Fast Approximation to Spherical Harmonics Rotation

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Abstract

We present a fast and simple approximation of spherical harmonic rotation which decreases the asymptotic complexity and achieves a speedup of four to six in practical applications. The rotation procedure's simplicity allows implementation in a pixel shader of standard graphics hardware. The main idea is to replace the general spherical harmonic rotation matrix by its truncated Taylor expansion, resulting in a sparse matrix. We show applications in global illumination (radiance interpolation) and GPU-based real-time shading (normal mapping on surfaces with arbitrary BRDFs illuminated by low-frequency environment lighting). Although the rotation approximation is accurate only for small rotation angles, we show this is not a serious limitation in our applications.

1 Introduction

When using spherical harmonics (SH) in rendering, one often faces the problem of aligning the illumination (represented by SH in the global coordinate frame) with the BRDF (represented by SH in the local frame at each surface point). The alignment is carried out through a SH rotation. However, even the fastest SH rotation procedure, the *zxzxz*-rotation [Kautz et al. 2002], is a bottleneck. Zonal harmonics [Sloan et al. 2005] allow fast rotation, but the slow nonlinear optimization needed to fit a given function limits their usefulness. We present a simple and fast SH rotation approximation which is four to six times faster than the *zxzxz*-rotation.

2 Our Method

Problem Statement. Given a vector of SH coefficients $\Lambda = \{\lambda_l^m\}$ representing a spherical function $L(\omega)$, find a vector of coefficients $\Upsilon = \{\upsilon_l^m\}$ representing the rotated function $L(\mathscr{R}^{-1}(\omega))$, where $\mathscr{R}$ is the desired rotation.

The rotation can be expressed as a linear transformation $\Upsilon = \mathbf{R}\Lambda$ with a block-sparse SH rotation matrix $\mathbf{R}$. The problem is how to construct $\mathbf{R}$ for a desired 3D rotation. Our approach is as follows. We decompose the desired rotation into the *zyz* Euler angles α, β and γ, *i.e.* $\mathbf{R} = \mathbf{R}_z(\alpha)\mathbf{R}_y(\beta)\mathbf{R}_z(\gamma)$. The two rotations around z are simple and efficient, but the rotation around y is not. We approximate $\mathbf{R}_y(\beta)$ with its second order Taylor expansion:

$$\mathbf{R}_y(\beta) \approx \mathbf{I} + \beta \frac{\mathrm{d}\mathbf{R}_y}{\mathrm{d}\beta}(0) + \frac{\beta^2}{2} \frac{\mathrm{d}^2\mathbf{R}_y}{\mathrm{d}\beta^2}(0),$$

where $\mathbf{I}$ is the identity matrix. Since the two derivative matrices, $\frac{\mathrm{d}\mathbf{R}_y}{\mathrm{d}\beta}(0)$ and $\frac{\mathrm{d}^2\mathbf{R}_y}{\mathrm{d}\beta^2}(0)$, have non-zero elements only on and around the main diagonal, evaluating this approximation is extremely efficient. The number of non-zero elements in those matrices is $O(n^2)$ (n is the SH order). Since rotation around z is also $O(n^2)$, the whole rotation procedure is $O(n^2)$ instead of the $O(n^3)$ complexity of previous rotation procedures. The rotation approximation is accurate only for small rotation angles around y. Yet, this is not a serious restriction in our applications.

3 Applications and Results

The first application is in radiance caching [Křivánek et al. 2005], a global illumination algorithm for glossy surfaces. Radiance caching computes indirect illumination at a sparse set of points over glossy surfaces, stores it in radiance cache, and interpolates elsewhere. The cached illumination at a point is represented by SH in the local coordinate frame. When interpolating, the cached illumination has to be rotated to the local coordinate frame of the location of interpolation—several rotations might be necessary for each pixel. Since the cached illumination is used for interpolation only in areas of small change of the surface normal, the angle of rotation is small and our rotation can be safely used without causing artifacts (see Figure 1).

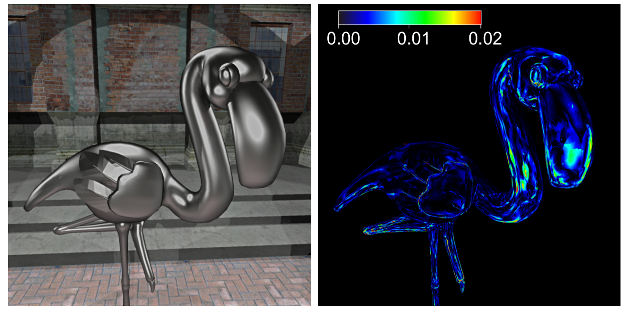

Figure 1: Left: Radiance caching rendering of a glossy flamingo (Phong BRDF, exp 15) obtained with our approximate rotation. SH order $n = 10$ is used. Right: Color coded difference between images with approximate and correct rotation, measured on a $[0,1]$ RGB scale (below the visual threshold of 1% for most pixels).

Figure 2: Detail of a normal mapped vase rendered with our SH rotation (right) and with the simplified normal mapping (left). Normal mapping with our SH rotation is more successful at conveying the shape approximated by the normal map.

The second application is in GPU-based shading on surfaces with arbitrary BRDFs under low-frequency lighting. We use our rotation to extend the shading technique of Kautz *et al.* [2002] with normal mapping, allowing fine surface detail with low polygon count. Both the environment lighting and the BRDFs are represented by SH. Our rotation is used to align the lighting in the local frame of triangle vertices with the local frame at each pixel, modulated by the normal map. Since the normal modulation is small, our rotation can be safely used. Figure 2 shows the quality improvements of using the rotation as compared to a simplified normal mapping which ignores the rotation. This application is permitted thanks to the simplicity of the rotation procedure: it fits in the pixel shader of the commodity graphics hardware (ATI Radeon 9800 Pro).

We see a potential in using the idea presented here for the rotation of functions represented by wavelets. Our rotation can also be used for normal mapping in pre-computed radiance transfer on surfaces with view-dependent BRDFs.

References

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