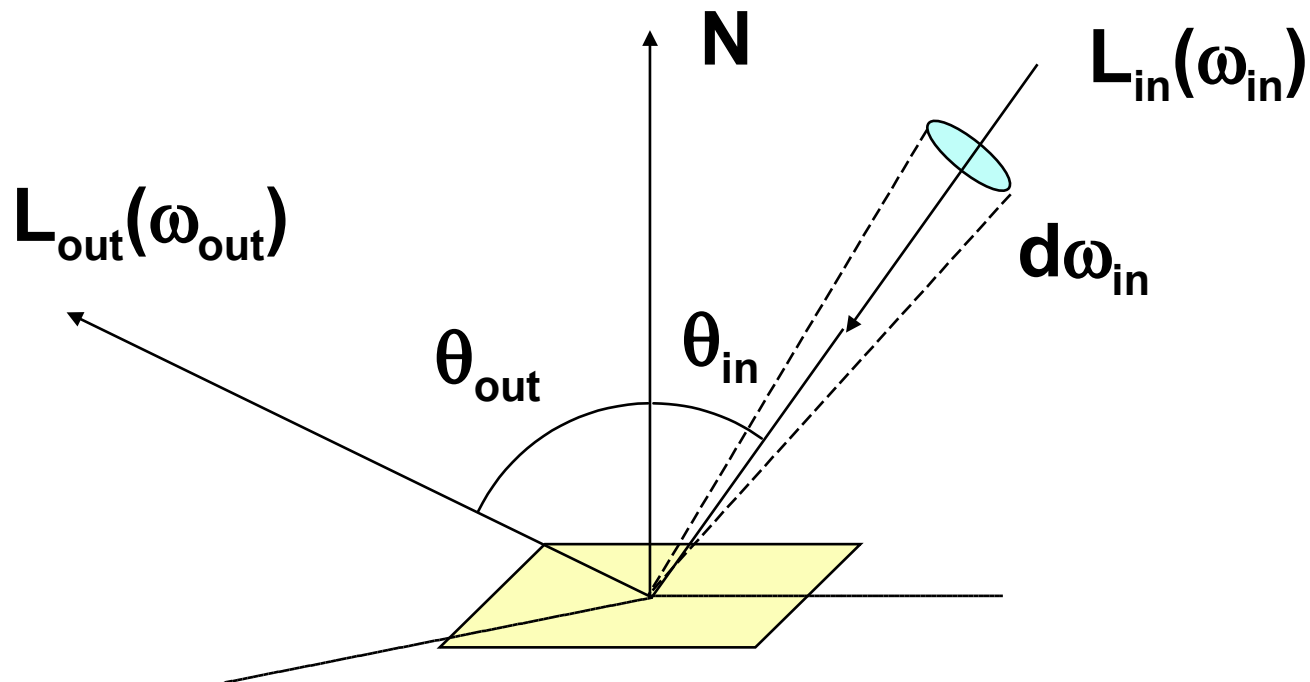


$$\underline{R} = \int_{A_2} \int_{\Omega} L_{\text{in}}(\mathbf{A}, \omega) \cdot \cos \theta \, d\omega \, dA = \underline{L_{\text{in}}} \cdot T$$



BRDF (reflectance function)

“Bidirectional Reflectance Distribution Function”



$$f(\omega_{in} \rightarrow \omega_{out}) = \frac{L_{out}(\omega_{out})}{L_{in}(\omega_{in}) \cdot \cos \theta_{in} \cdot d\omega_{in}} \quad [\text{sr}^{-1}]$$



Helmholtz law (reciprocity)

→ for **real** surfaces (physically plausible):

$$\mathbf{f}(\omega_{\text{in}} \rightarrow \omega_{\text{out}}) = \mathbf{f}(\omega_{\text{out}} \rightarrow \omega_{\text{in}})$$

→ general **BRDF** needs not be **isotropic** (invariant to rotation around surface normal)

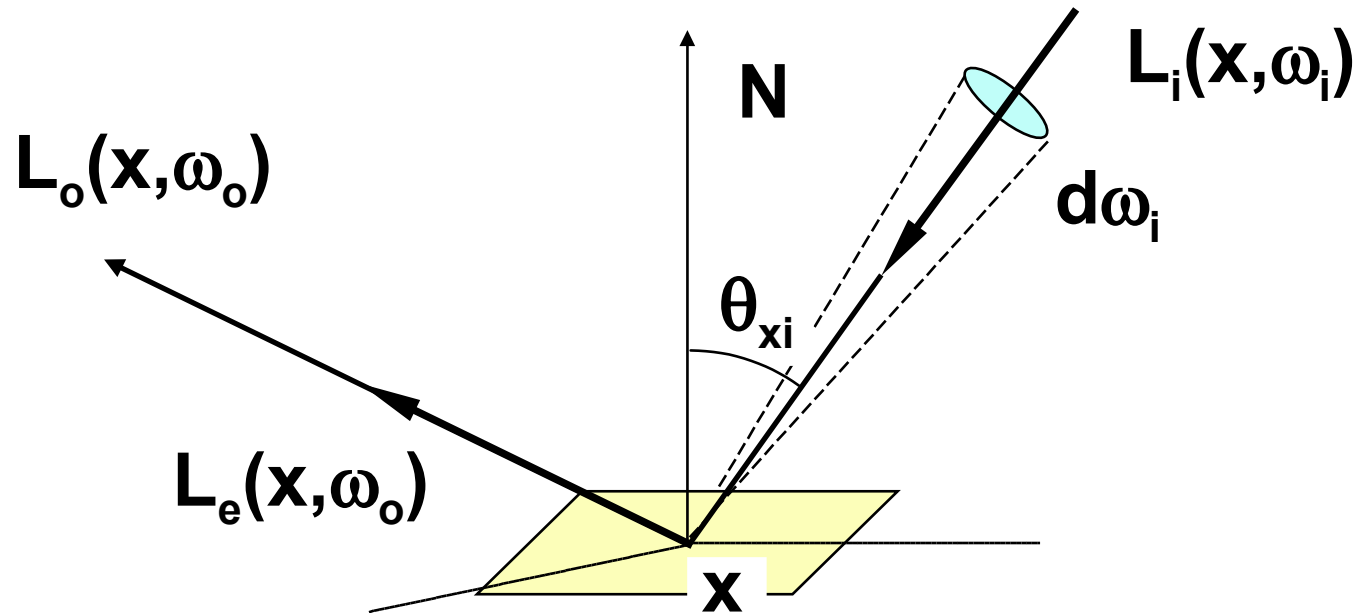
– metal surfaces polished in one direction, ..

$$\mathbf{f}(\theta_{\text{in}}, \phi_{\text{in}}, \theta_{\text{out}}, \phi_{\text{out}}) \neq \mathbf{f}(\theta_{\text{in}}, \phi_{\text{in}} + \phi, \theta_{\text{out}}, \phi_{\text{out}} + \phi)$$



Local equation (OVTIGRE)

“Outgoing, Vacuum, Time-Invariant, Gray Radiance Equation”

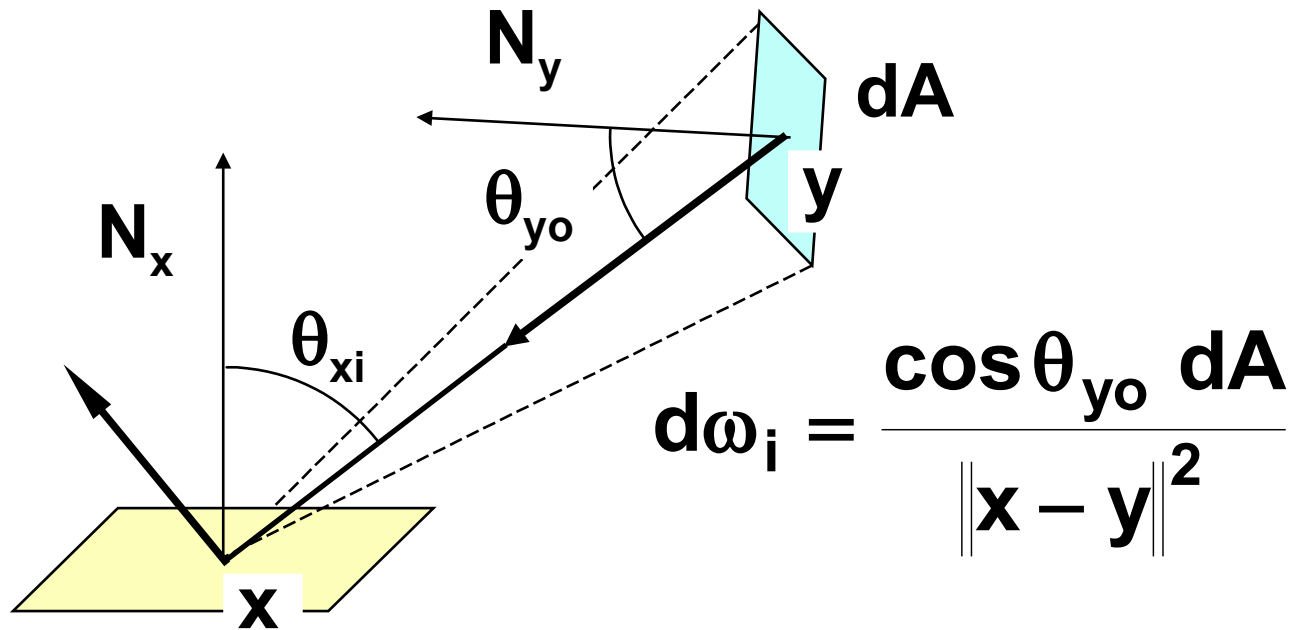


own radiant exitance

$$\mathbf{L}_o(\mathbf{x}, \omega_o) = \mathbf{L}_e(\mathbf{x}, \omega_o) + \int \mathbf{f}(\mathbf{x}, \omega_i \rightarrow \omega_o) \cdot \mathbf{L}_i(\mathbf{x}, \omega_i) \cdot \cos \theta_{xi} d\omega_i$$



Radiance received from a surface



Geometric term:
$$G(y, x) = \frac{\cos \theta_{y_o} \cos \theta_{x_i}}{\|x - y\|^2}$$



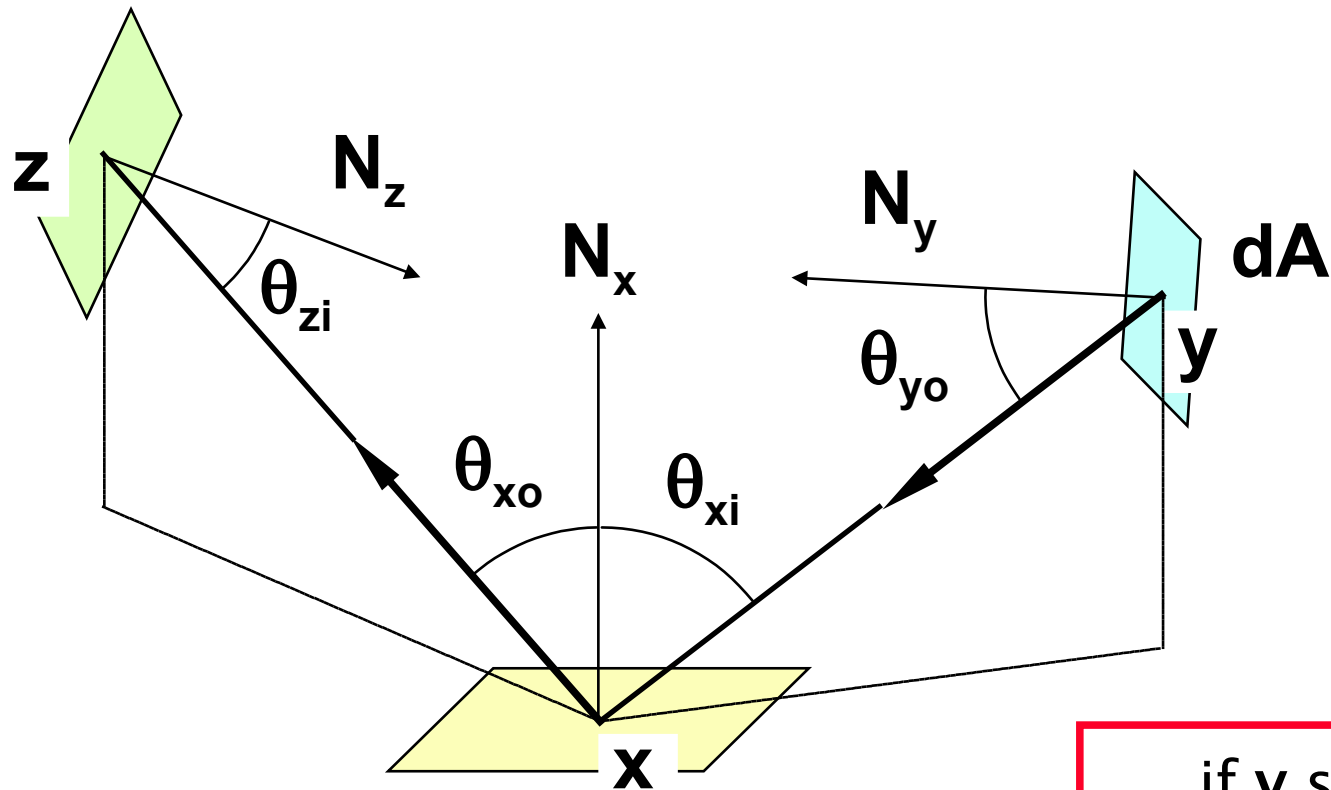
Radiance received from a surface

$$\begin{aligned} L_o(\mathbf{x}, \omega_o) &= \text{integral over all angles} \\ &= L_e(\mathbf{x}, \omega_o) + \int_{\Omega} \mathbf{f}(\mathbf{x}, \omega_i \rightarrow \omega_o) \cdot L_i(\mathbf{x}, \omega_i) \cdot \cos \theta_{xi} \, d\omega_i = \\ &= L_e(\mathbf{x}, \omega_o) + \int_S \mathbf{f}(\mathbf{x}, \omega_i \rightarrow \omega_o) \cdot L_o(\mathbf{y}, -\omega_i) \cdot \mathbf{G}(\mathbf{y}, \mathbf{x}) \, dA \\ & \quad \text{integral over an emitting surface} \end{aligned}$$

(assumption: the whole surface S is visible from \mathbf{x})



Reflected light



Terminology: $\underline{L}(\mathbf{y}, \mathbf{x}) = L_o(\mathbf{y}, \mathbf{x} - \mathbf{y}) = L_i(\mathbf{x}, \mathbf{y} - \mathbf{x})$ if y sees x

$\underline{f}(\mathbf{y}, \mathbf{x}, \mathbf{z}) = f(\mathbf{x}, (\mathbf{y} - \mathbf{x}) \rightarrow (\mathbf{z} - \mathbf{x}))$



Indirect radiance equation

$$V(\mathbf{y}, \mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{y} \text{ sees } \mathbf{x} \\ 0 & \text{else} \end{cases}$$

$$\underline{L(\mathbf{x}, \mathbf{z})} = \underline{L_e(\mathbf{x}, \mathbf{z})} + \int_S \underline{f(\mathbf{y}, \mathbf{x}, \mathbf{z})} \cdot L(\mathbf{y}, \mathbf{x}) \cdot \underline{G(\mathbf{y}, \mathbf{x})} \cdot V(\mathbf{y}, \mathbf{x}) \, dA$$

own (emitted)
radiant exitance

BRDF

geometric
terms



Radiosity equation

- assumption – **ideal diffuse (Lambertian)** surface:
 - **BRDF** is not dependent on incoming/outgoing angles
 - outgoing radiance $\mathbf{L}(\mathbf{y}, \omega)$ independent on direction ω

$$\mathbf{L}(\mathbf{x}, \mathbf{z}) = \mathbf{L}_e(\mathbf{x}, \mathbf{z}) + \mathbf{f}(\mathbf{x}) \cdot \int_S \mathbf{L}(\mathbf{y}, \mathbf{x}) \cdot \mathbf{G}(\mathbf{y}, \mathbf{x}) \cdot \mathbf{V}(\mathbf{y}, \mathbf{x}) \, dA$$

$$\mathbf{L}(\mathbf{x}, \mathbf{z}) = \mathbf{B}(\mathbf{x}) / \pi, \quad \mathbf{L}_e(\mathbf{x}, \mathbf{z}) = \mathbf{E}(\mathbf{x}) / \pi, \quad \mathbf{f}(\mathbf{x}) = \rho(\mathbf{x}) / \pi$$

$$\mathbf{B}(\mathbf{x}) = \mathbf{E}(\mathbf{x}) + \rho(\mathbf{x}) \cdot \int_S \mathbf{B}(\mathbf{y}) \cdot \frac{\mathbf{G}(\mathbf{y}, \mathbf{x}) \cdot \mathbf{V}(\mathbf{y}, \mathbf{x})}{\pi} \, dA$$



Discrete solution

$$\mathbf{B}(\mathbf{x}) = \mathbf{E}(\mathbf{x}) + \rho(\mathbf{x}) \cdot \int_{\mathcal{S}} \mathbf{B}(\mathbf{y}) \cdot \mathbf{g}(\mathbf{y}, \mathbf{x}) \, dA$$

where $\mathbf{g}(\mathbf{y}, \mathbf{x}) = \frac{\mathbf{G}(\mathbf{y}, \mathbf{x}) \cdot \mathbf{V}(\mathbf{y}, \mathbf{x})}{\pi}$

- ♦ solution \mathbf{B} is infinit-dimensional
- ➔ discretization of the task:
 - **Monte-Carlo** ray-tracing (dependent on camera)
 - classical **radiosity** (finite/boundary elements FEM)



General radiosity method

- ① object surfaces divided into set of **elements**
- ② definition of **knot points** on elements
 - **radiosity** will be computed there
- ③ choice of an **approximation method** and error metric
 - basis functions for convex blend from knot points
- ④ **coefficients** of linear equation system
 - “form-factors”



General radiosity method

- 5 solution of **linear equation system**
 - result: radiosity in knot points

- 6 reconstruction of values on **whole surfaces**
 - linear blends using basis functions and knot point radiosities

- 7 **rendering** of results (arbitrary view)
 - light is proportional to radiosity

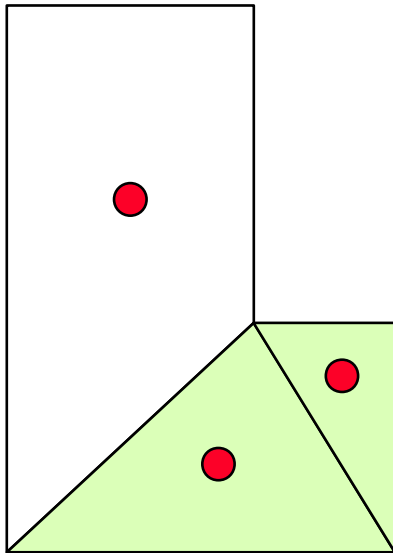


Remarks

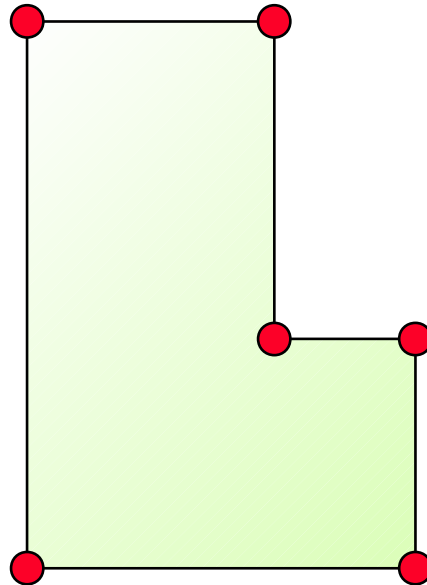
- ♦ step ③ is performed in **algorithm design** phase
 - does not appear in an implementation
- ♦ some **advanced methods** do not strictly follow the sequence ① to ⑦
 - sometimes a computation flow goes back to some previous phase, some phases can be iterated,..



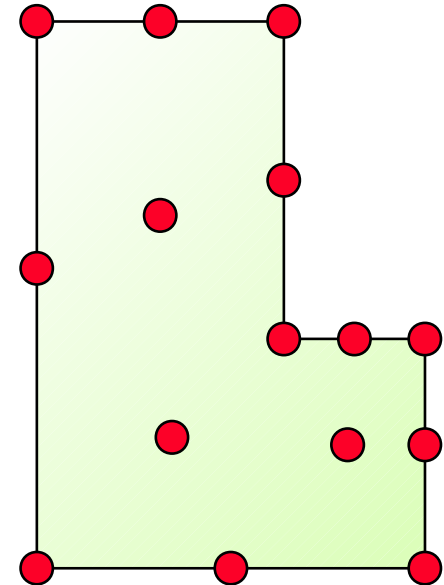
Radiosity approximation



constant
(knots in
centers)



bilinear
(knots in
vertices)



quadratic
(more knots
in centers..)



Constant elements

- on every **element A_i** constant reflectivity is assumed ρ , radiosity **B** – average of **$B(\mathbf{x})$** :
 - terminology: ρ_i, B_i for $i = 1 \dots N$

$$B(\mathbf{x}) = E(\mathbf{x}) + \rho(\mathbf{x}) \cdot \int_S B(\mathbf{y}) \cdot g(\mathbf{y}, \mathbf{x}) dA$$

average over
area A_i

↓

$$B_i = E_i + \rho_i \cdot \frac{1}{A_i} \int_{A_i} \left[\sum_{j=1}^N B_j \int_{A_j} g(\mathbf{y}, \mathbf{x}) dA_j \right] dA_i$$

radiosity received in point \mathbf{x} (lying on A_i)



Basic radiosity equation

switching sum and integral:

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^N B_j \cdot \frac{1}{A_i} \int_{A_i} \int_{A_j} g(y, x) dA_j dA_i$$

geometric term – form factor F_{ij}
(part of energy irradiated from A_i received directly by A_j)

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^N B_j F_{ij} \quad \left[\frac{W}{m^2} \right]$$



Intuitive derivation

$$B_i A_i = E_i A_i + \rho_i \cdot \sum_{j=1}^N B_j A_j F_{ji} \quad [w]$$

emitted power = own power + reflected power

reciprocal rule:

$$A_j F_{ji} = A_i F_{ij}$$

$$B_i A_i = E_i A_i + \rho_i \cdot \sum_{j=1}^N B_j F_{ij} A_i \quad \Big| \cdot A_i^{-1}$$

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^N B_j F_{ij} \quad \left[\frac{w}{m^2} \right]$$



System of linear equations

$$\underline{B_i} - \rho_i \cdot \sum_{j=1}^N \underline{B_j} F_{ij} = E_i \quad i = 1..N$$

$$\begin{bmatrix} 1 - \rho_1 F_{1,1} & -\rho_1 F_{1,2} & \dots & -\rho_1 F_{1,N} \\ -\rho_2 F_{2,1} & 1 - \rho_2 F_{2,2} & \dots & -\rho_2 F_{2,N} \\ \dots & \dots & \dots & \dots \\ -\rho_N F_{N,1} & -\rho_N F_{N,2} & \dots & 1 - \rho_N F_{N,N} \end{bmatrix} \begin{bmatrix} \underline{B_1} \\ \underline{B_2} \\ \dots \\ \underline{B_N} \end{bmatrix} = \begin{bmatrix} \underline{E_1} \\ \underline{E_2} \\ \dots \\ \underline{E_N} \end{bmatrix}$$

vector of unknown vars $[B_i]$



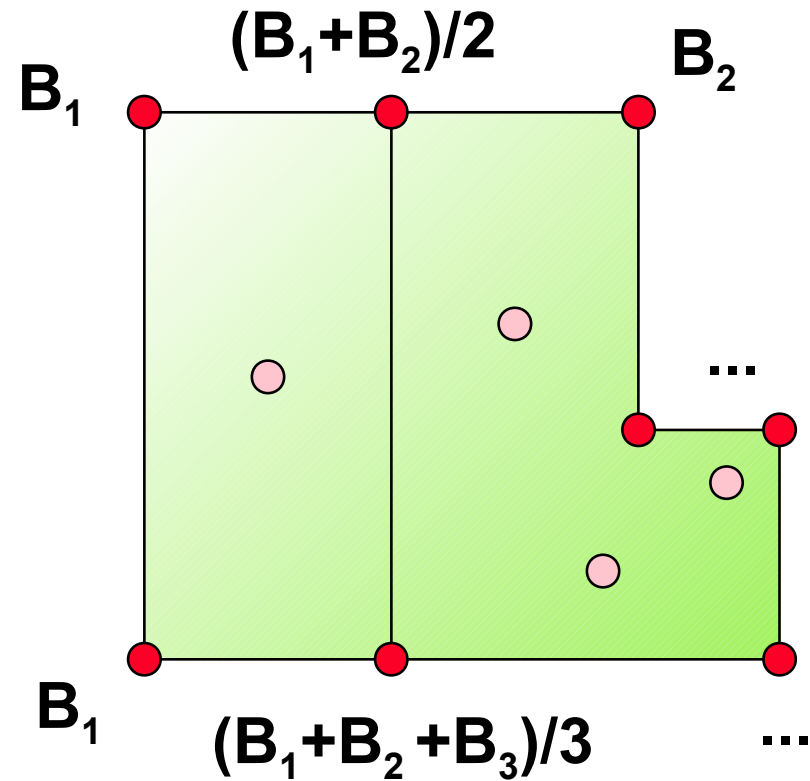
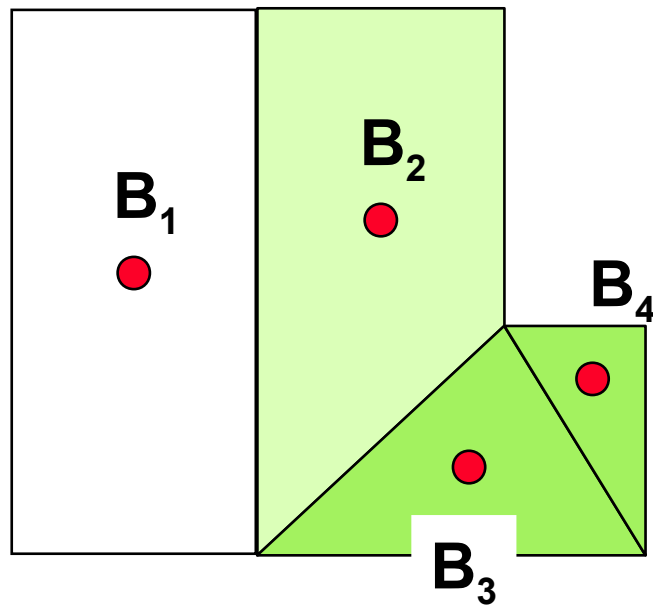
System of linear equations

- for **planar (convex) surfaces**: $F_{ii} = 0$
 - the diagonal contain only unit values
- **nondiagonal items** are usually very small (abs value)
 - matrix is “diagonally dominant”
 - ⇒ system is stable and can be solved by **iterative methods** (Jacobi, Gauss-Seidel)
- for **light change (light sources)** $[E_i]$ system needs not to be fully re-computed, only reverse phase could be done

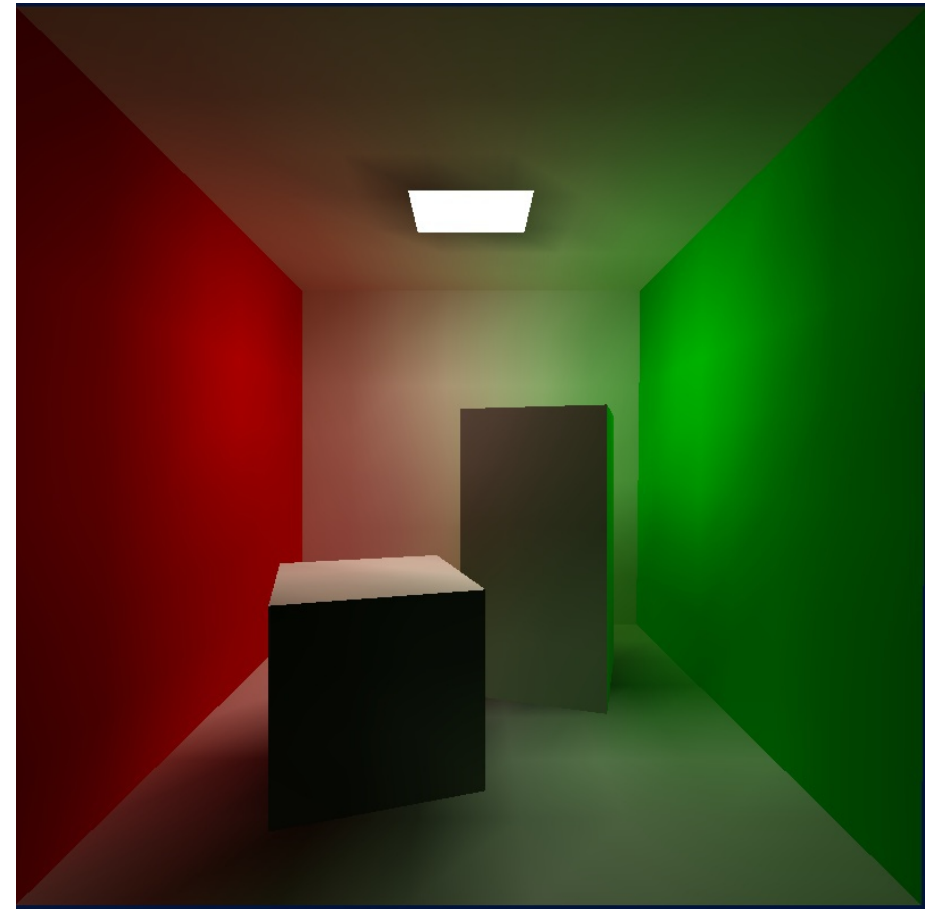
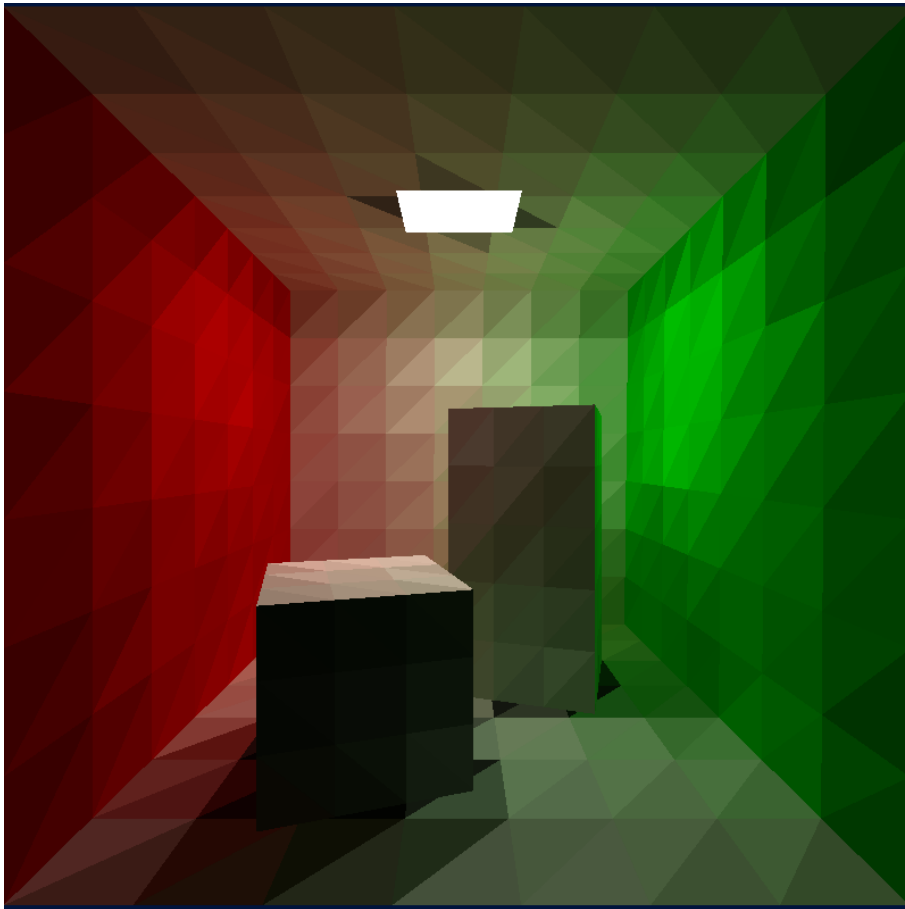


Radiosity to vertices

Even in constant element approach usage of some color interpolation method is recommended (**Gouraud**)



Linear color interpolation





References

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- **J. Foley, A. van Dam, S. Feiner, J. Hughes:** *Computer Graphics, Principles and Practice*, 793-804