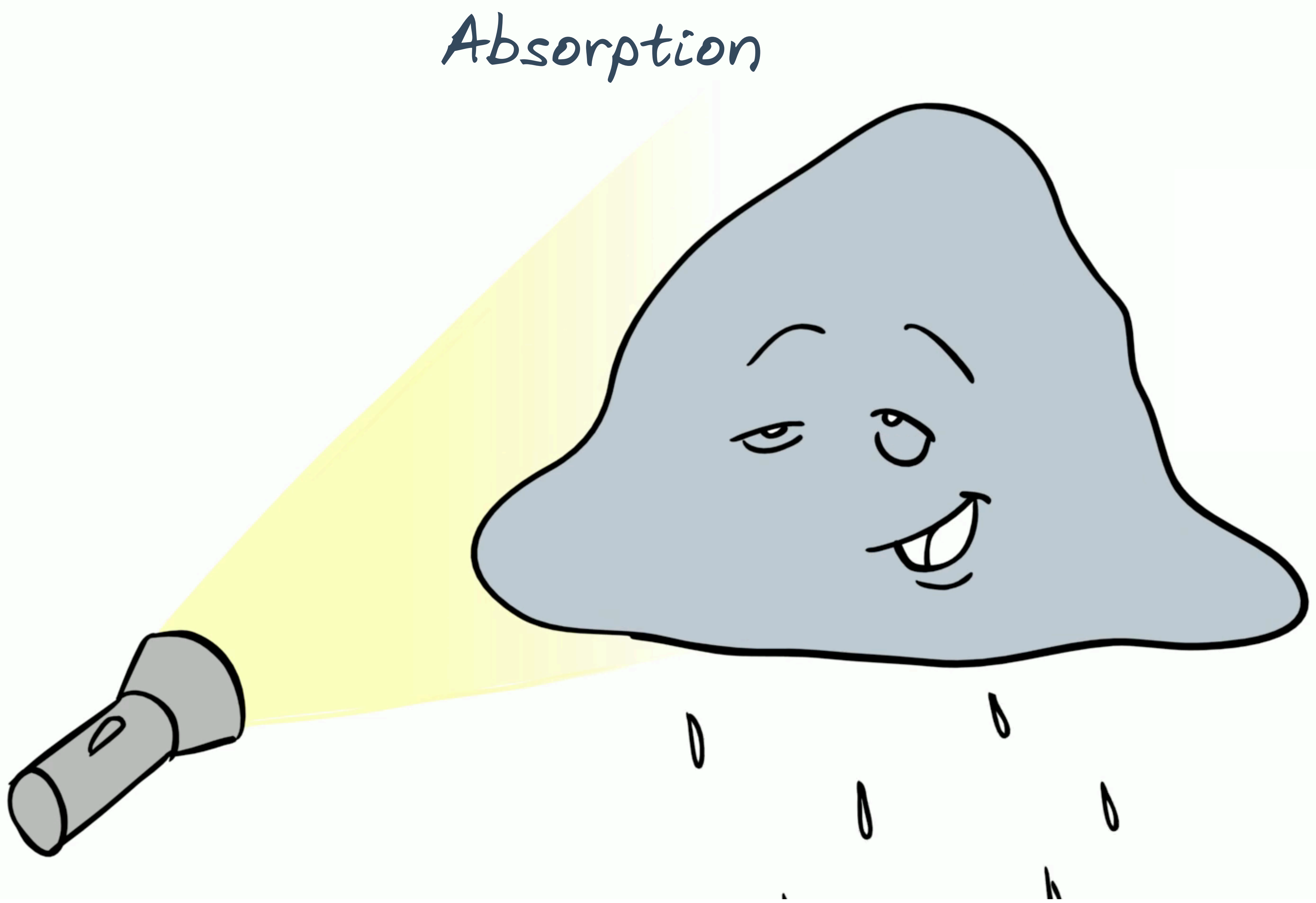
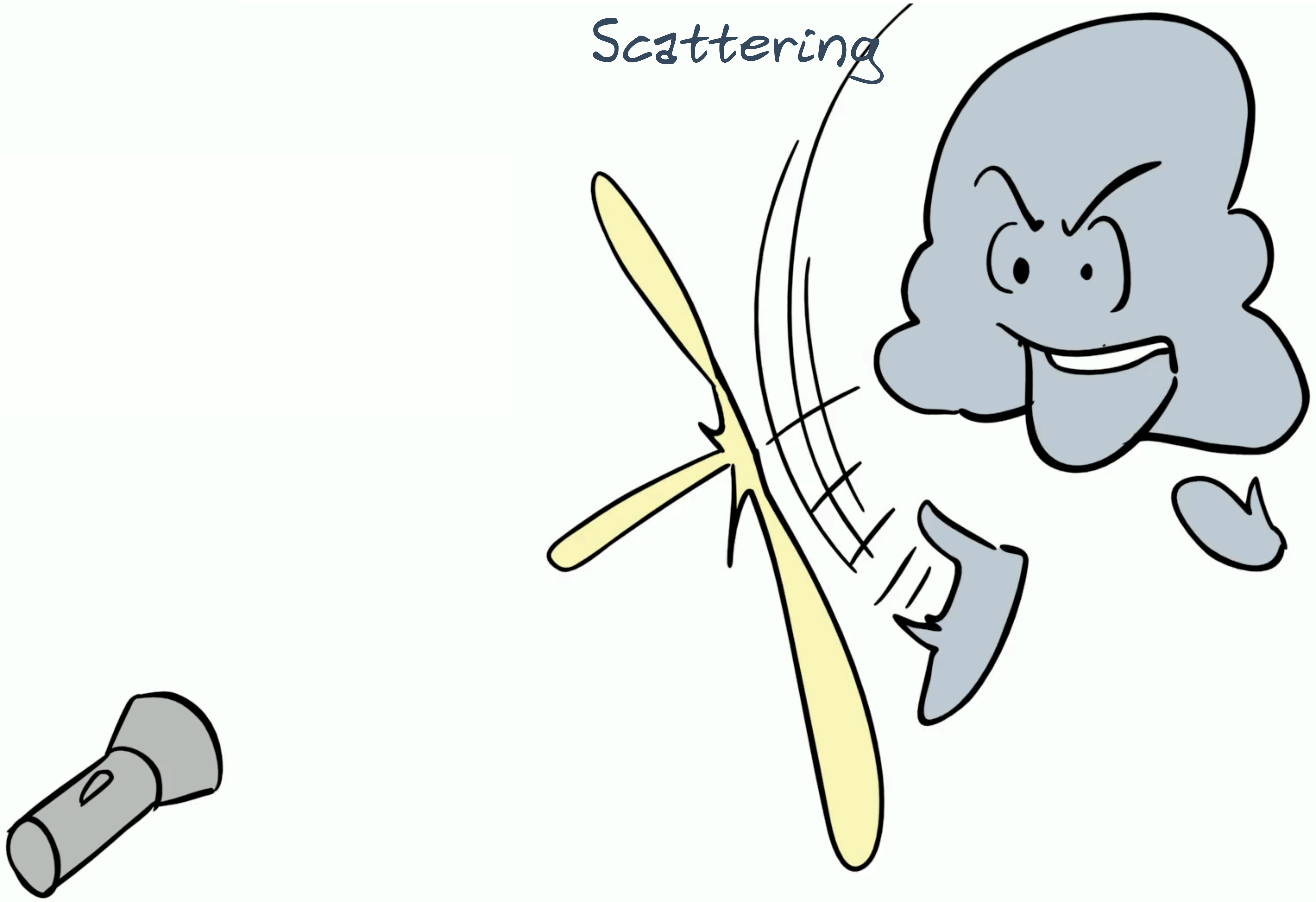
FUNDAMENTALS





FUNDAMENTALS

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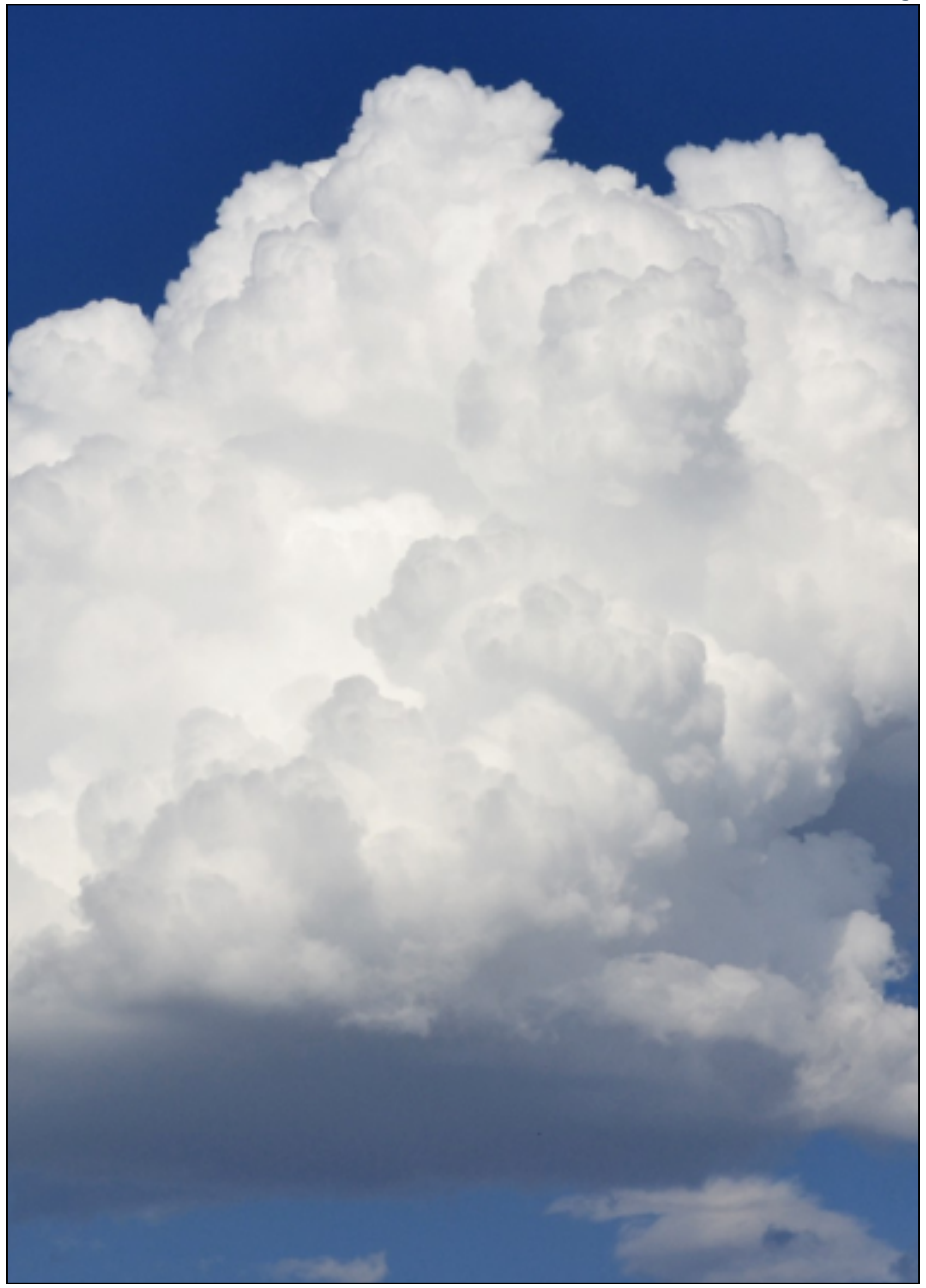
Absorption

http://commons.wikimedia.org

Scattering



http://coclouds.com

Emission

http://wikipedia.org

RADIATIVE TRANSFER

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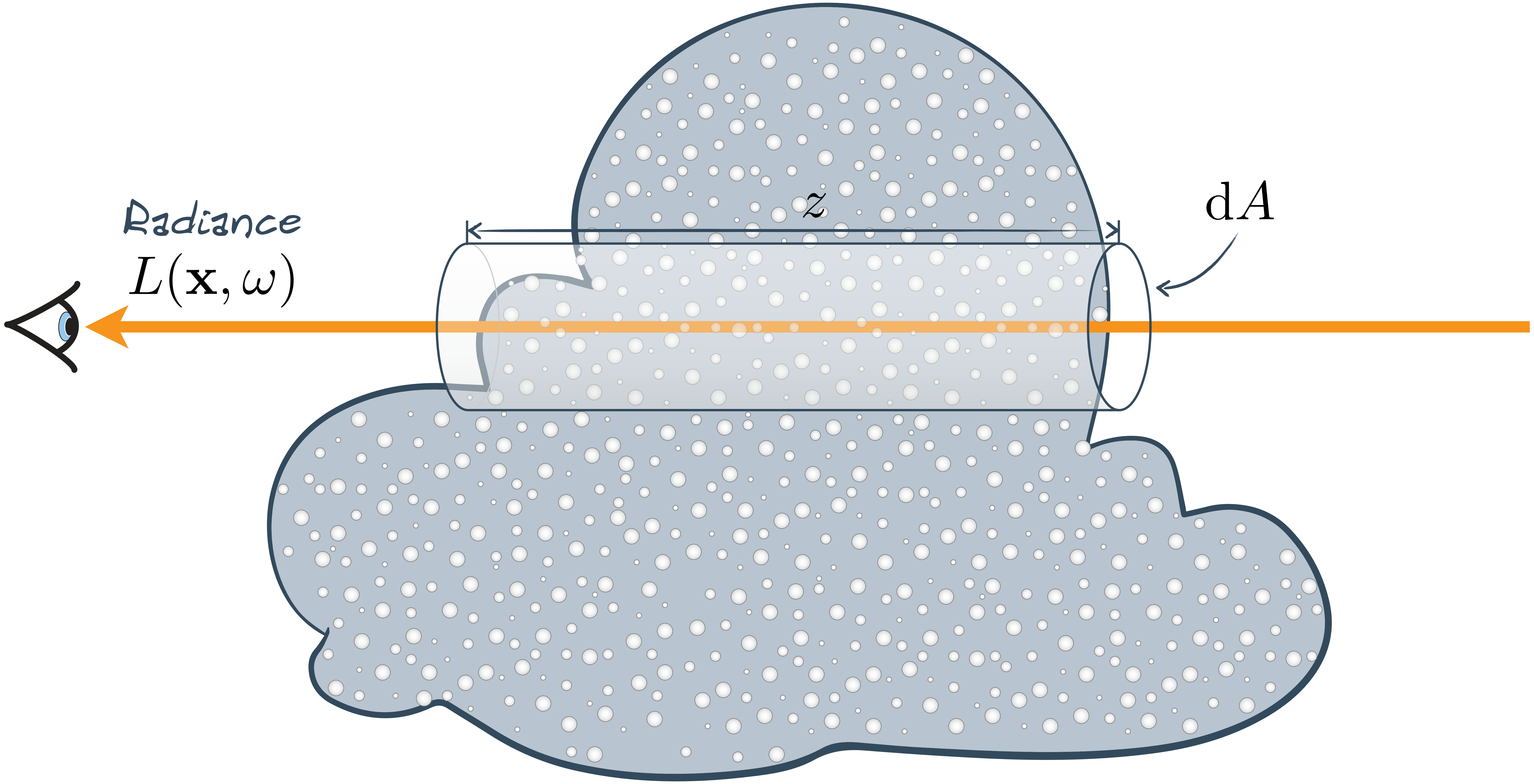
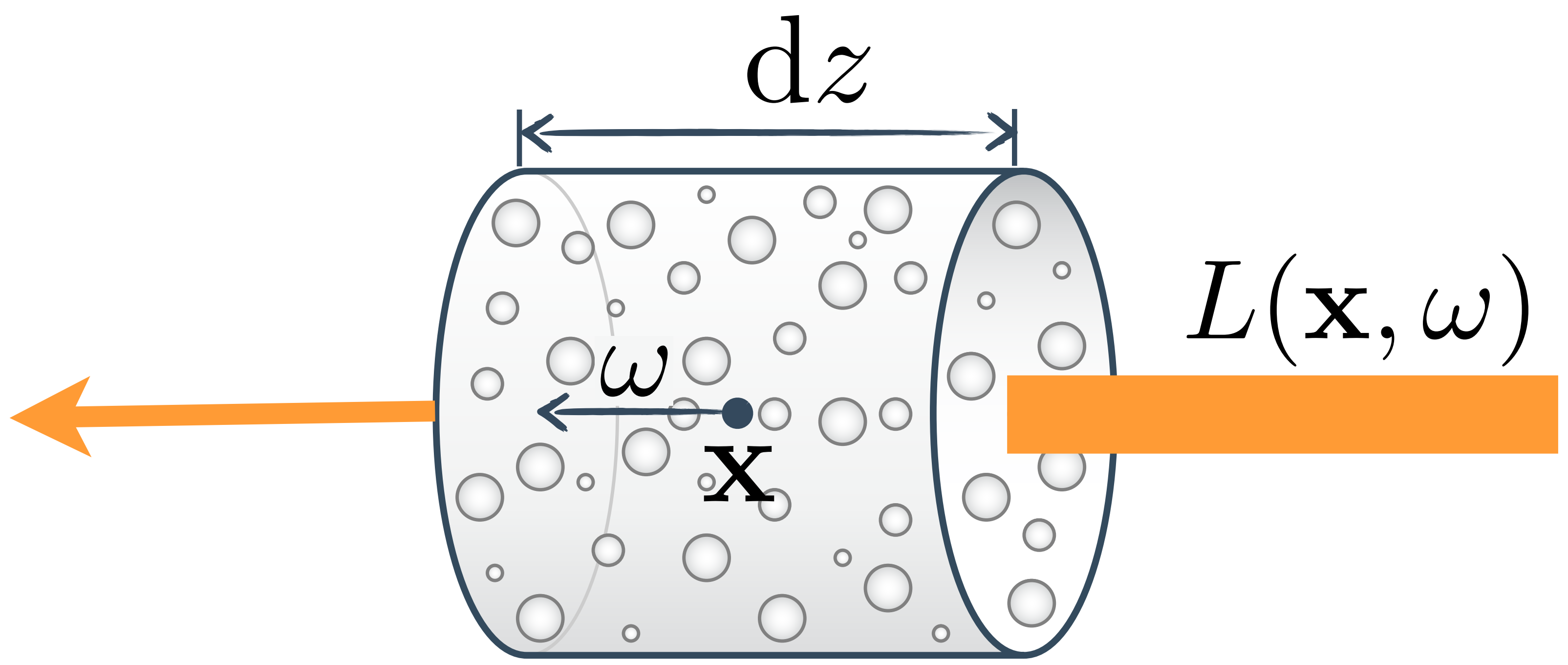


ABSORPTION



$$\frac{\mathrm{d}L}{\mathrm{d}z} = -\mu_\mathrm{a}(\mathbf{x})L(\mathbf{x}, \omega)$$

μ_a - absorption coefficient

OUT-SCATTERING

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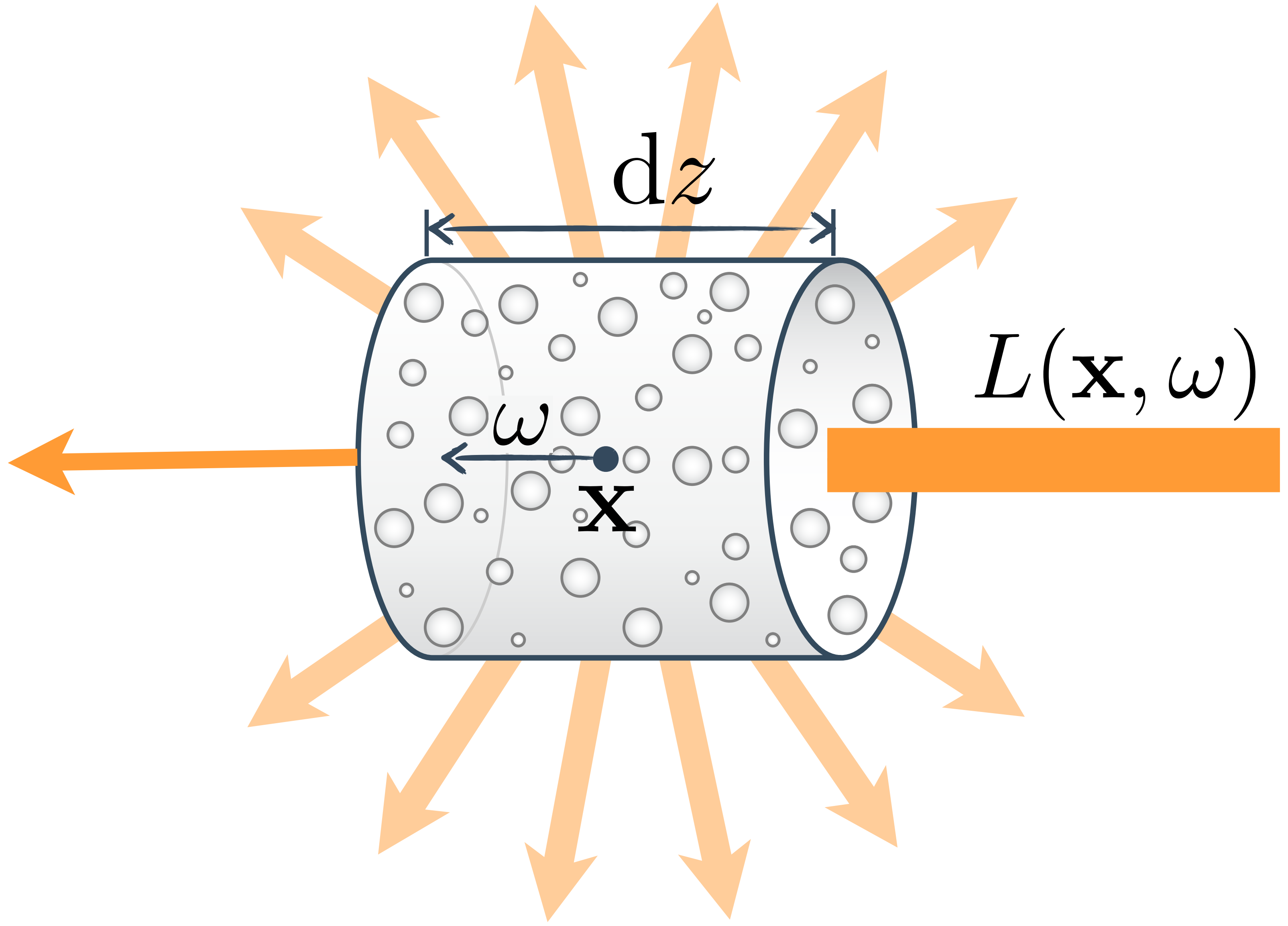

$$\frac{\mathrm{d}L}{\mathrm{d}z} = -\mu_{\mathrm{s}}(\mathbf{x}) L(\mathbf{x}, \omega)$$

μ_{s} – scattering coefficient

IN-SCATTERING

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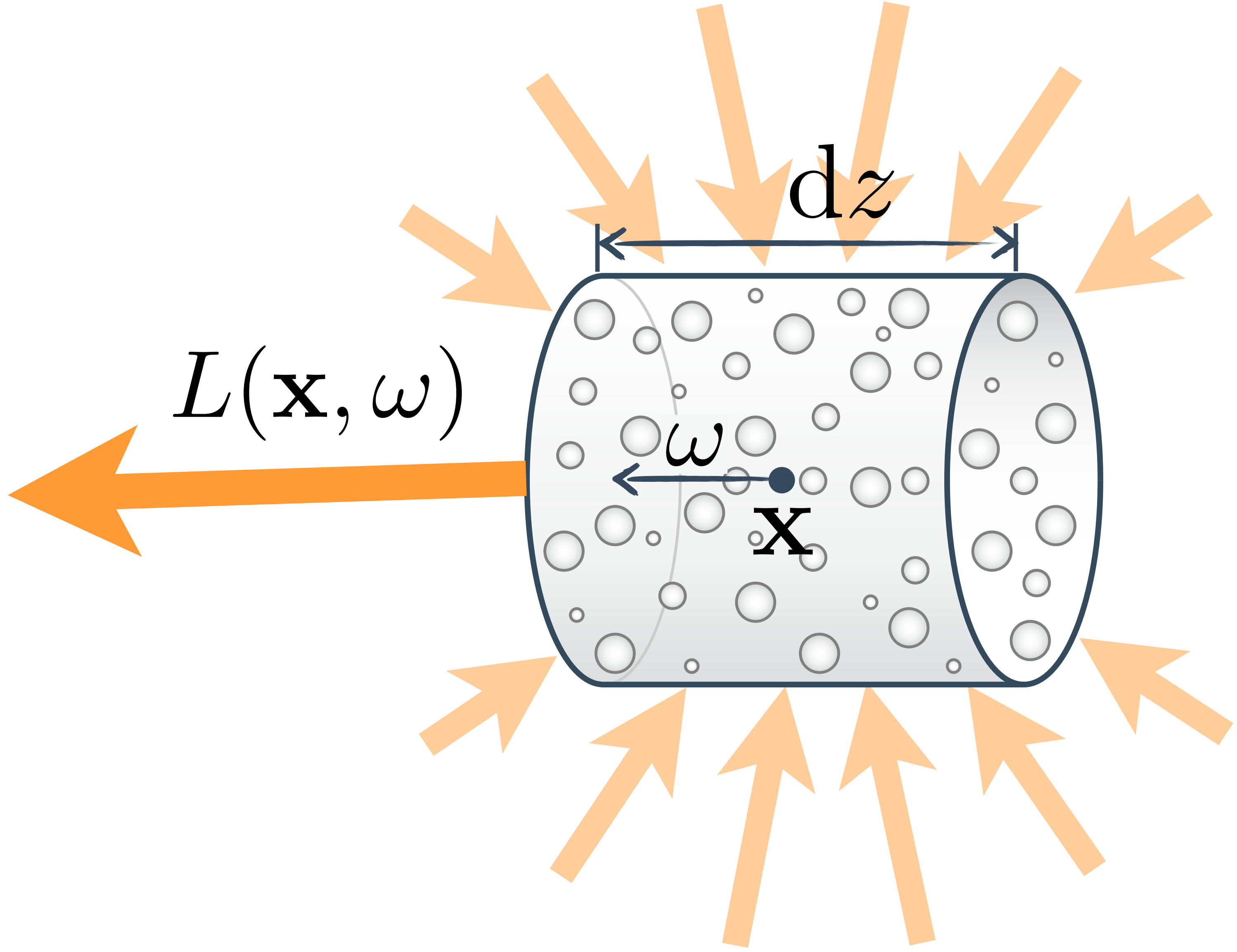


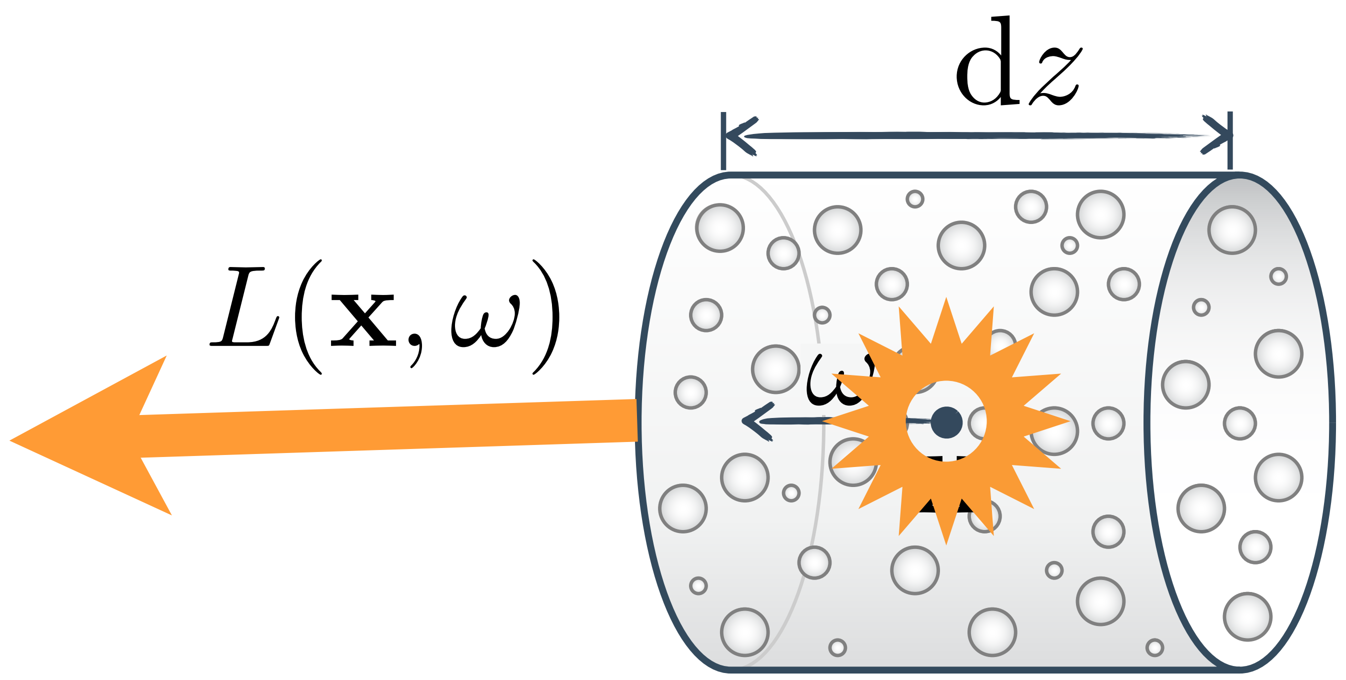
In-scattered radiance

$$L_s(\mathbf{y}, \omega) = \int_{S^2} f_p(\omega, \bar{\omega}) L(\mathbf{y}, \bar{\omega}) \mathrm{d}\bar{\omega}$$

$$\frac{\mathrm{d}L}{\mathrm{d}z} = \mu_s(\mathbf{x}) L_s(\mathbf{x}, \omega)$$

μ_s - *scattering coefficient*

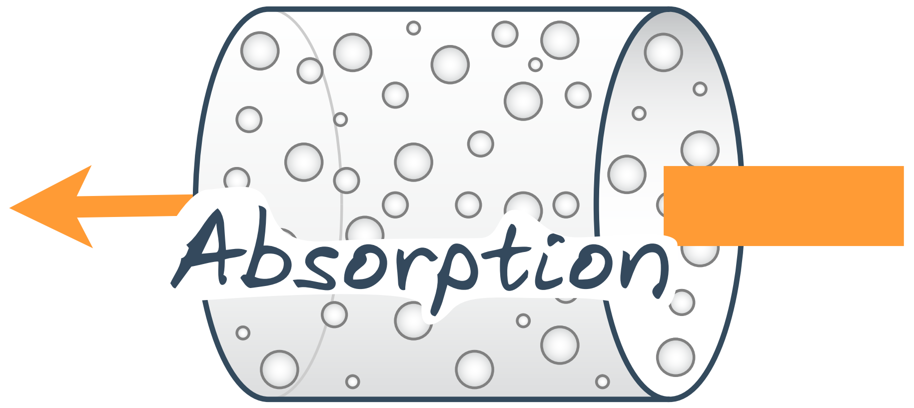
EMISSION



$$\frac{\mathrm{d}L}{\mathrm{d}z} = \mu_{\mathrm{a}}(\mathbf{x}) L_{\mathrm{e}}(\mathbf{x}, \omega)$$

L_{e} - emitted radiance

RADIATIVE TRANSFER EQUATION

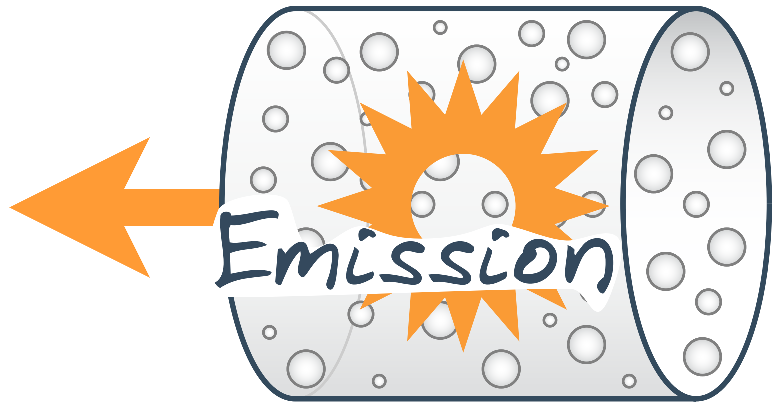


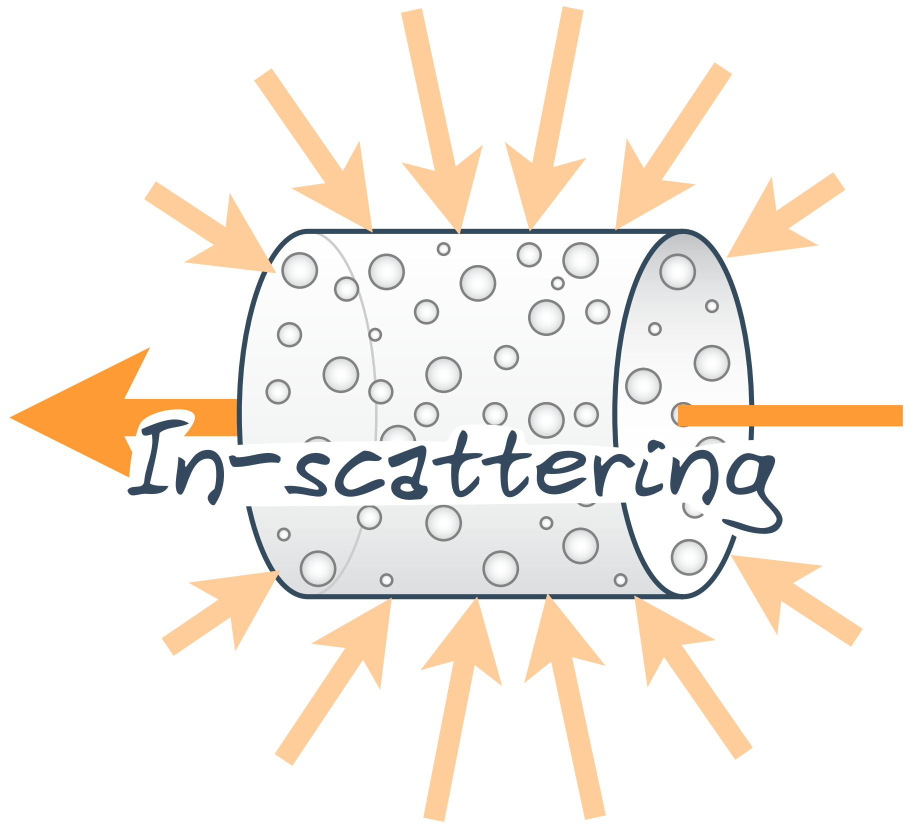
$$\frac{\mathrm{d}L(\mathbf{x},\omega)}{\mathrm{d}z} = -\mu_{\mathrm{a}}(\mathbf{x})L(\mathbf{x},\omega) \; -\mu_{\mathrm{s}}(\mathbf{x})L(\mathbf{x},\omega)$$

Losses

$$+\mu_{\mathrm{a}}(\mathbf{x})L_{\mathrm{e}}(\mathbf{x},\omega) + \mu_{\mathrm{s}}(\mathbf{x})L_{\mathrm{s}}(\mathbf{x},\omega)$$

Gains





[Chandrasekhar 1960]

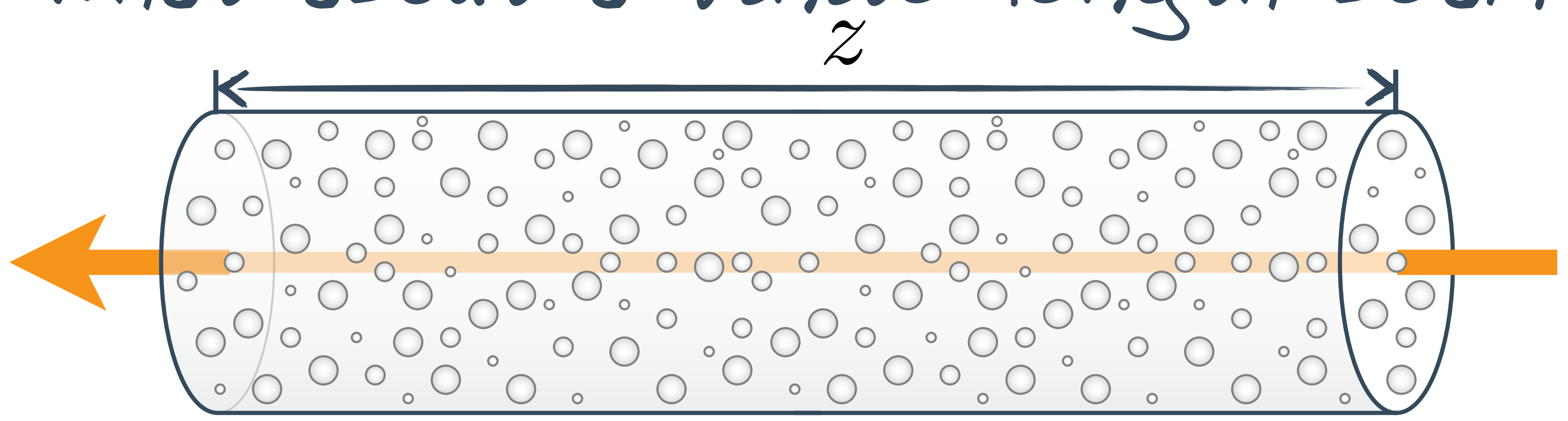
RADIATIVE TRANSFER

Extinction coefficient $\mu_t(\mathbf{x}) = \mu_a(\mathbf{x}) + \mu_s(\mathbf{x})$

$$\frac{\mathrm{d}L(\mathbf{x},\omega)}{\mathrm{d}z} = -\mu_t(\mathbf{x})L(\mathbf{x},\omega) \quad \text{Losses}$$

$$+\mu_a(\mathbf{x})L_e(\mathbf{x},\omega) + \mu_s(\mathbf{x})L_s(\mathbf{x},\omega) \quad \text{Gains}$$

What about a finite-length beam?



[Chandrasekhar 1960]

RTE — INTEGRAL FORM

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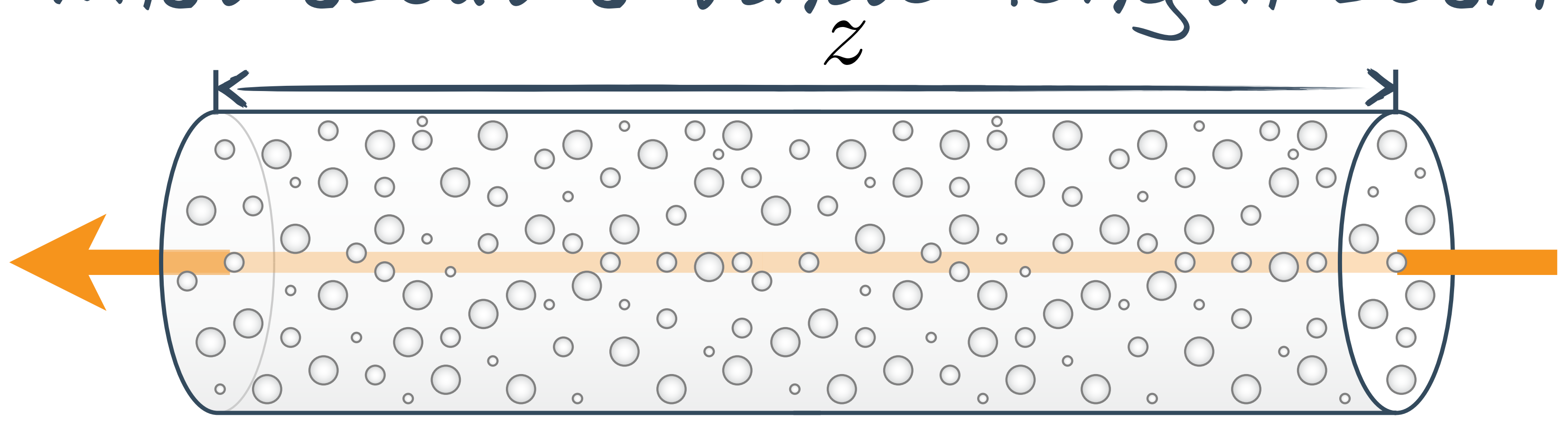
$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\Big[\mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\Big]\,\mathrm{d}y$$

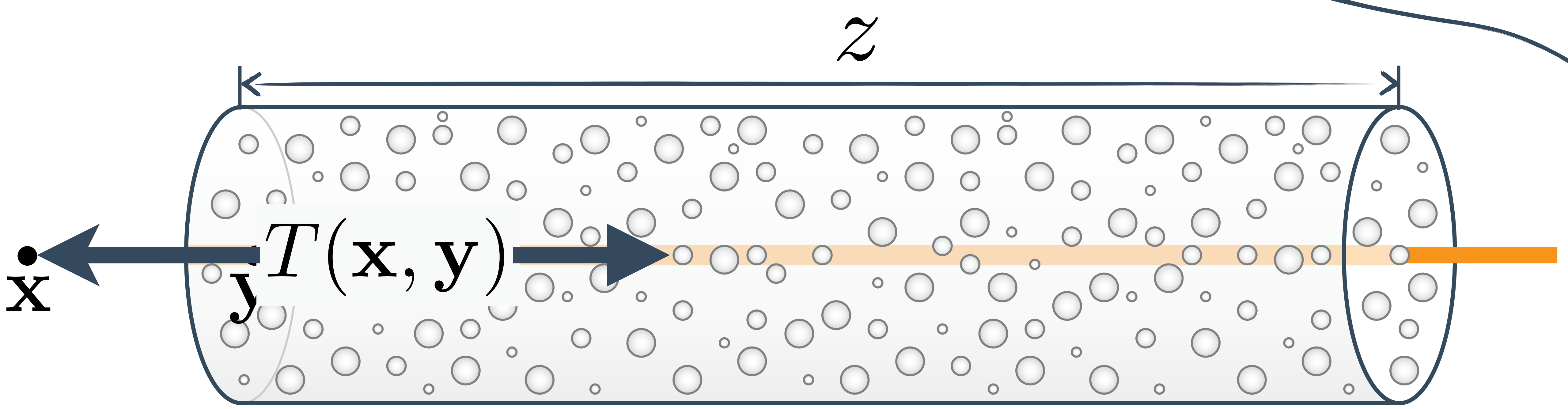
What about a finite-length beam?



RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y}) \left[\mu_a(\mathbf{y}) L_e(\mathbf{y},\omega) + \mu_s(\mathbf{y}) L_s(\mathbf{y},\omega) \right] \mathrm{d}y$$

Transmittance $T(\mathbf{x},\mathbf{y}) = e^{-\int_0^y \mu_t(\mathbf{s})\mathrm{d}s}$ is the fraction of light that makes it from y to x



Optical thickness

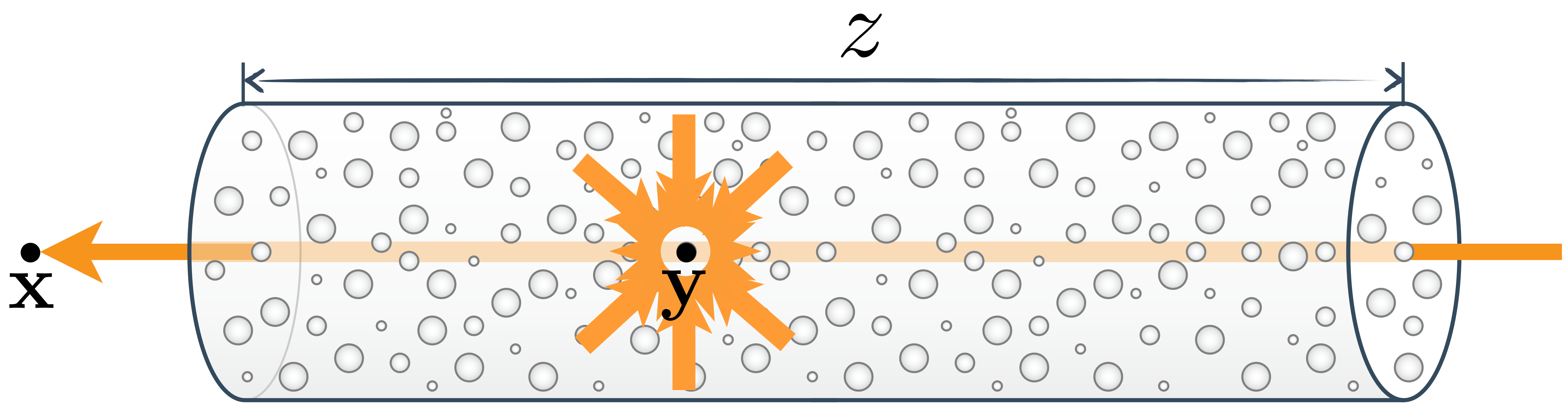
$$\tau(\mathbf{x},\mathbf{y}) = \int_0^y \mu_t(\mathbf{s})\mathrm{d}s$$

RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y}) \left[\mu_\mathrm{a}(\mathbf{y}) L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y}) L_\mathrm{s}(\mathbf{y},\omega) \right] \mathrm{d}y$$

Emission

In-scattering



RTE — INTEGRAL FORM

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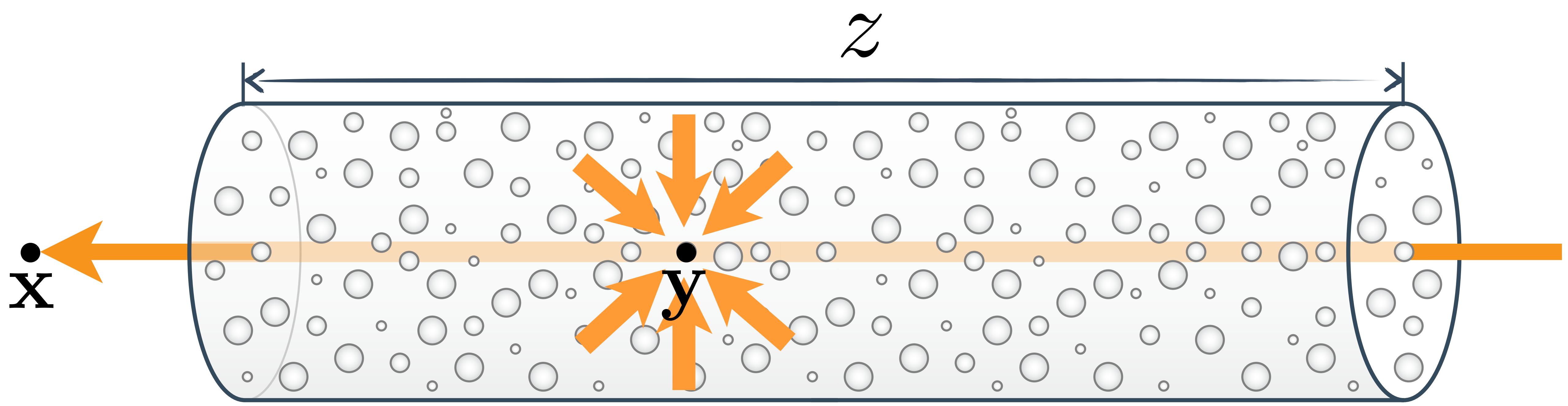
$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\left[\mu_a(\mathbf{y})L_e(\mathbf{y},\omega) + \mu_s(\mathbf{y})L_s(\mathbf{y},\omega)\right]\mathrm{d}y$$

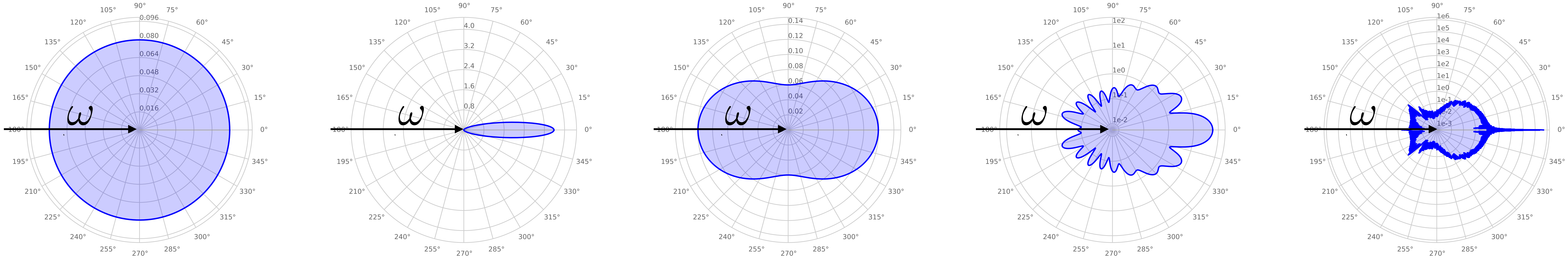
$$L_s(\mathbf{y},\omega) = \int_{S^2} f_p(\omega,\bar{\omega})L(\mathbf{y},\bar{\omega})\mathrm{d}\bar{\omega}$$

Phase function



PHASE FUNCTION

$$f_{\mathrm{p}}(\omega, \bar{\omega})$$



Isotropic | Henyey-Greenstein | Rayleigh | Lorenz-Mie small particles | Lorenz-Mie large particles

PHASE FUNCTION

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Backward scattering PF	Forward scattering PF
	
Smoke	Steam

PHASE FUNCTION

Isotropic PF

Forward scattering PF

RTE — INTEGRAL FORM

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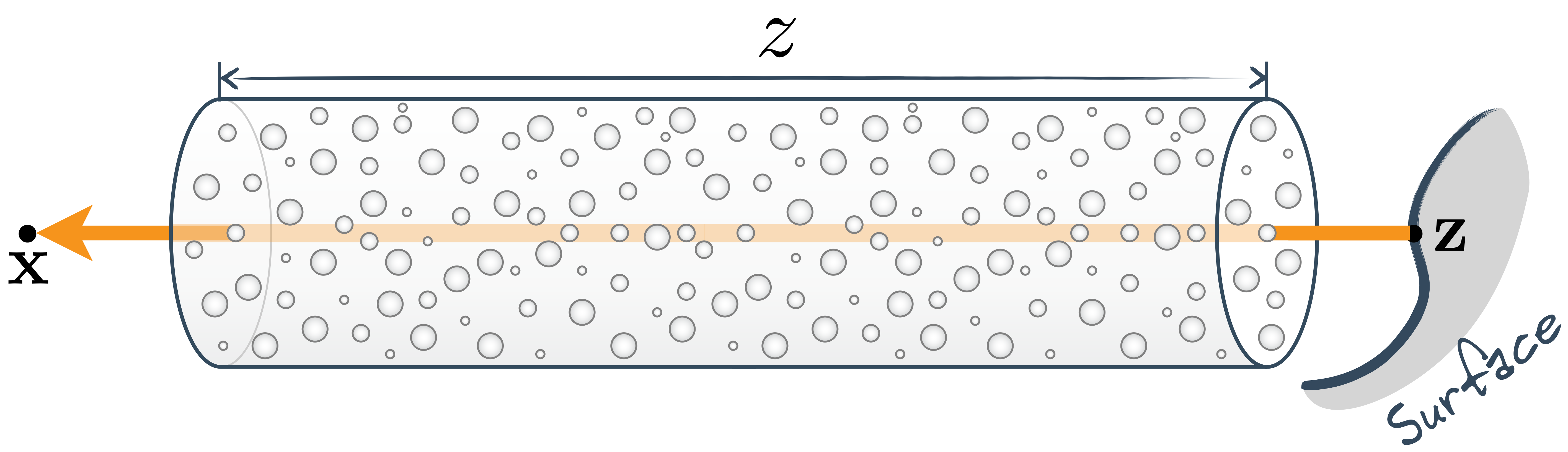
$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\Big[\mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\Big]\mathrm{d}y$$
$$+T(\mathbf{x},\mathbf{z})L_\mathrm{o}(\mathbf{z},\omega)$$

Background radiance



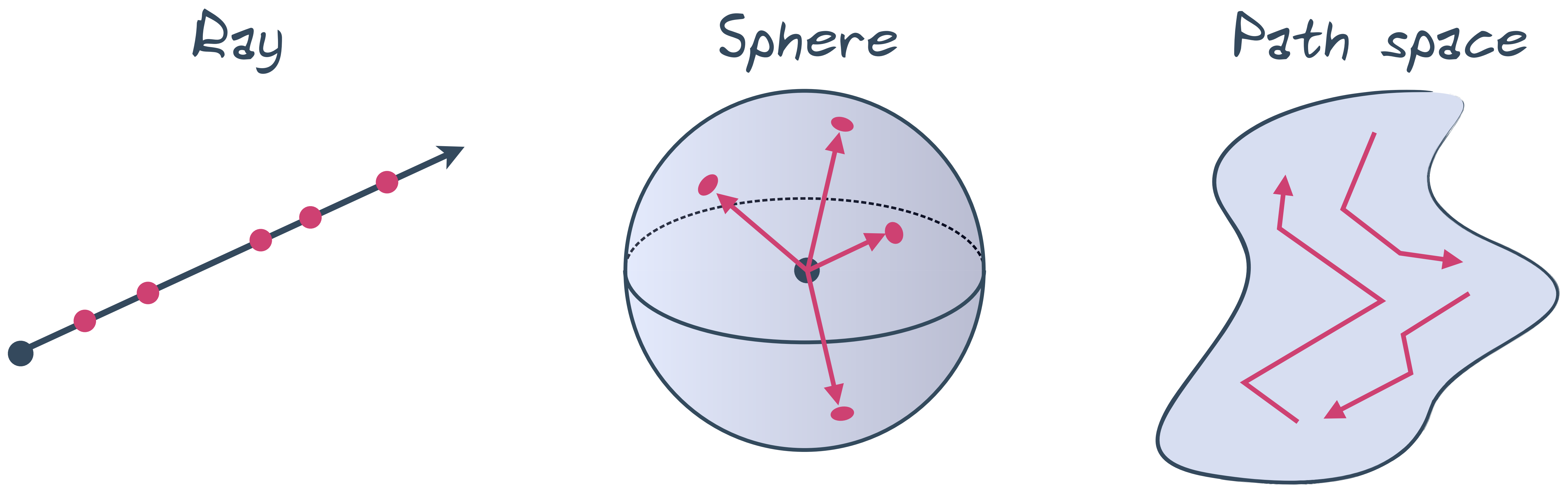
VOLUME RENDERING EQUATION

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\Big[\mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\Big]\mathrm{d}y$$
$$+T(\mathbf{x},\mathbf{z})L_\mathrm{o}(\mathbf{z},\omega)$$

How do we solve it?

MONTE CARLO INTEGRATION

$$F = \int_{\mathcal{D}} f(x)\,\mathrm{d}x$$



$$\langle F \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

Probability density function (PDF)

VRE ESTIMATOR

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$$\langle L(\mathbf{x},\omega)\rangle = \frac{T(\mathbf{x},\mathbf{y})}{p(y)}\left[\mu_{\mathrm{a}}(\mathbf{y})L_{\mathrm{e}}(\mathbf{y},\omega)+\mu_{\mathrm{s}}(\mathbf{y})L_{\mathrm{s}}(\mathbf{y},\omega)\right]$$

$$+\frac{T(\mathbf{x},\mathbf{z})}{P(z)}L_{\mathrm{o}}(\mathbf{z},\omega)$$

$p(y)$ - probability density of distance y

$P(z)$ - probability of exceeding distance z

VRE ESTIMATOR

Transmittance estimation

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T(\mathbf{x},\mathbf{y})}{p(y)} \left[\mu_{\mathrm{a}}(\mathbf{y}) L_{\mathrm{e}}(\mathbf{y},\omega) + \mu_{\mathrm{s}}(\mathbf{y}) L_{\mathrm{s}}(\mathbf{y},\omega)\right]$$

$$+ \frac{T(\mathbf{x},\mathbf{z})}{P(z)} L_{\mathrm{o}}(\mathbf{z},\omega)$$

Distance sampling