Bi-Directional Polarised Light Transport

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Motivation

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State of Polarisation Support

- Goal: Support polarised light in **bi-directional** light transport algorithms

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- Previous implementations:
  - only for **uni-directional** path-tracer
  - details already gathered [Wilkie et al 2012]

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- Can we use it for bi-directional light transport?

Contribution

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- Radiometry of polarised light transport

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- Reference implementation of bi-directional algorithms

Proposed theory

Why New Theory?

- Goal: Generalize

Rendering eq.: $$L_{\mathsf{o}} = L_{\mathsf{e}} + \int_{S_2} f(\omega_{\mathsf{i}}, \omega_{\mathsf{o}}) L_{\mathsf{i}}(\omega_{\mathsf{i}})\, d\omega_{\mathsf{i}}^{\perp}$$

Measurement eq.: $$I = \int_{S_2} W_{\mathsf{e}}(\omega) L_{\mathsf{i}}(\omega)\, d\omega^{\perp}$$

Why New Theory?

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Measurement eq.: $$I = \int_{S_2} W_\mathsf{e}(\omega) L_\mathsf{i}(\omega) \, d\omega^\perp$$

- What is radiance L, BSDF f and importance W in the context of polarised light?
- Radiance:

classical: $$L : \underbrace{\mathcal{M} \times S^2}_{\text{ray space}} \to \mathbb{R}$$

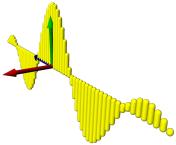
polarised: $$\boldsymbol{L} : \mathcal{M} \times S^2 \to ?$$

Brief Overview of Polarised Light

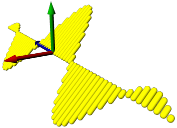
Stokes Vector

- We describe polarised light with Stokes vector $\boldsymbol{S}$

$$\boldsymbol{S} = (S_0, S_1, S_2, S_3)$$



$\begin{pmatrix}1 & 1 & 0 & 0\end{pmatrix}$



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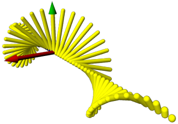


$\begin{pmatrix}1 & 0 & 0 & 1\end{pmatrix}$

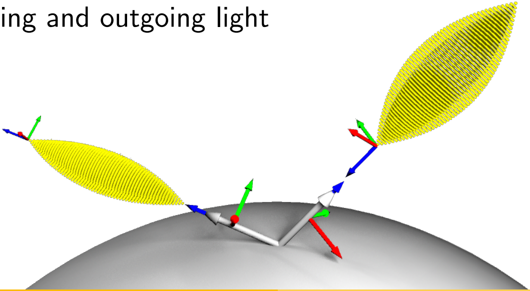
- The Stokes vector depend on the **choice of the coordinate system**

Mueller Matrix

- Muller matrix M describes what happens to the light when it bounces of the surface

$$\begin{pmatrix} S'_0 \\ S'_1 \\ S'_2 \\ S'_3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

- Its components **depend on the coordinate frame** of the incoming and outgoing light



Proposed Theory

Radiance

- Radiance in **polarised** light transport

$$\boldsymbol{L} : \mathcal{M} \times S^2 \to ?$$

Radiance

- First guess:

$$\boldsymbol{L} : \mathcal{M} \times S^2 \to \mathbb{R}^4$$

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Bad choice: missing coordinate frame

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- **Solution**:

$$\boldsymbol{L} : (\boldsymbol{x}, \omega) \to [\boldsymbol{S}, F]$$

Radiance

- First guess:

$$\boldsymbol{L} : \mathcal{M} \times S^2 \to \not{\mathbb{R}^4}$$

Bad choice: missing coordinate frame

- **Solution**:

$$\boldsymbol{L} : (\boldsymbol{x}, \omega) \to [\boldsymbol{S}, F]$$

- **Stokes space** $\mathbb{S}_\omega$: space of all pairs, Stokes vector $\boldsymbol{S}$ and its coordinate frame F

$$\boldsymbol{L} : \mathcal{M} \times S^2 \to \mathbb{S}_\omega$$

BSDF

- BSDF in **polarised** light transport

$$\boldsymbol{f} : \mathcal{M} \times S^2 \times S^2 \to ?$$

BSDF

- First guess:

$$\boldsymbol{f} : \mathcal{M} \times S^2 \times S^2 \to \mathbb{R}^{4\times 4}$$

BSDF

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- **Solution**:

$$\boldsymbol{f} : (\boldsymbol{x}, \omega_{\mathsf{i}}, \omega_{\mathsf{o}}) \to [M, F_{\mathsf{i}}, F_{\mathsf{o}}]$$

BSDF

- First guess:

$$\boldsymbol{f} : \mathcal{M} \times S^2 \times S^2 \rightarrow \cancel{\mathbb{R}^{4\times 4}}$$

Bad choice: missing coordinate frames

- **Solution**:

$$\boldsymbol{f} : (\boldsymbol{x}, \omega_\mathsf{i}, \omega_\mathsf{o}) \rightarrow [M, F_\mathsf{i}, F_\mathsf{o}]$$

- **Mueller space** $\mathbb{M}_{\omega_\mathsf{i}}^{\omega_\mathsf{o}}$: space of all triplets, Mueller matrix M, incoming frame F_i and outgoing frame F_o

$$\boldsymbol{f} : \mathcal{M} \times S^2 \times S^2 \rightarrow \mathbb{M}_{\omega_\mathsf{i}}^{\omega_\mathsf{o}}$$

Operations on $\mathbb{S}_\omega$ and $\mathbb{M}_{\omega_\mathsf{i}}^{\omega_\mathsf{o}}$

- Define integration and multiplication

$$\boldsymbol{L}_\mathsf{o} = \boldsymbol{L}_\mathsf{e} + \int_{S_2} \boldsymbol{f}(\omega_\mathsf{i}, \omega_\mathsf{o}) * \boldsymbol{L}_\mathsf{i}(\omega_\mathsf{i})\, d\omega_\mathsf{i}^\perp$$

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- Multiplication

$$[M, F_\mathsf{i}, F_\mathsf{o}] * [\boldsymbol{S}, F] = [M \underbrace{F_\mathsf{i} F^{-1}}_{\text{transformation from } F \text{ to } F_\mathsf{i}} \boldsymbol{S}, F_\mathsf{o}]$$

Operations on $\mathbb{S}_\omega$ and $\mathbb{M}_{\omega_i}^{\omega_o}$

- Define integration and multiplication

$$\boldsymbol{L}_o = \boldsymbol{L}_e + \int_{S_2} \boldsymbol{f}(\omega_i, \omega_o) * \boldsymbol{L}_i(\omega_i)\, d\omega_i^{\perp}$$

- Multiplication

$$[M, F_i, F_o] * [\boldsymbol{S}, F] = [M \underbrace{F_i F^{-1}}_{\text{transformation from } F \text{ to } F_i} \boldsymbol{S}, F_o]$$

- Integration $\iff$ addition

$$[\boldsymbol{S}, F] + [\boldsymbol{T}, G] = [\boldsymbol{S} + \underbrace{F G^{-1}}_{\text{transformation from } G \text{ to } F} \boldsymbol{T}, F]$$

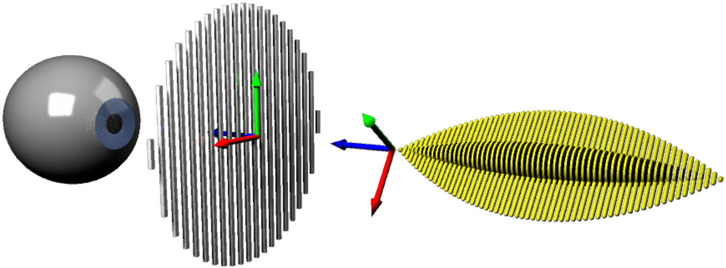
Importance

- Measurement equation

$$\boldsymbol{I} = \int_{S_2} \boldsymbol{W}_{\mathsf{e}}(\omega) \boldsymbol{L}_{\mathsf{i}}(\omega)\, d\omega^{\perp}$$

- Importance $\iff$ camera/eye sensitivity

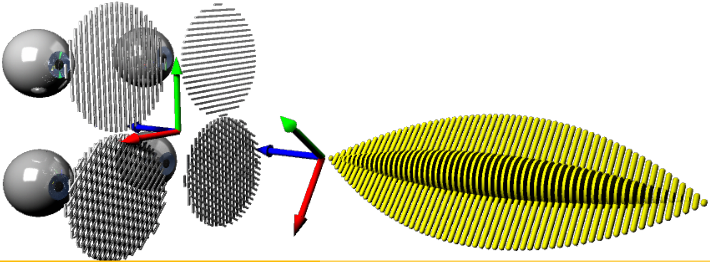
$$\boldsymbol{S}' = \underbrace{FG^{-1}}_{\text{frame transformation}}\boldsymbol{S} \qquad I = \frac{1}{2}\begin{pmatrix}1 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}S'_0\\S'_1\\S'_2\\S'_3\end{pmatrix}$$



Measurement

$$\boldsymbol{S}' = \underbrace{FG^{-1}}_{\text{frame transformation}}\boldsymbol{S}$$

$$\begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} S'_0 \\ S'_1 \\ S'_2 \\ S'_3 \end{pmatrix}$$



Importance

- Importance for polarised light transport

$$\boldsymbol{W}^T : (\boldsymbol{x}, \omega) \rightarrow [W^T, F]$$

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- **Importance space** $\bar{\mathbb{I}}_{\omega_\mathrm{i}}$: space of all pairs, measurement matrix W^T and its coordinate frame F

$$\boldsymbol{W}^T : \mathcal{M} \times S^2 \rightarrow \bar{\mathbb{I}}_\omega$$

Importance

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- **Importance space** $\bar{\mathbb{I}}_{\omega_{\mathsf{i}}}$: space of all pairs, measurement matrix W^T and its coordinate frame F

$$\boldsymbol{W}^T : \mathcal{M} \times S^2 \rightarrow \bar{\mathbb{I}}_{\omega}$$

- Define multiplication

$$\boldsymbol{I} = \int_{S_2} \boldsymbol{W}_{\mathsf{e}}^T(\omega) * \boldsymbol{L}_{\mathsf{i}}(\omega)\, d\omega^{\perp}$$

Summary of Theory

- We have defined radiance, BSDF and importance in the context of the polarising light transport
- Rendering and measurement equations are now well defined
- Path integral formulation is now well defined too

Reference implementation

Target platform

- Extending SmallUPBP
- Non-polarising algorithms already implemented.
- Better presents the necessary changes.

Polarisation-Capable Uni-Directional Path Tracing

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- $\mathbb{S}_\omega \Rightarrow \text{Light}$, $\mathbb{M}_{\omega_o}^{\omega_i} \Rightarrow \text{Attenuation}$
- Light sources and BSDF return them
- Potential reordering of expressions

Polarisation-Capable Uni-Directional Path Tracing

- $\mathbb{S}_\omega \Rightarrow \text{Light}, \mathbb{M}_{\omega_o}^{\omega_i} \Rightarrow \text{Attenuation}$
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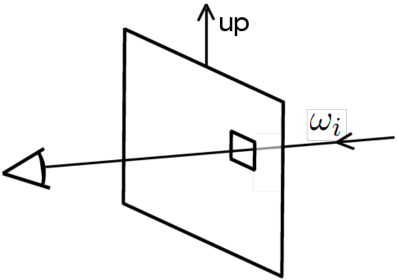
- Matches standad polarisation support
- Data type separation
- Clear changes to light transport code itself

- Matrix representation of importance
- Measuring Stokes components – identity matrix
- Coordinate frame based on camera orientation



- Non-polarising BDPT can disregard path direction with BRDF

$$f(\omega_i \to \omega_o) = f(\omega_o \to \omega_i)$$

- What about polarising BDPT:

$$\mathbf{f}(\omega_i \to \omega_o) \stackrel{?}{=} \mathbf{f}(\omega_o \to \omega_i)$$

- Non-polarising BDPT can disregard path direction with BRDF

$$f(\omega_i \to \omega_o) = f(\omega_o \to \omega_i)$$

- What about polarising BDPT:

$$\mathbf{f}(\omega_i \to \omega_o) \neq \mathbf{f}(\omega_o \to \omega_i)$$

- Matrices match, but frames do not.
  - incoming frame must match incoming direction
  - outgoing frame must match outgoing direction
- Information of path direction propagated

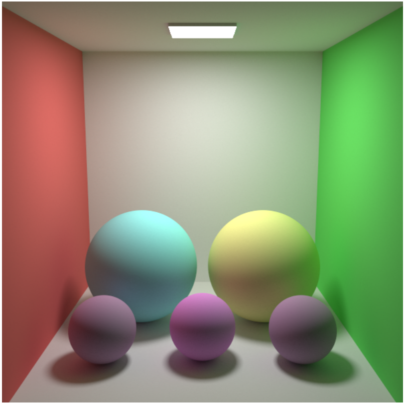
Polarisation-Capable Photon Mapping

- Tracing works the same
- Type Light stored in photon map
- Photons transformed through BSDF into view direction
- Averaging leads to eligible operations

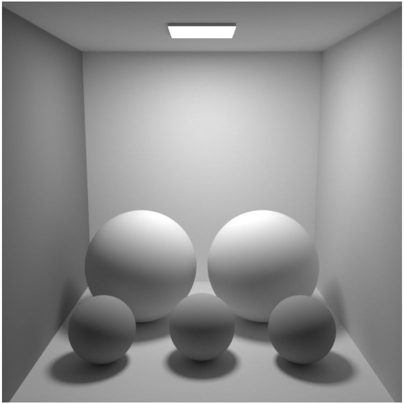
Polarisation-Capable Volumetric Bi-Directional Path Tra

- Equivalent changes to BDPT and VPT
- Transmittance accumulation + bi-directional = problem
- Accumulation dependant on path direction

Results and optimizations



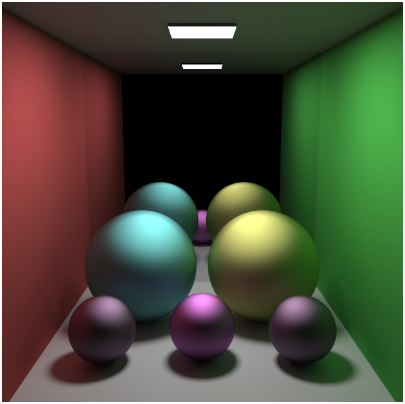
S_0



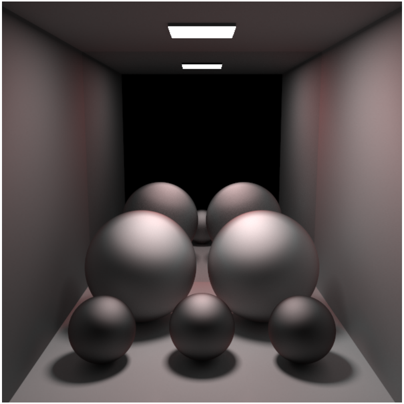
degree of polarisation
overlaid in red

Test Scenes

glossy



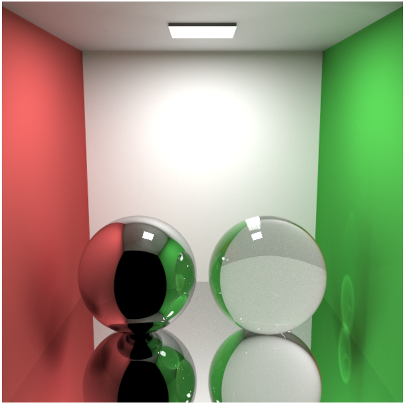
S_0



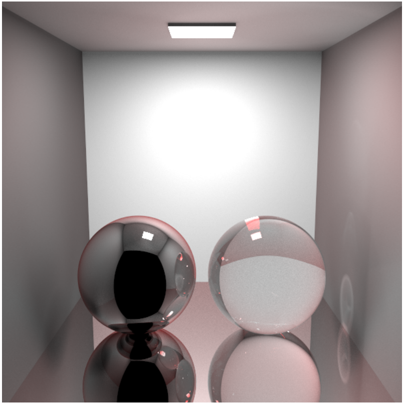
degree of polarisation overlaid in red

Test Scenes

glass



S_0



degree of polarisation
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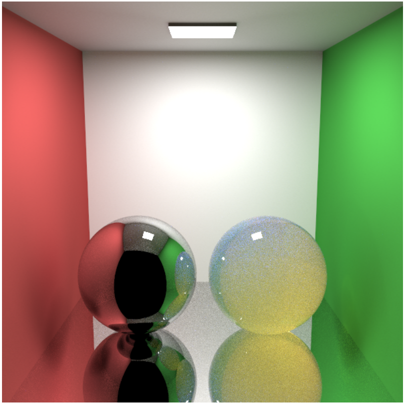
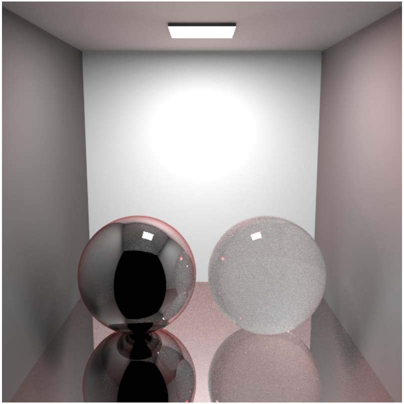
Test Scenes

Volumetric glass



S_0

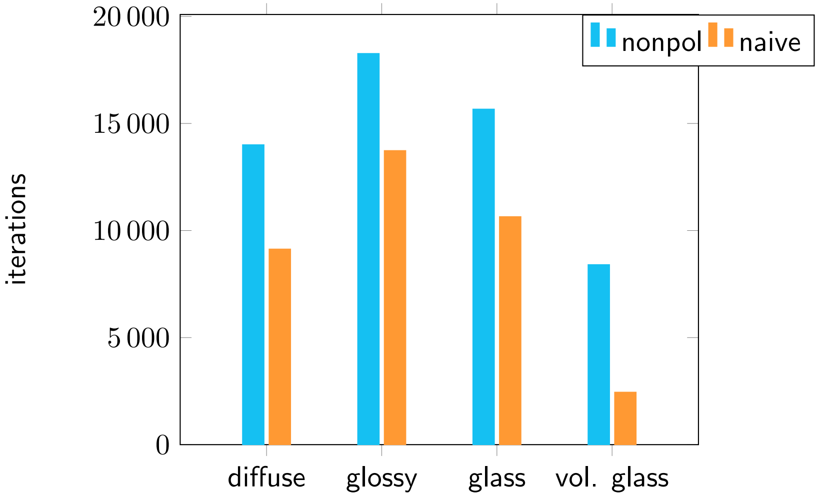


degree of polarisation overlaid in red

Bathroom scene

S_0

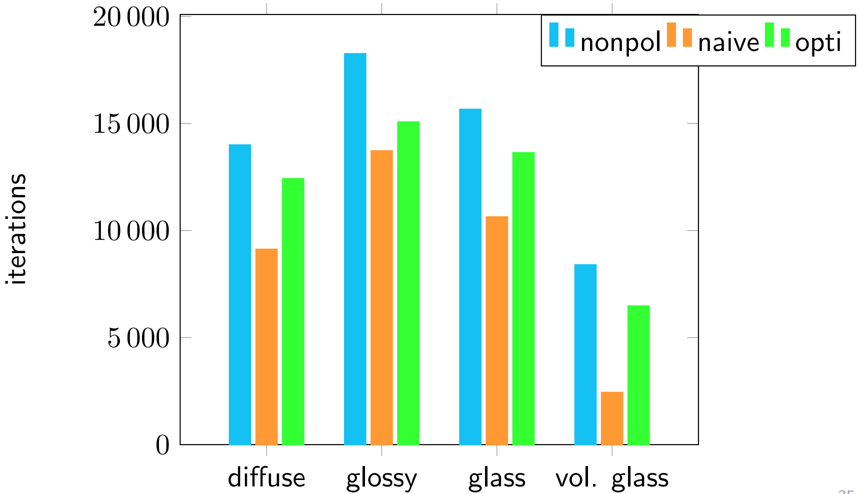
degree of polarisation
overlaid in red


20 000
15 000
10 000
5 000
0
iterations
nonpol
naive
diffuse
glossy
glass
vol. glass

Efficiency

- Polarisation support brings overhead
- Comparing with non-polarising version
- Optimizations on data types
  - unpolarised Light
  - depolarising Attenuation
  - plain Attenution

Efficiency



20 000
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Conclusion

- Reformulated the radiometry for polarised light
- Developed reference implmenetation of BDPT, VBDPT and VCM
- Examined the efficiency of polarisation support and suggested optimizations

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Thank you.

Acknowledgement

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