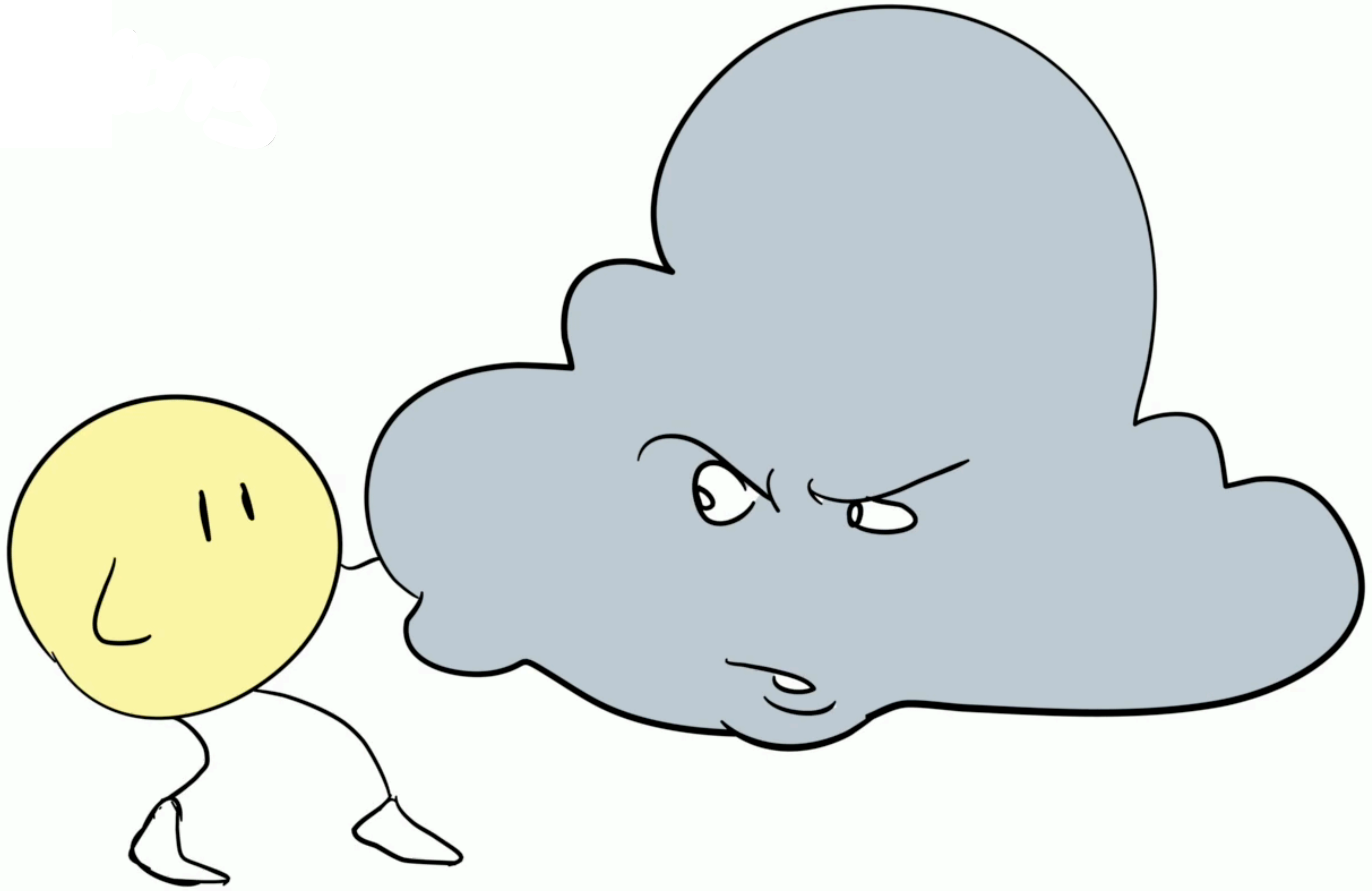
DISTANCE SAMPLING

How far will photon travel before interacting with the medium?



DISTANCE SAMPLING

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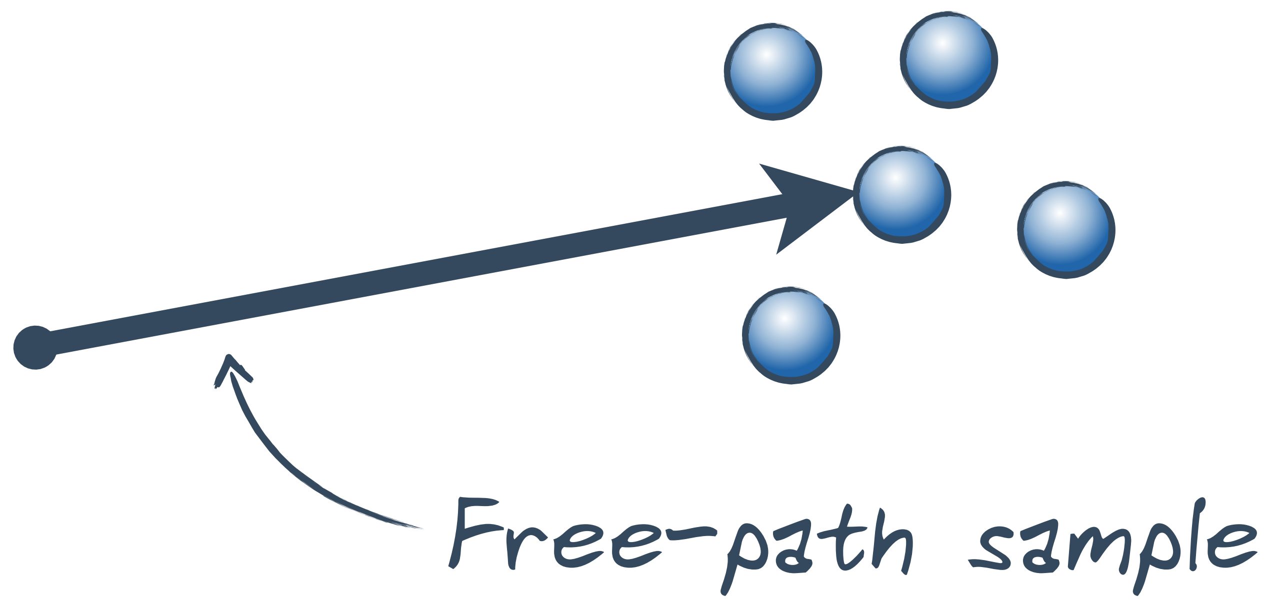
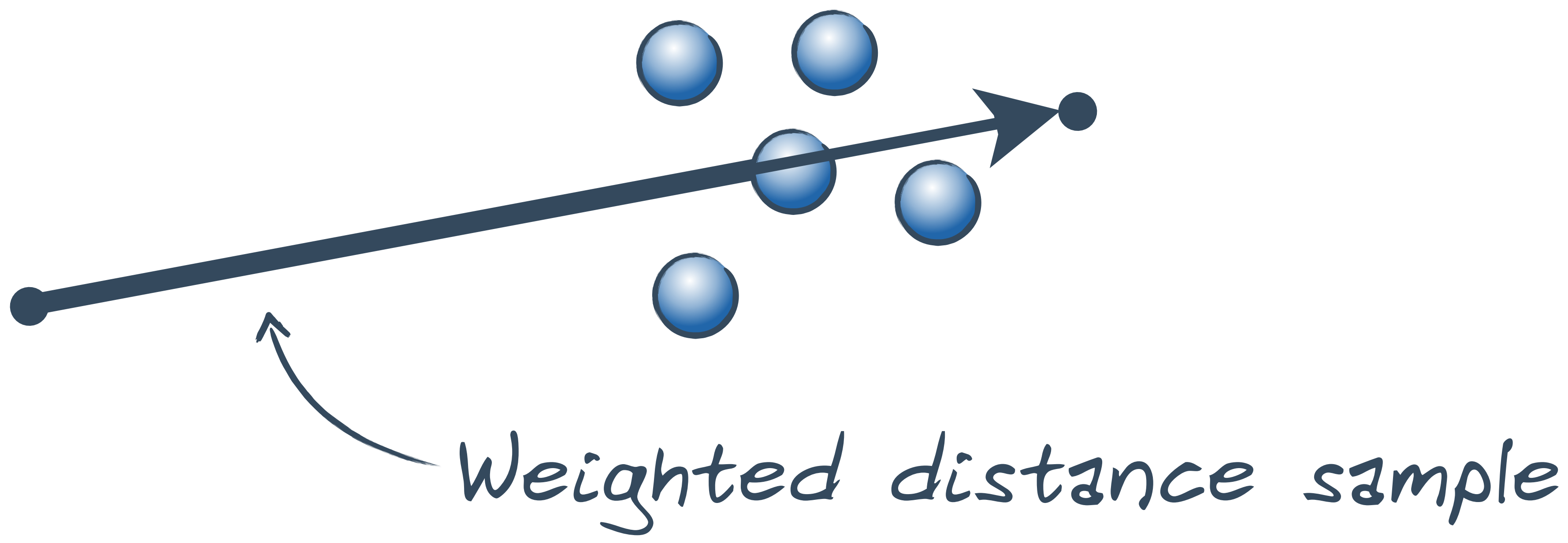
ANALOG methods

- Adhere to physical process
- Produce **free-path** samples
- Energy of particles **unchanged**

NON-ANALOG methods

- Deviate from physical process
- Produce **arbitrary distance** samples
- Particles (photons) are **weighted**





FREE-PATH SAMPLING

How to sample the flight distance to the next interaction?

$$T(t) = e^{-\int_0^t \mu_t(s)\mathrm{d}s} = P(X > t)$$

Random vari

$$P(X \leq t) = F(t)$$

CDF

Partition of unity

$$F(t) = 1 - T(t)$$

Recipe for generating samples

Losses expressed in differential form:

$$\frac{\mathrm{d}L(\mathbf{x},\omega)}{\mathrm{d}z} = -\mu_t(\mathbf{x})L(\mathbf{x},\omega)$$

Radiance gathered along a ray:

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})L_o(\mathbf{y},\omega)\mathrm{d}y$$

Transmittance:

$$T(t) = e^{-\int_0^t \mu_t(s)\mathrm{d}s}$$

FREE-PATH SAMPLING

Cumulative distribution function (**CDF**)

$$F(t) = 1 - T(t) = 1 - e^{-\tau(t)}$$

Probability density function (**PDF**)

$$p(t) = \frac{\mathrm{d}F(t)}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(1 - e^{-\tau(t)}\right) = \mu_{\mathrm{t}}(t)e^{-\tau(t)}$$

Inverted cumulative distr. function (**CDF^{-1}**)

$$\xi = 1 - e^{-\tau(t)}$$

Solve for t

$$\int_0^t \mu_{\mathrm{t}}(s)\mathrm{d}s = -\ln(1-\xi)$$

Approaches for finding t:

1) ANALYTIC (closed-form CDF^{-1})

2) SEMI-ANALYTIC (regular tracking)

3) APPROXIMATE (ray marching)

ANALYTIC APPROACH

Inverted cumulative distr. function (**CDF[-1]**)

$$\int_0^t \mu_t(s)\mathrm{d}s = -\ln(1-\xi)$$

Some simple volumes permit closed-form solutions

Example: **homogeneous** medium ($\mu_t(\mathbf{x}) = \mu_t$)

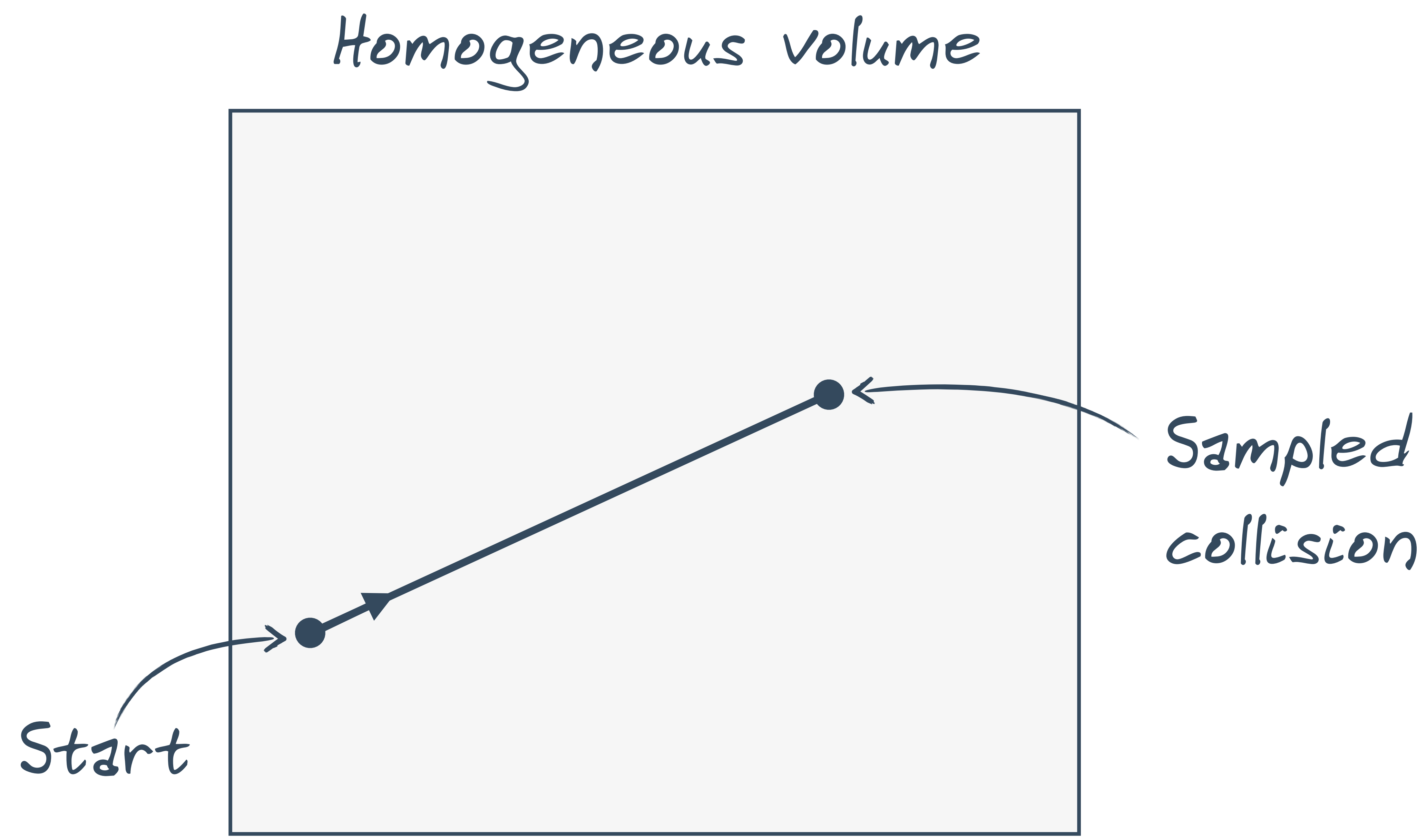
Opt. thickness *Inverted CDF*

$$\int_0^t \mu_t(s)\mathrm{d}s = t\mu_t \quad \Rightarrow \quad F^{-1}(\xi) = -\frac{\ln(1-\xi)}{\mu_t}$$

ANALYTIC APPROACH

Inverted cumulative distr. function (**CDF**$^{-1}$)

$$\int_0^t \mu_t(s)\mathrm{d}s = -\ln(1-\xi)$$



Sampling in homogeneous vol:

1) Draw a random number ξ
2) Set $t = -\frac{\ln(1-\xi)}{\mu_t}$
3) Set $p(t) = \mu_t e^{-t\mu_t}$

REGULAR TRACKING (SEMI-ANALYTIC)

For piecewise-simple (e.g. piecewise-constant), summation replaces integration

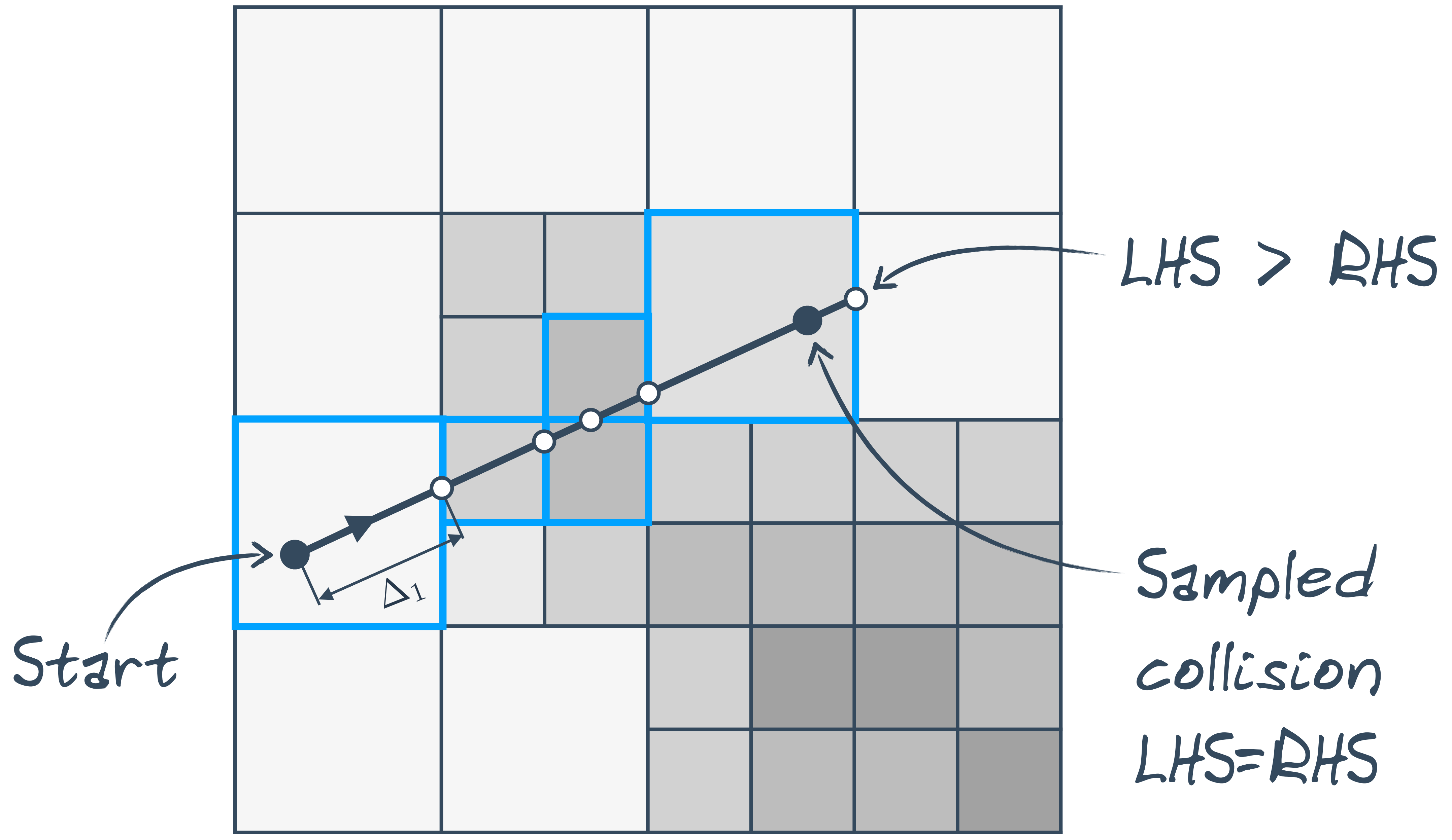
$$\int_0^t \mu_{\mathrm{t}}(s)\mathrm{d}s = -\ln(1-\xi)$$

$$\sum_{i=1}^{k} \mu_{\mathrm{t},i}\,\Delta_i = -\ln(1-\xi)$$

Regular tracking:

1) Draw a random number ξ
2) While LHS < RHS
   move to the next intersection
3) Find the exact location
   in the last segment analytically

(Hierarchical) voxel grid



REGULAR TRACKING (SEMI-ANALYTIC)

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Regular tracking:

1) Draw a random number ξ
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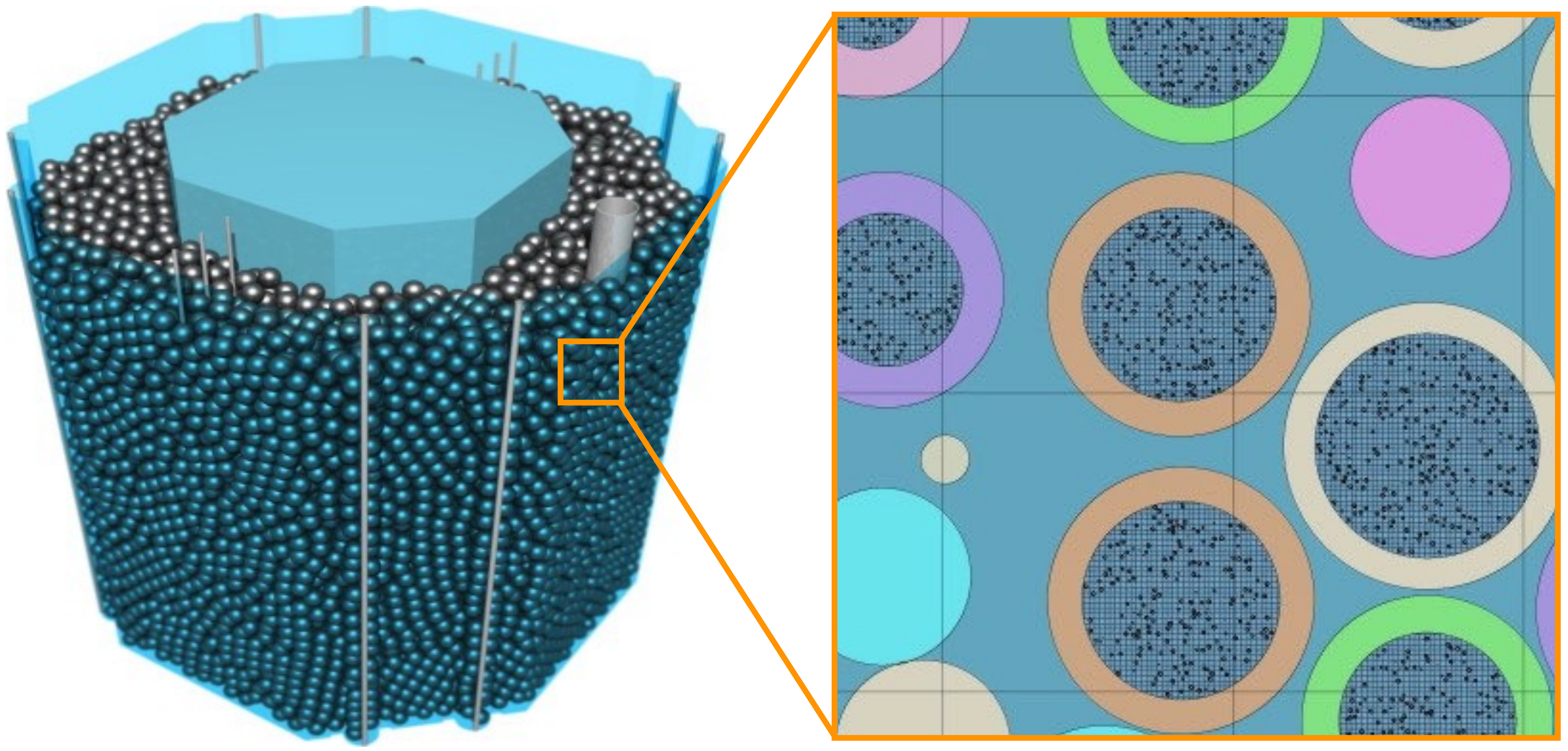
   move to the next intersection
3) Find the exact location
   in the last segment analytically

Pebble-bed reactor



Images courtesy of Rintala et al. [2015]

Finding the intersections can be expensive...

RAY MARCHING

Find the collision distance approximately

$$\int_0^t \mu_t(s)\mathrm{d}s = -\ln(1-\xi)$$

$$\neq$$

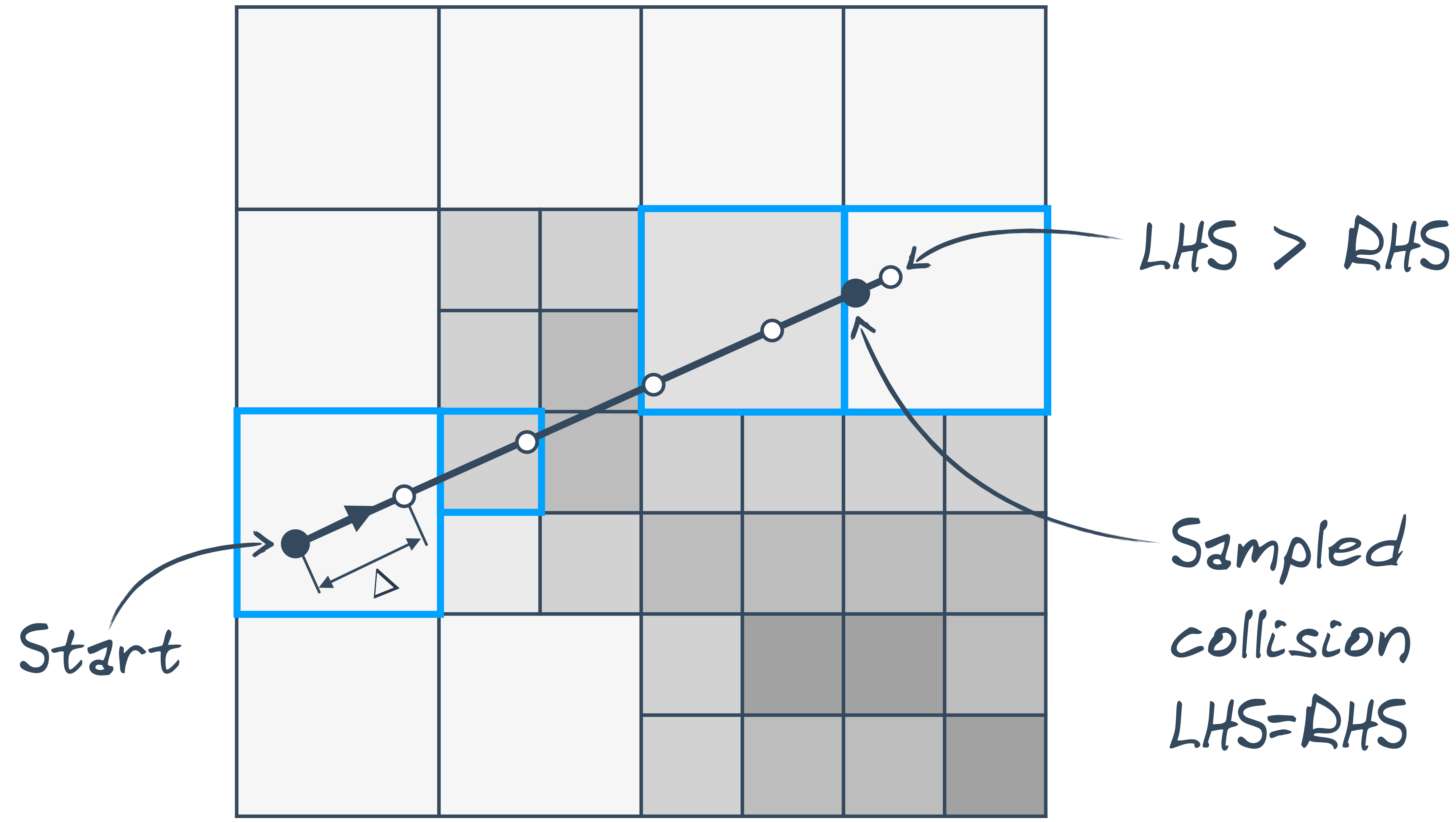
$$\sum_{i=1}^{k} \mu_{t,i}\,\Delta = -\ln(1-\xi)$$

Constant step

Ray marching:

1) Draw a random number ξ
2) While LHS < RHS
make a (fixed-size) step
3) Find the exact location
in the last segment analytically

(Hierarchical) voxel grid



RAY MARCHING

Find the collision distance approximately

$$\int_0^t \mu_t(s)\mathrm{d}s = -\ln(1-\xi)$$

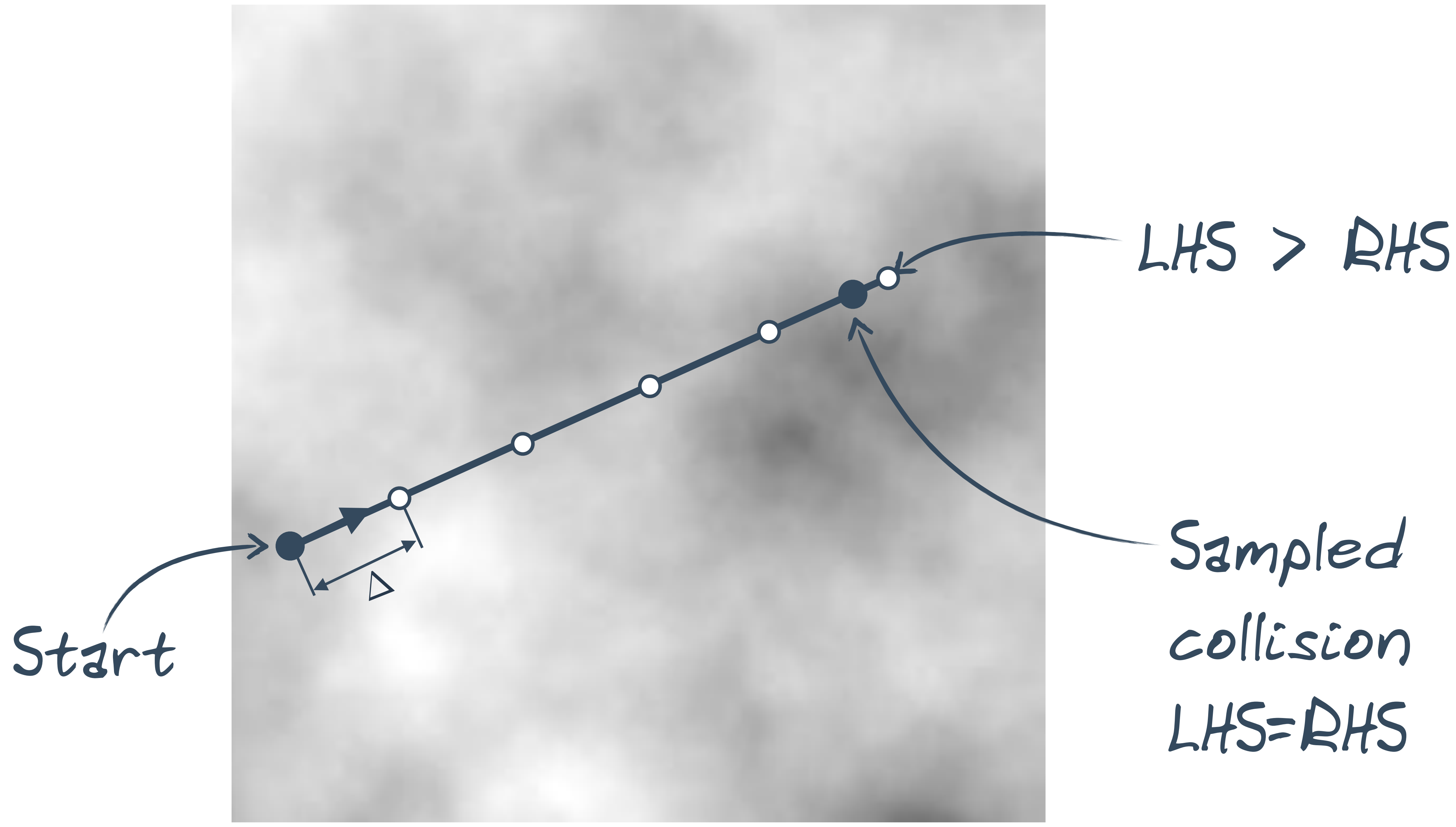
$$\sum_{i=1}^{k} \mu_{t,i}\,\Delta = -\ln(1-\xi)$$

Constant step

Ray marching:

1) Draw a random number ξ
2) While LHS < RHS
   make a (fixed-size) step
3) Find the exact location
   in the last segment analytically

General volume



RAY MARCHING

Find the collision distance approximately

$$\int_0^t \mu_{\mathrm{t}}(s)\mathrm{d}s = -\ln(1-\xi)$$

$$\neq$$

$$\sum_{i=1}^{k} \mu_{\mathrm{t},i}\,\Delta = -\ln(1-\xi)$$

Constant step

Ray marching:

1) Draw a random number ξ
2) While LHS < RHS
   make a (fixed-size) step
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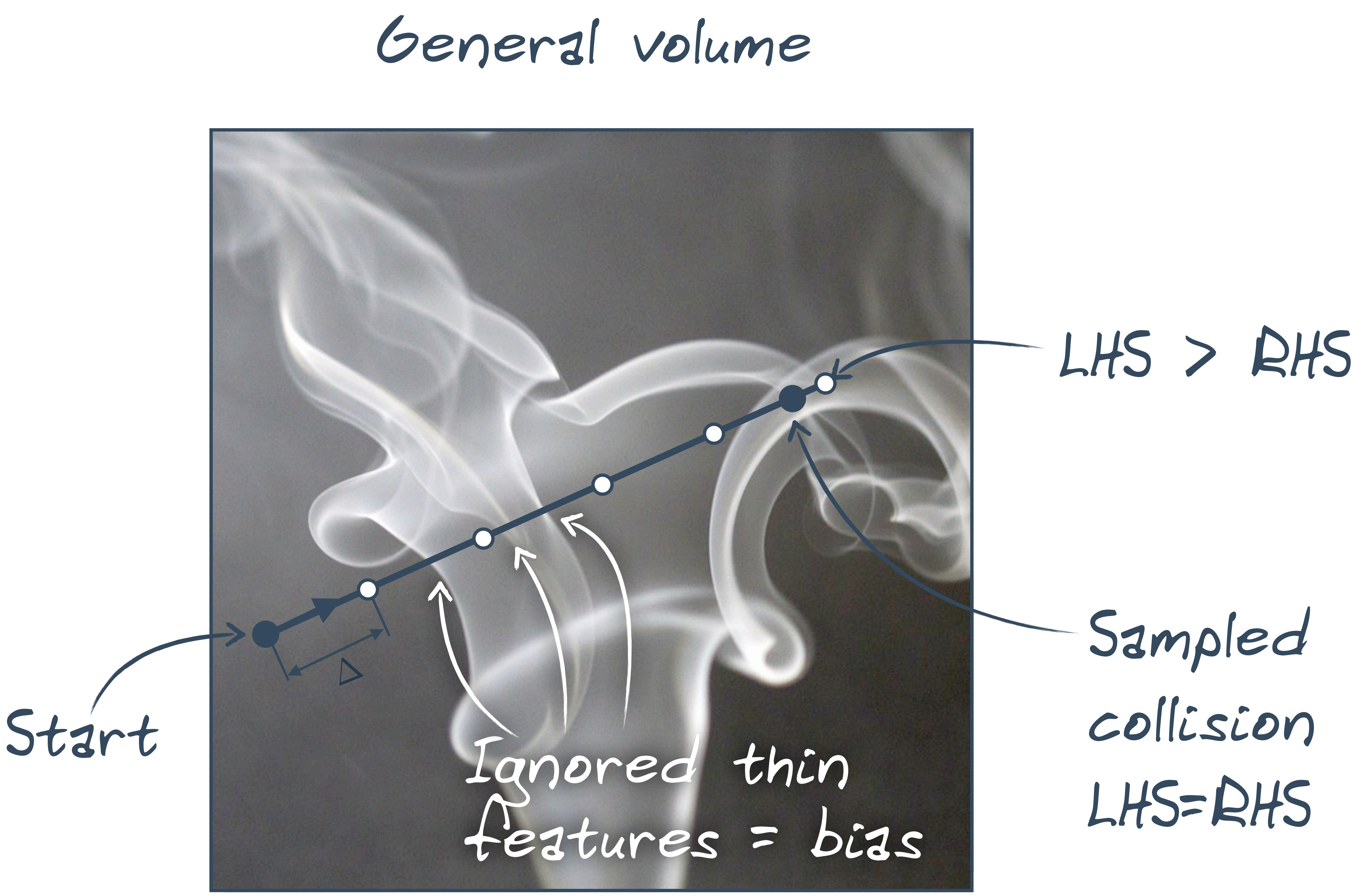
   in the last segment analytically



FREE-PATH SAMPLING

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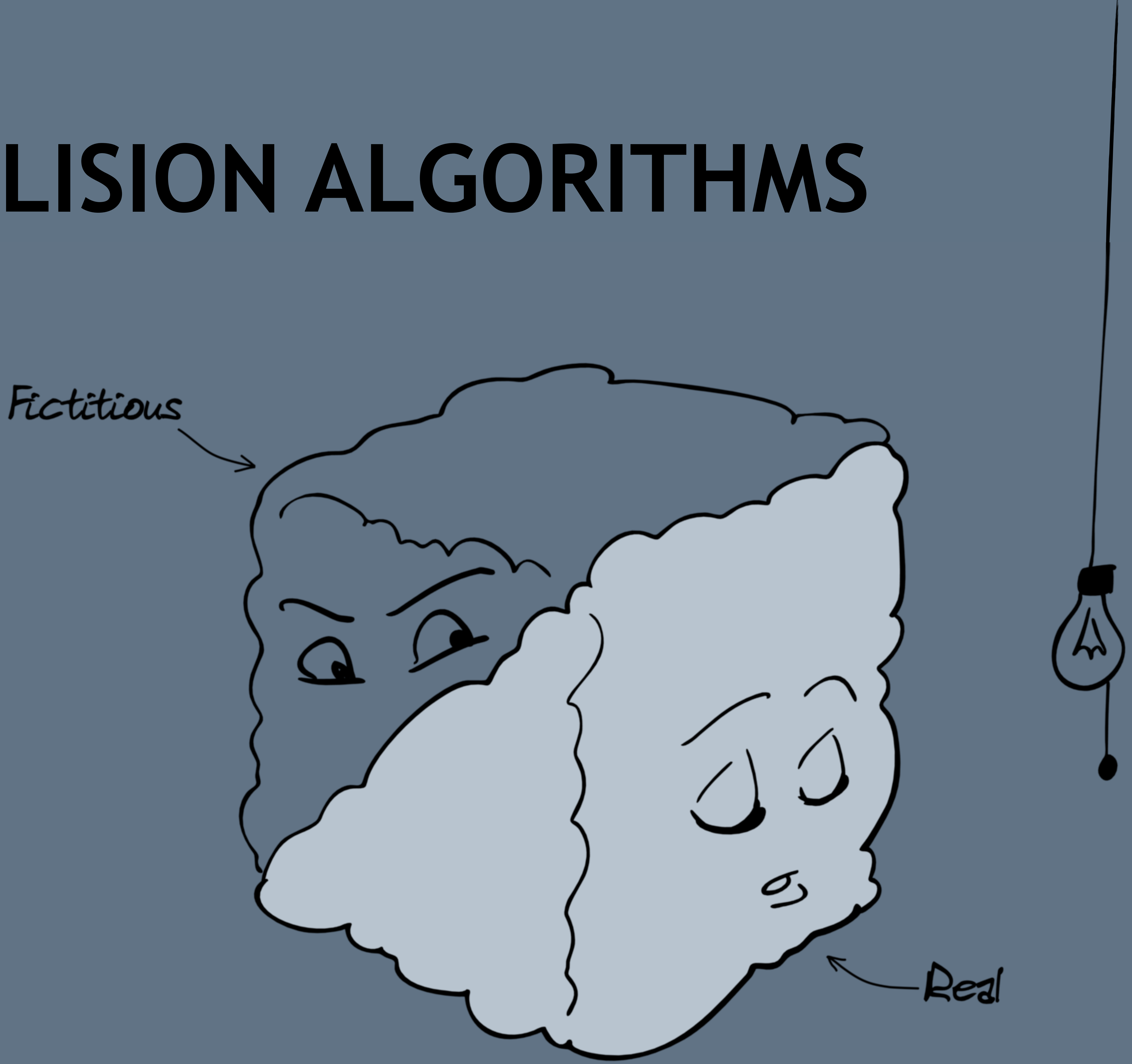
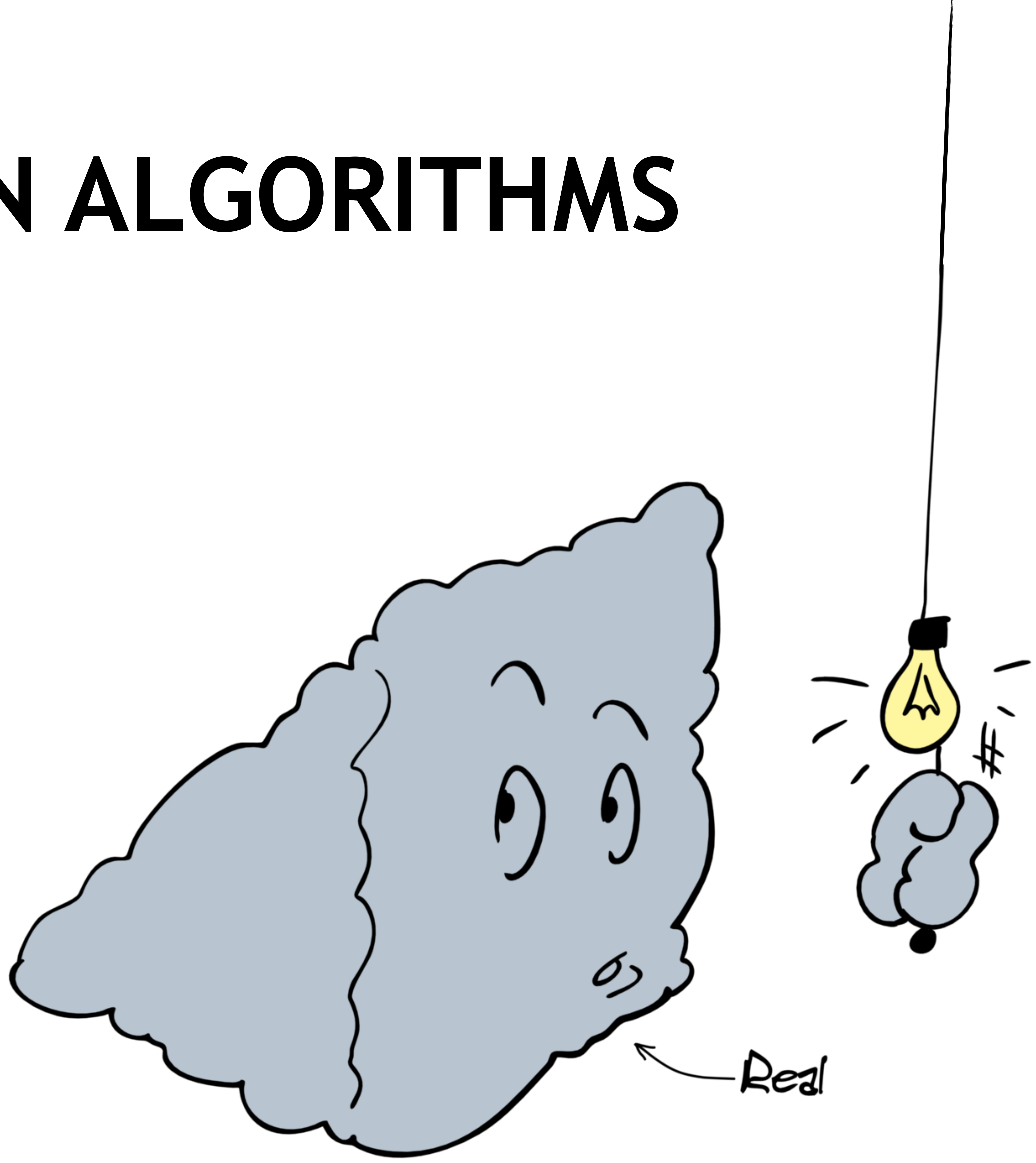
ANALYTIC CDF^{-1}	REGULAR TRACKING	RAY MARCHING
Efficient & simple, limited to few volumes	Iterative, inefficient if free paths cross many boundaries	Iterative, inaccurate (or inefficient) for media with high frequencies
Simple volumes (e.g. homogeneous)	Piecewise-simple volumes	Any volume
Unbiased	Unbiased	Biased

Common approach: sample optical thickness, find corresponding distance

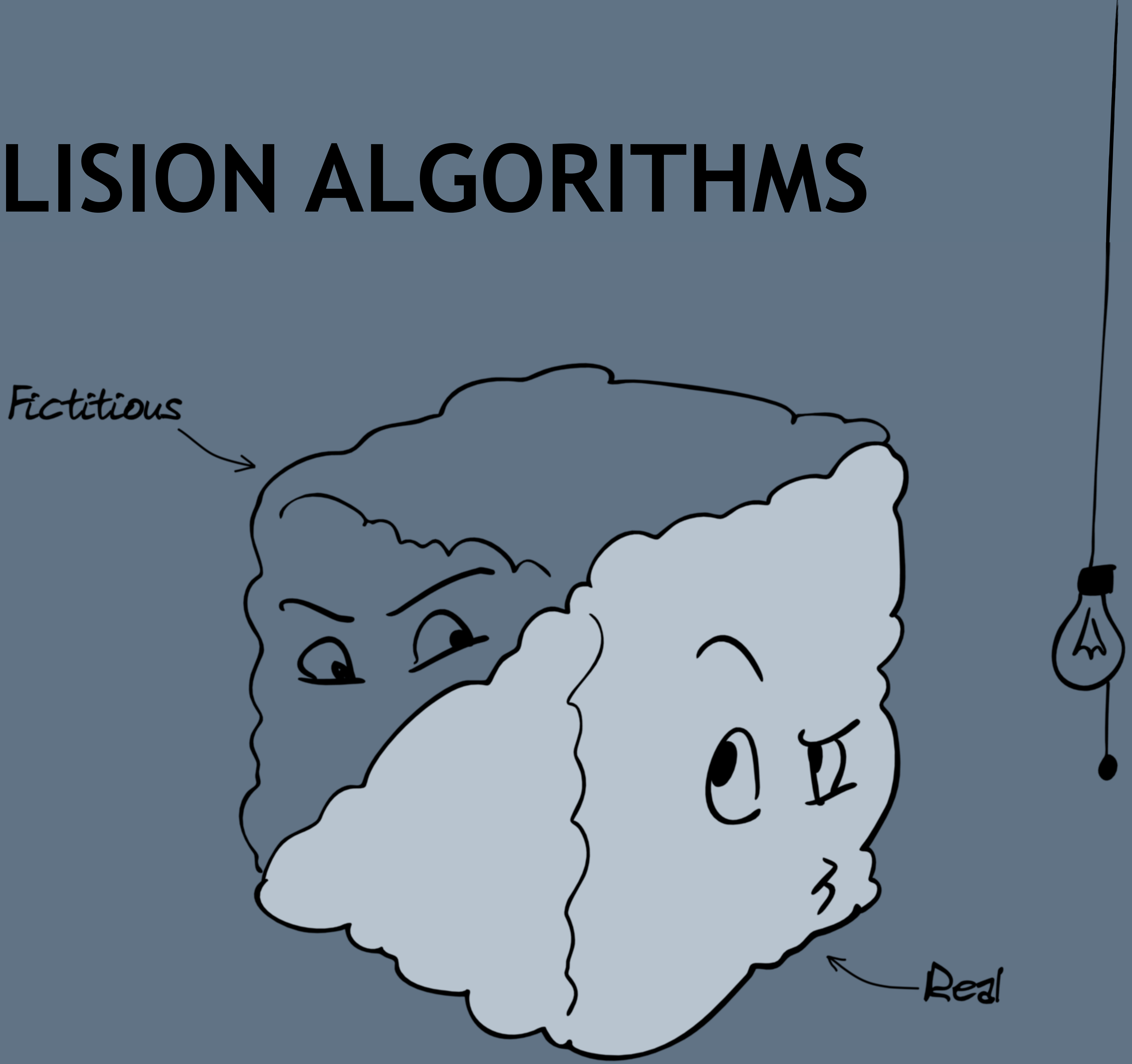
NULL-COLLISION ALGORITHMS



NULL-COLLISION ALGORITHMS



NULL-COLLISION ALGORITHMS



NULL-COLLISION ALGORITHMS

Origins in neutron transport and plasma physics, unbiased sampling

Applied in rendering since 2008 [Raab et al. 2008]

FREE-PATH sampling:

- **Delta tracking** (a.k.a Woodcock tracking)
- **Weighted delta tracking**
- **Decomposition tracking**
- Spectral tracking

Discussed by Jo later

TRANSMITTANCE estimation:

- Delta tracking
- (Residual) ratio tracking
- Next-flight delta/ratio tracking

Discussed together w/ other transmittance estimators

DELTA TRACKING

WEIGHTED (DELTA) TRACKING

DECOMPOSITION TRACKING

DELTA TRACKING

a.k.a. Woodcock tracking, pseudo scattering, hole tracking, null-collision method,…

PHYSICALLY-BASED interpretation

- Correctness motivated by intuitive physical arguments:
  Butcher and Messel [1958, 1960],
  Zerby et al. [1961], Bertini [1963],
  Woodcock et al. [1965], Skullerud [1968],
  …

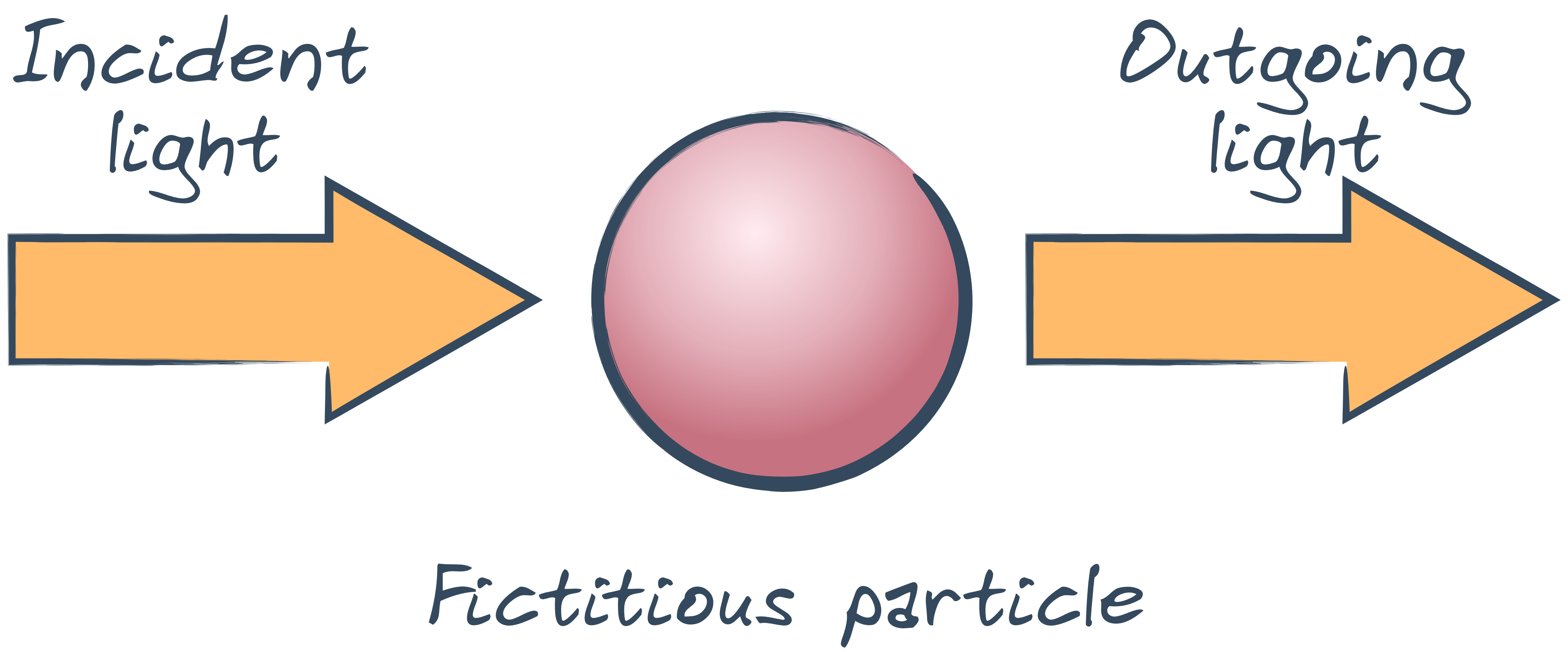
MATHEMATICAL formalisms

- Proofs: Miller [1967], Coleman [1968]
- Integral formulation: Galtier et al. [2013]

PHYSICAL INTERPRETATION

Add **FICTITIOUS MATTER** to homogenize heterogeneous extinction

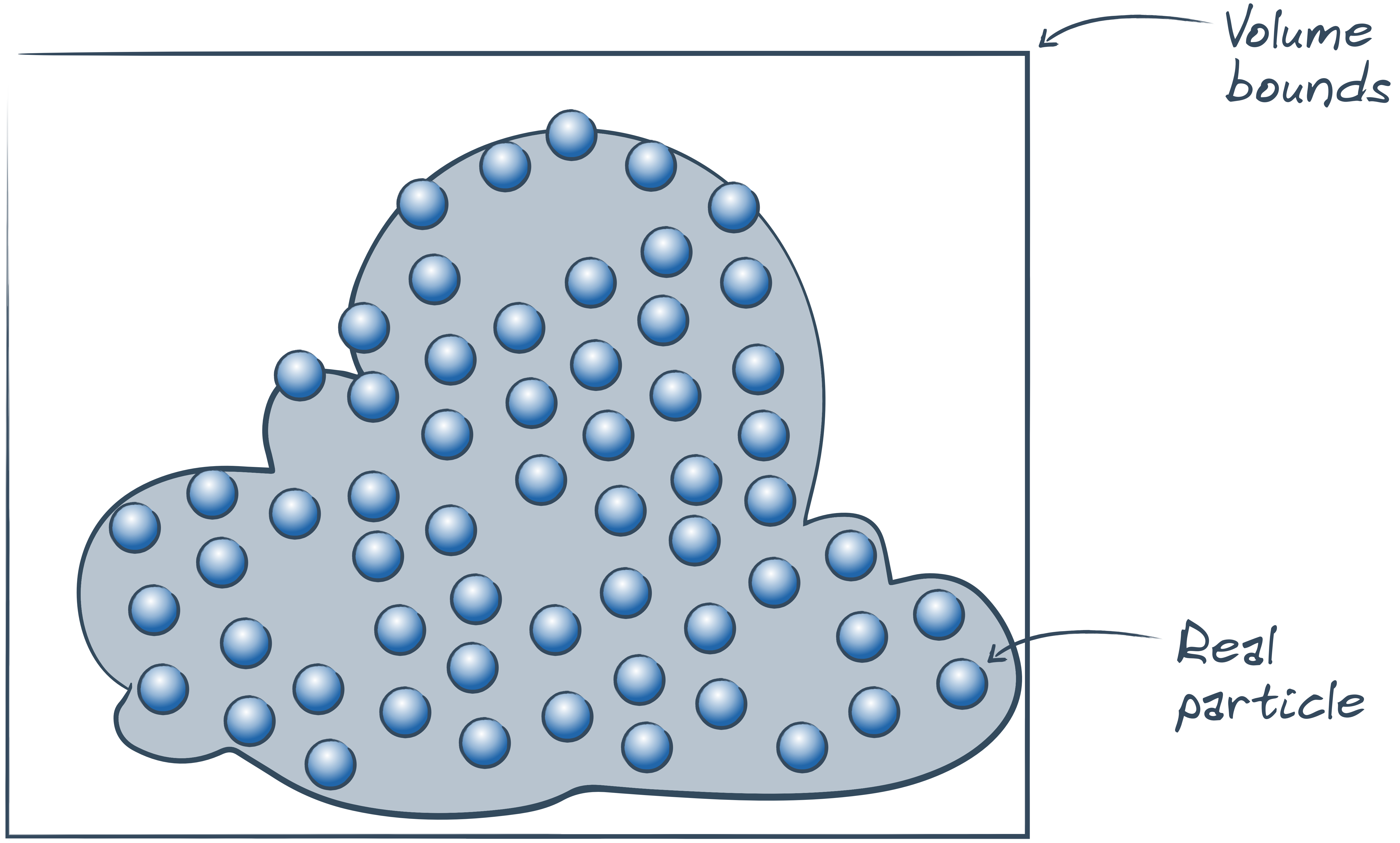
- albedo $\alpha(\mathbf{x}) = 1$
- phase function $f_{\mathrm{p}}(\omega, \bar{\omega}) = \delta(\omega - \bar{\omega})$



Presence of fictitious matter
does not impact light transport

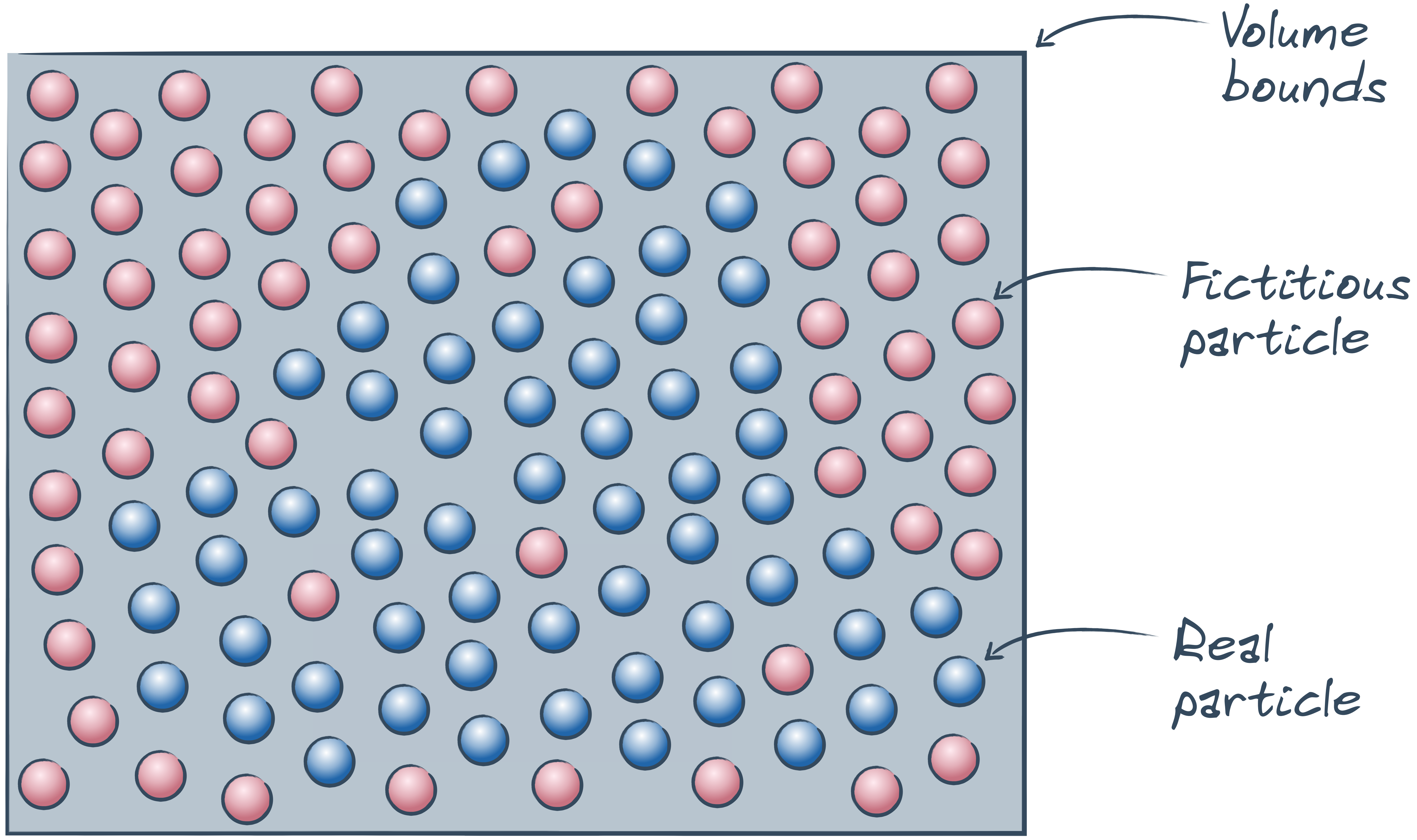
PHYSICAL INTERPRETATION

HOMOGENIZATION



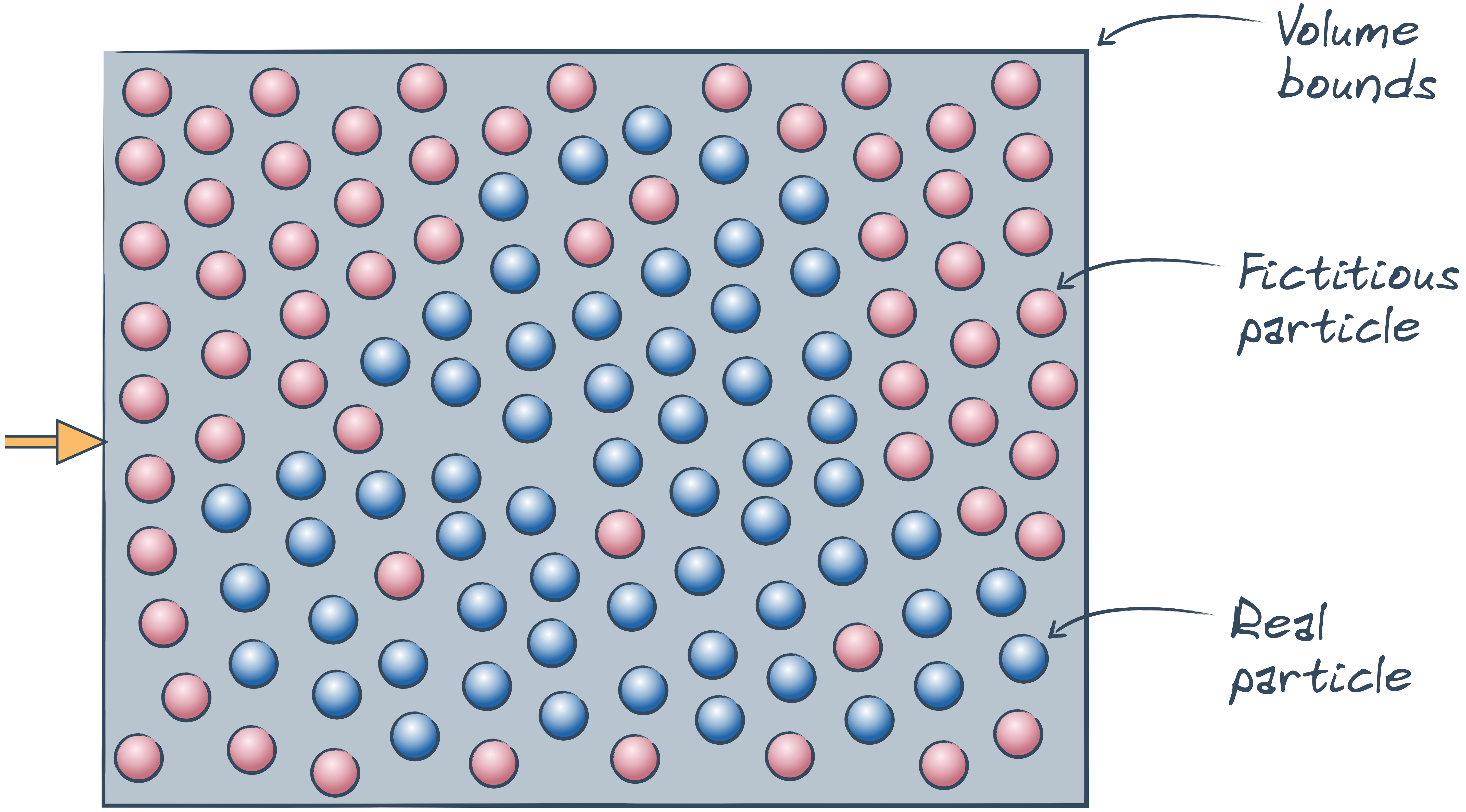
PHYSICAL INTERPRETATION

HOMOGENIZATION



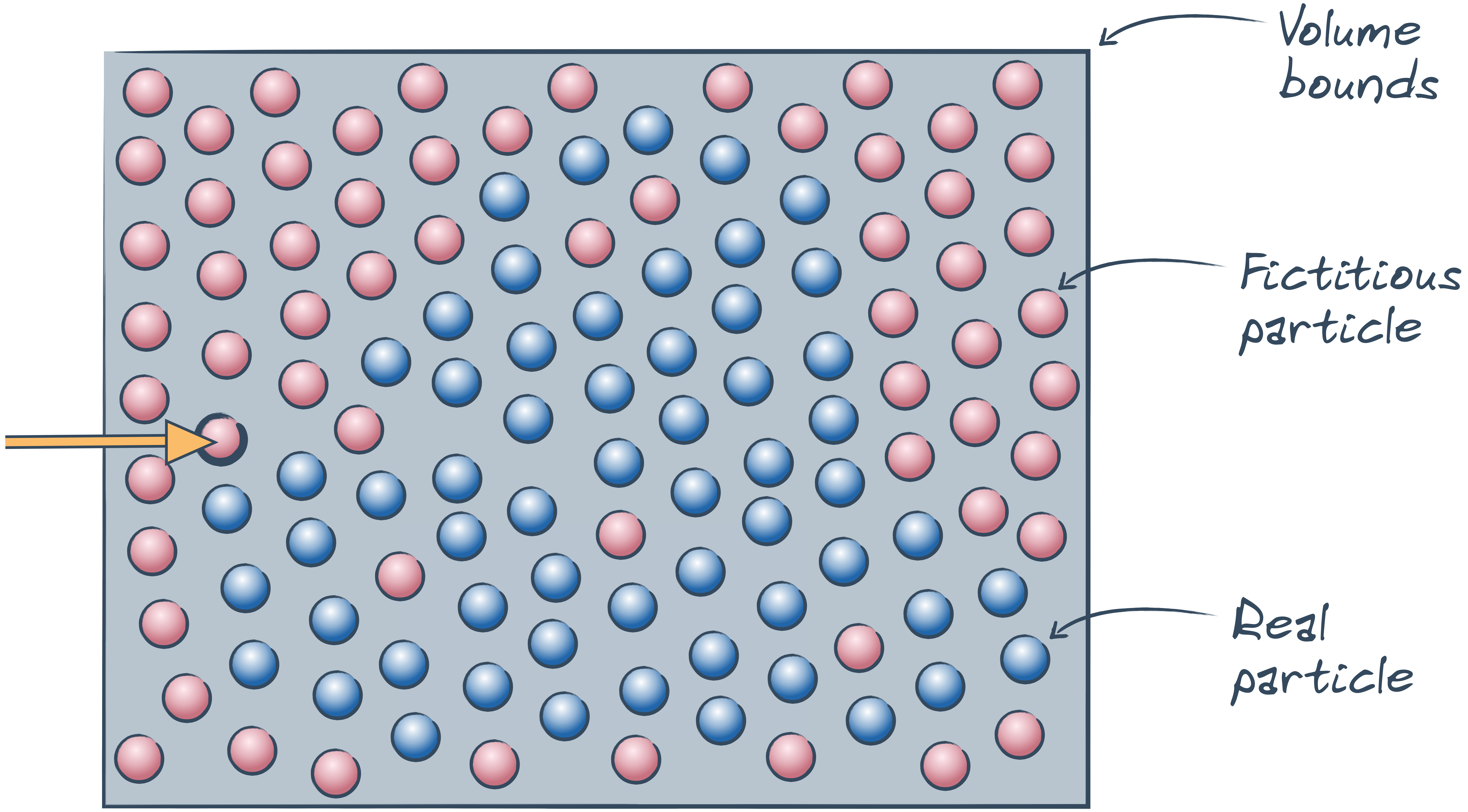
PHYSICAL INTERPRETATION

HOMOGENIZATION



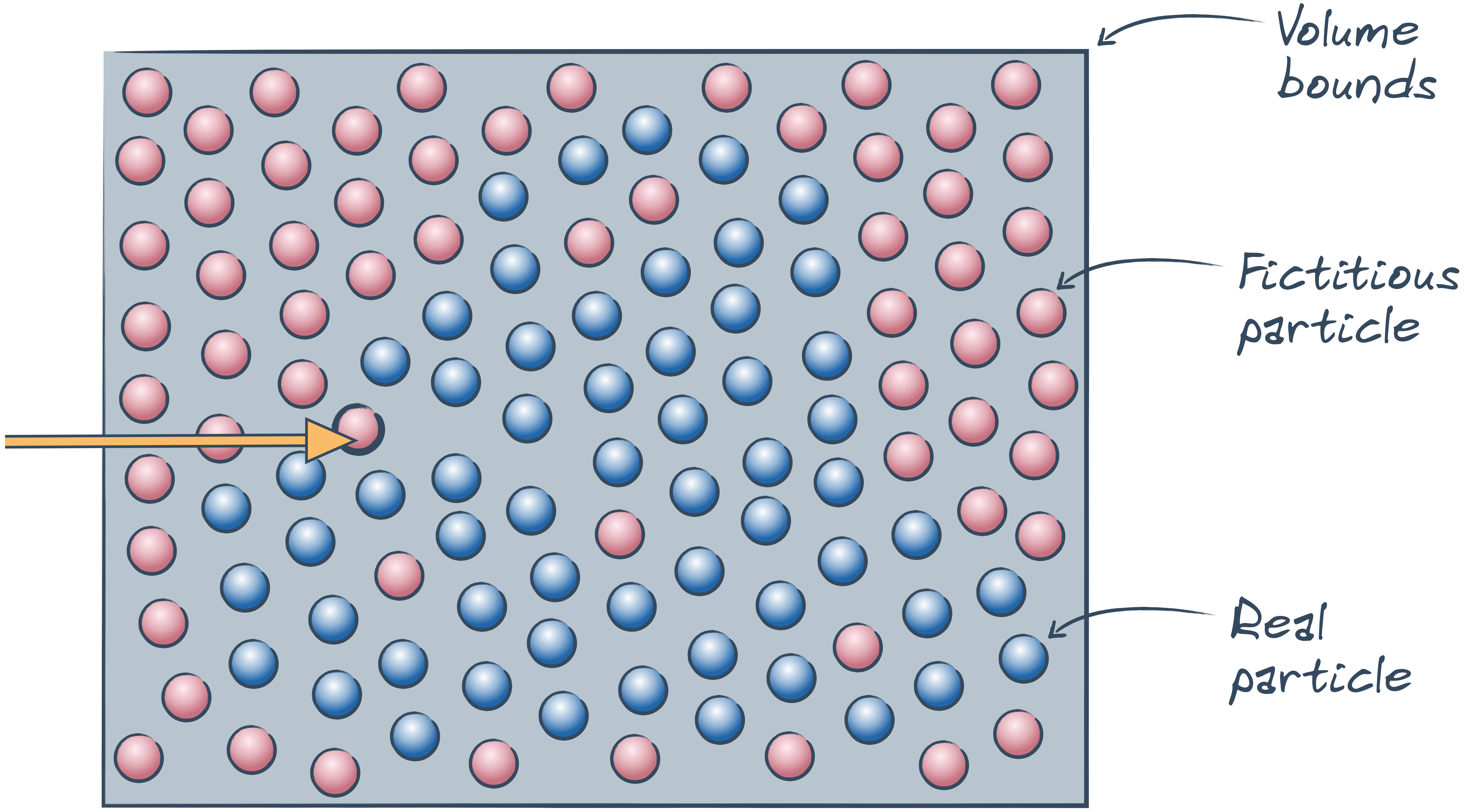
PHYSICAL INTERPRETATION

HOMOGENIZATION



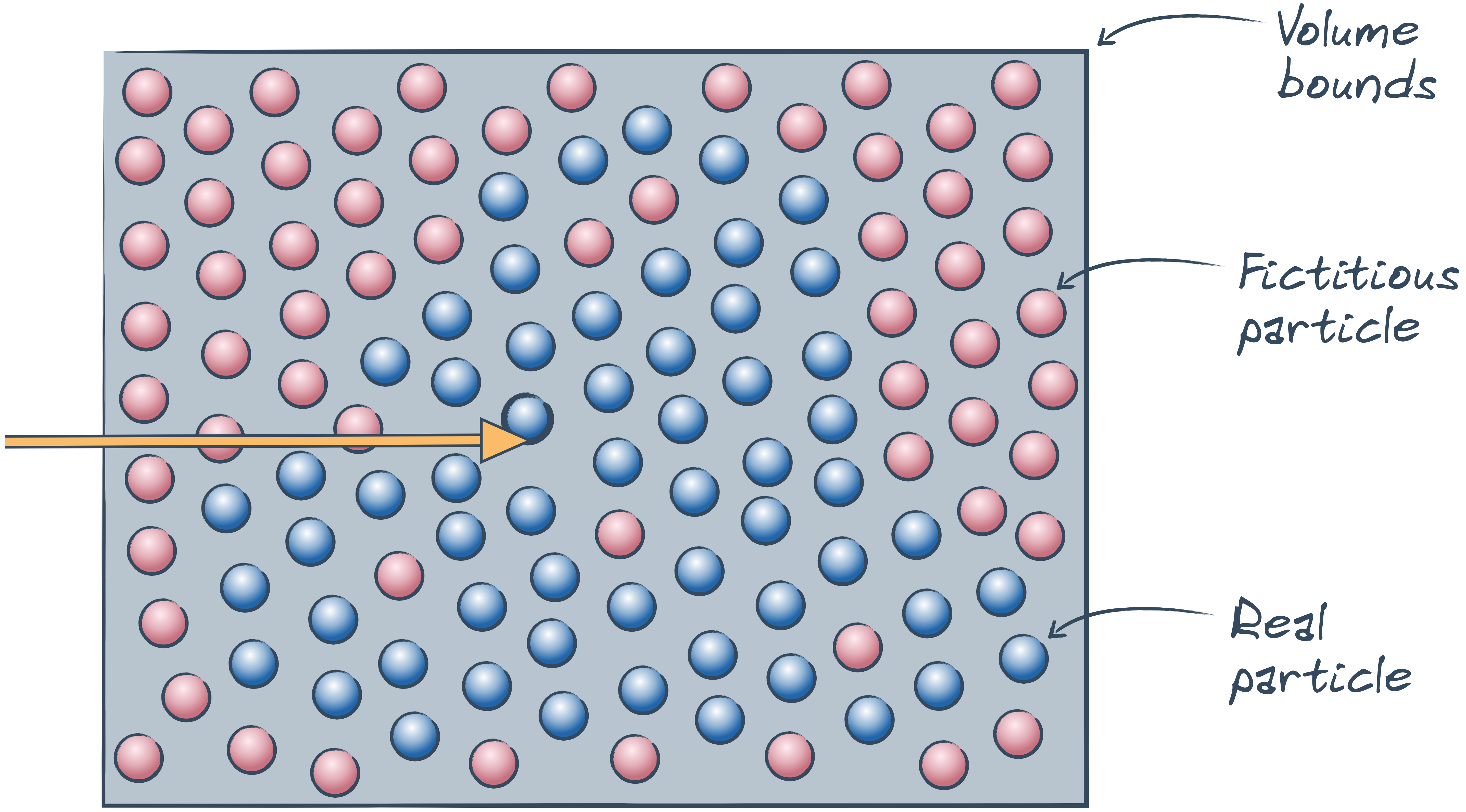
PHYSICAL INTERPRETATION

HOMOGENIZATION



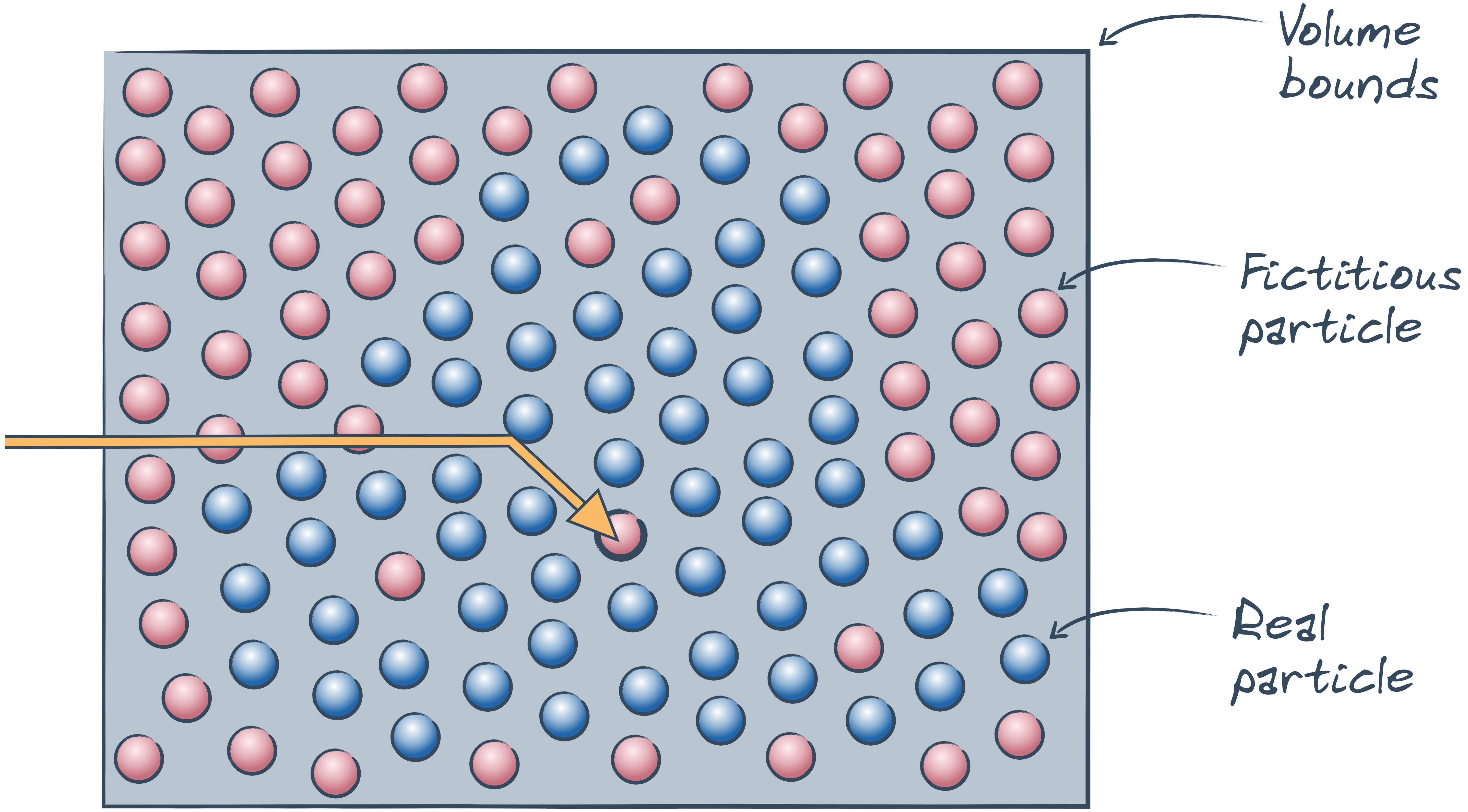
PHYSICAL INTERPRETATION

HOMOGENIZATION



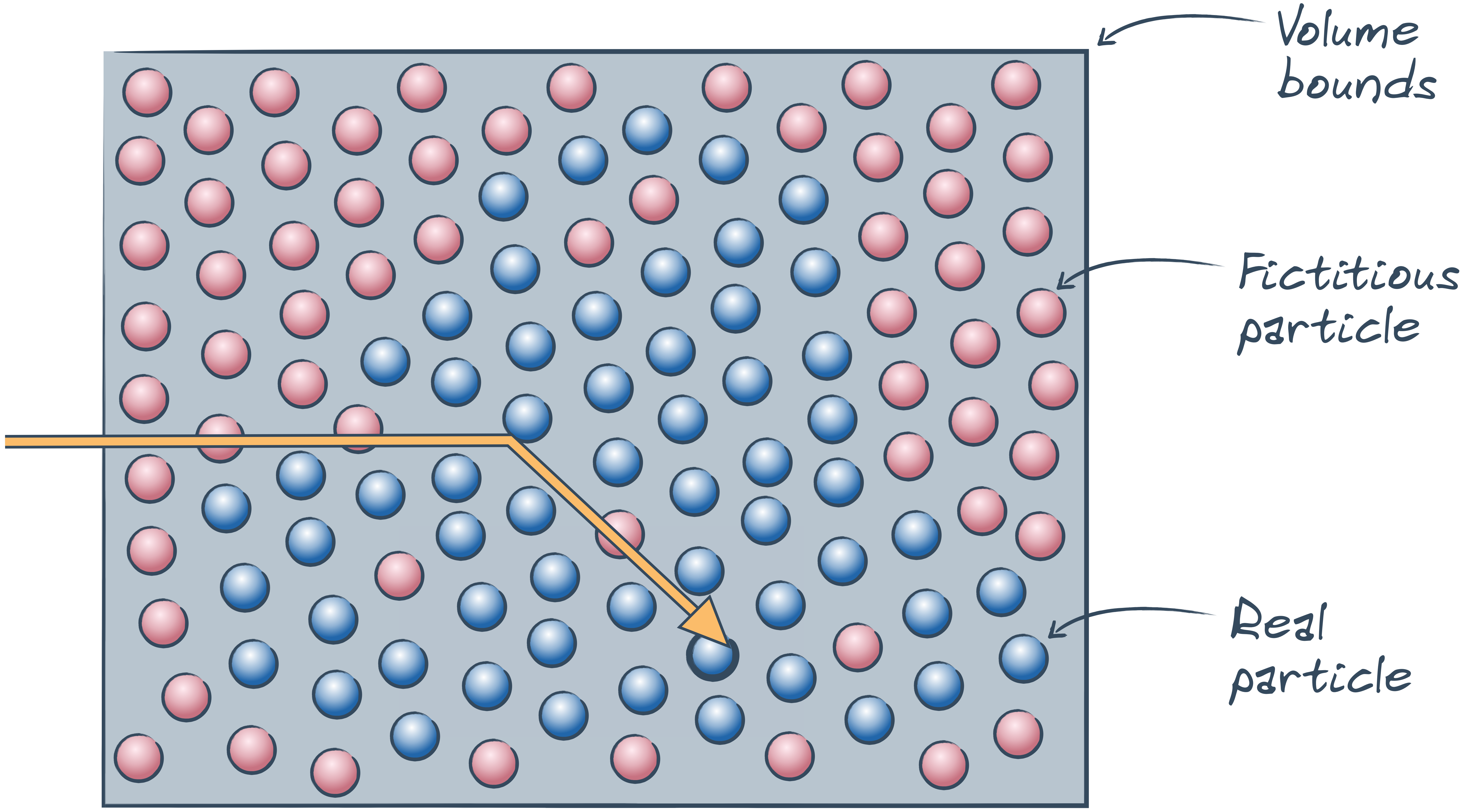
PHYSICAL INTERPRETATION

HOMOGENIZATION



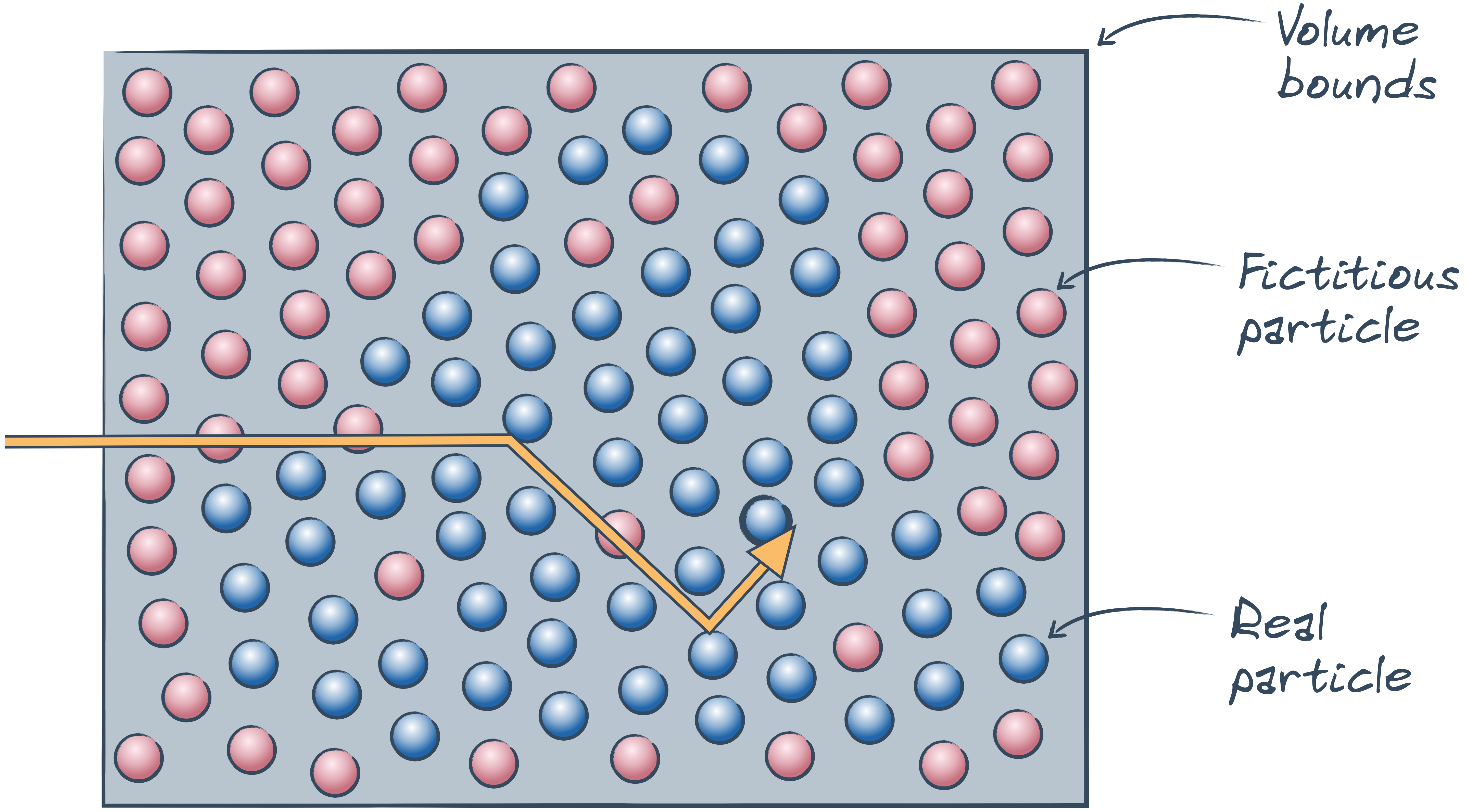
PHYSICAL INTERPRETATION

HOMOGENIZATION



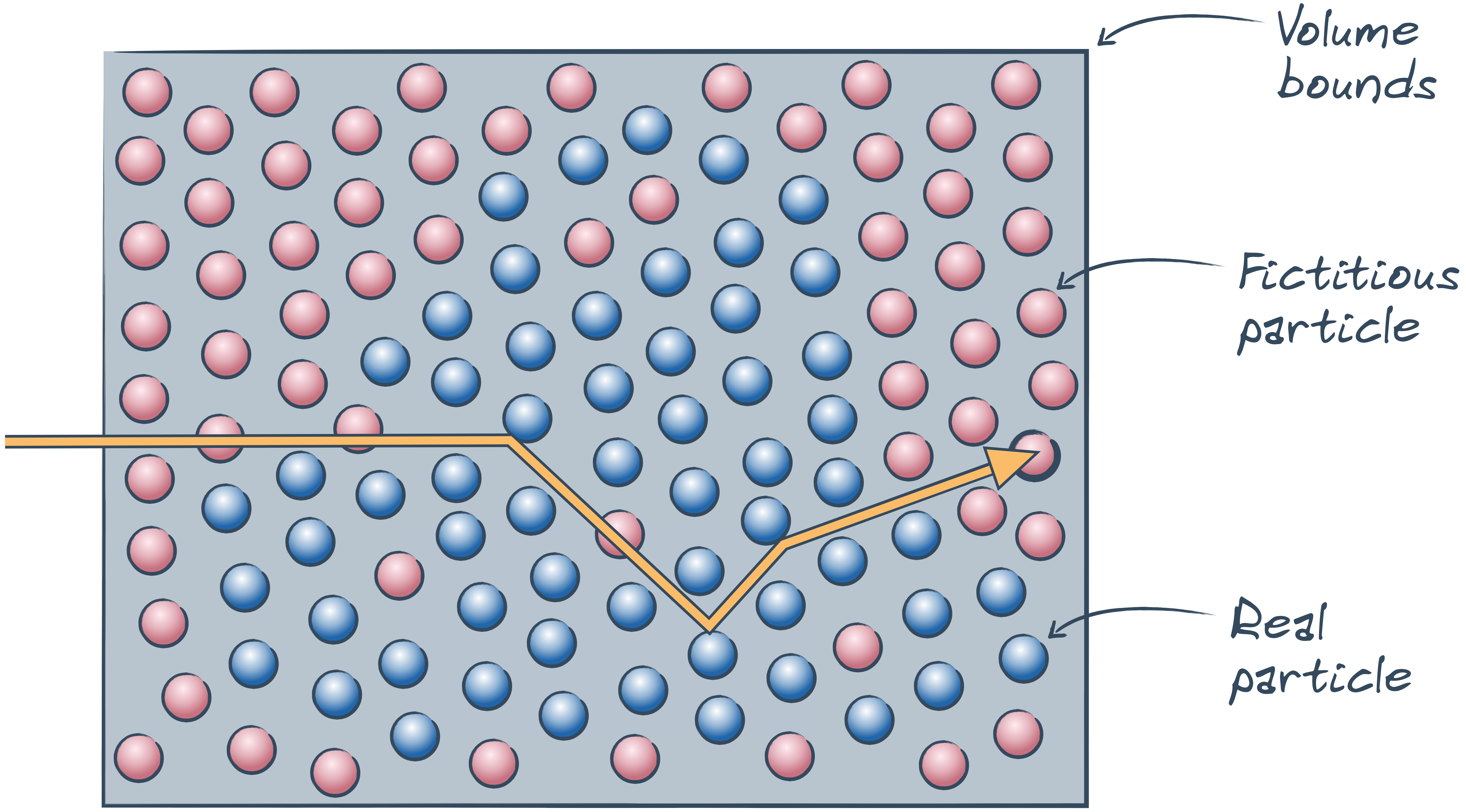
PHYSICAL INTERPRETATION

HOMOGENIZATION



PHYSICAL INTERPRETATION

HOMOGENIZATION



PHYSICAL INTERPRETATION

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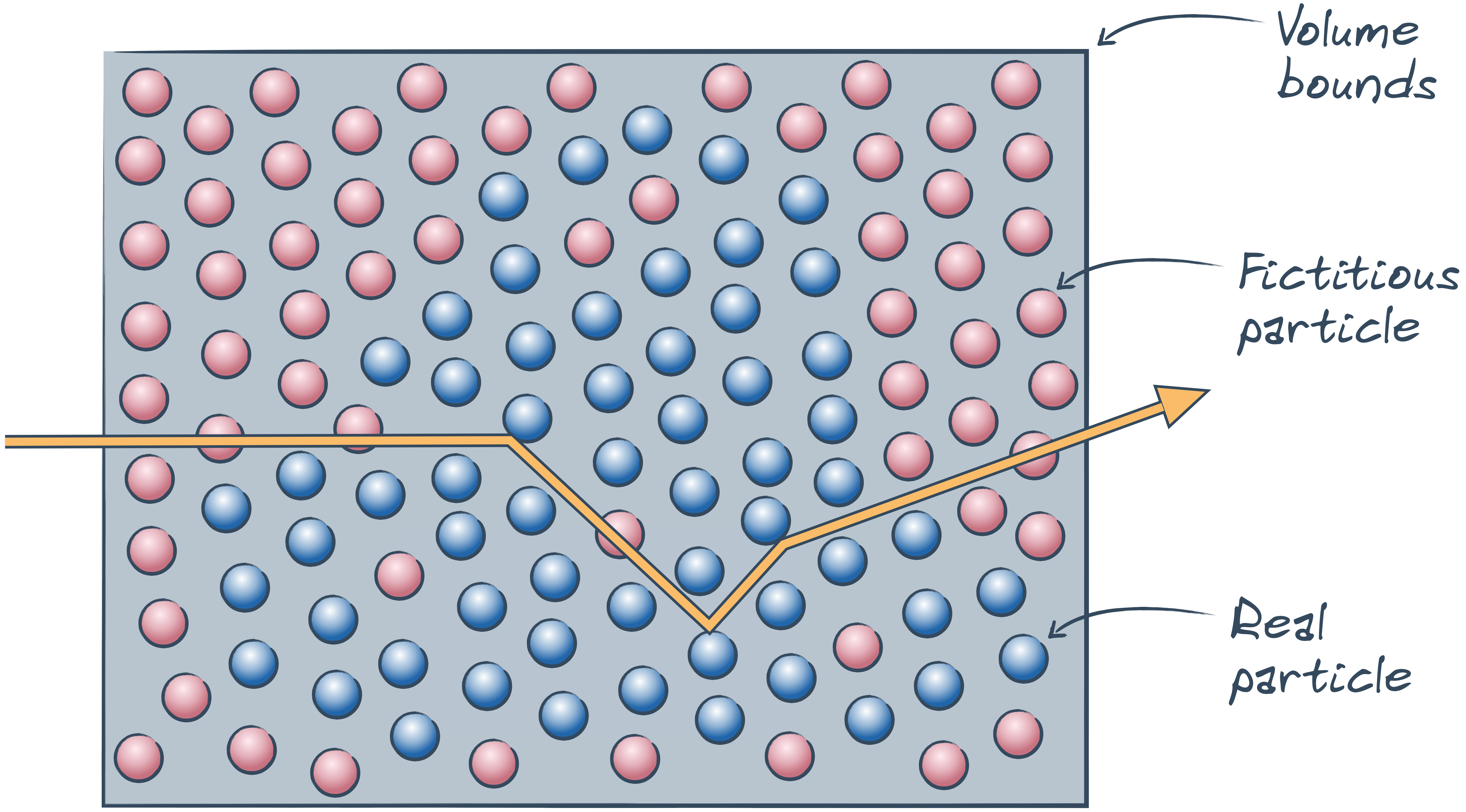
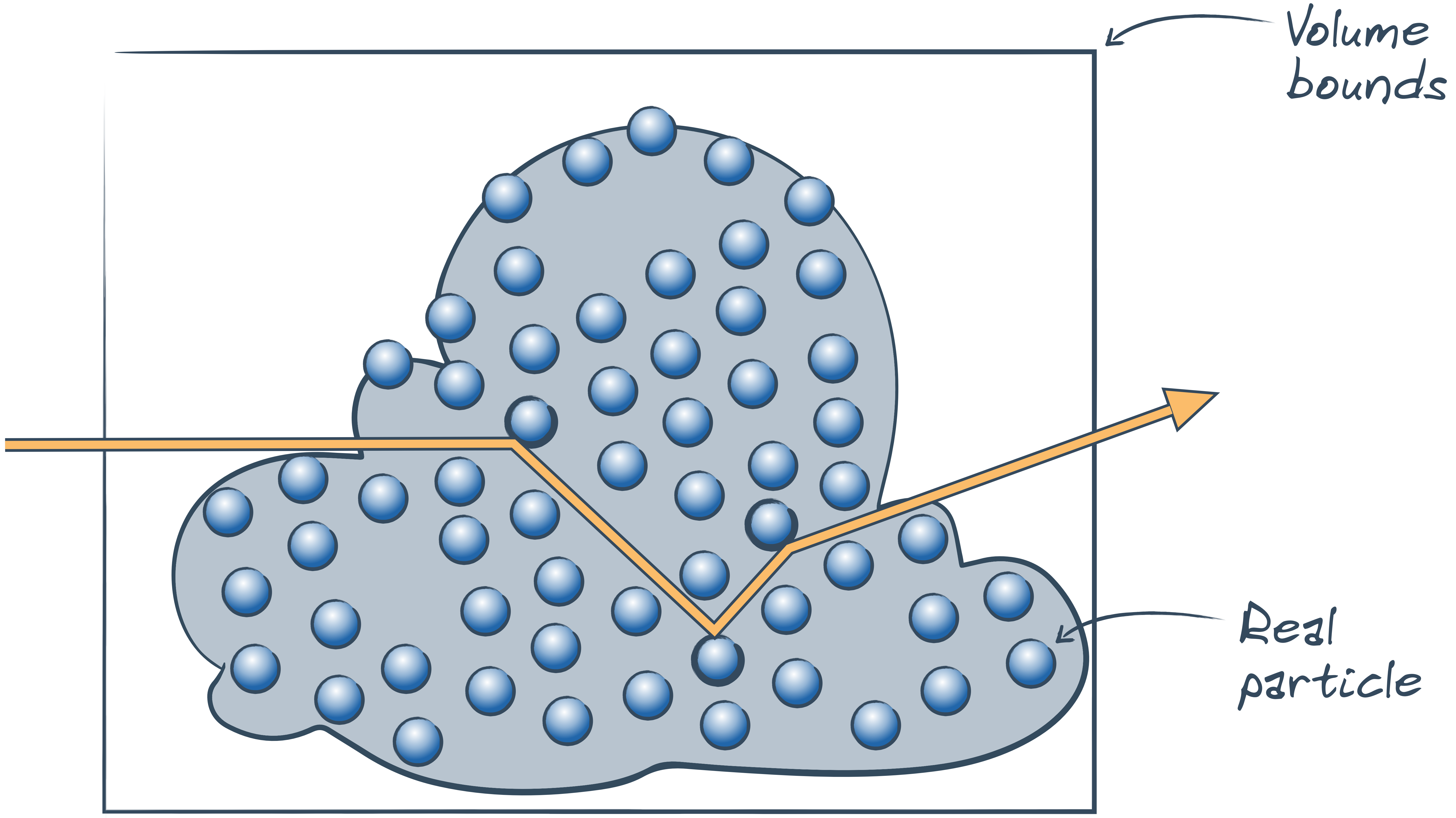
HOMOGENIZATION

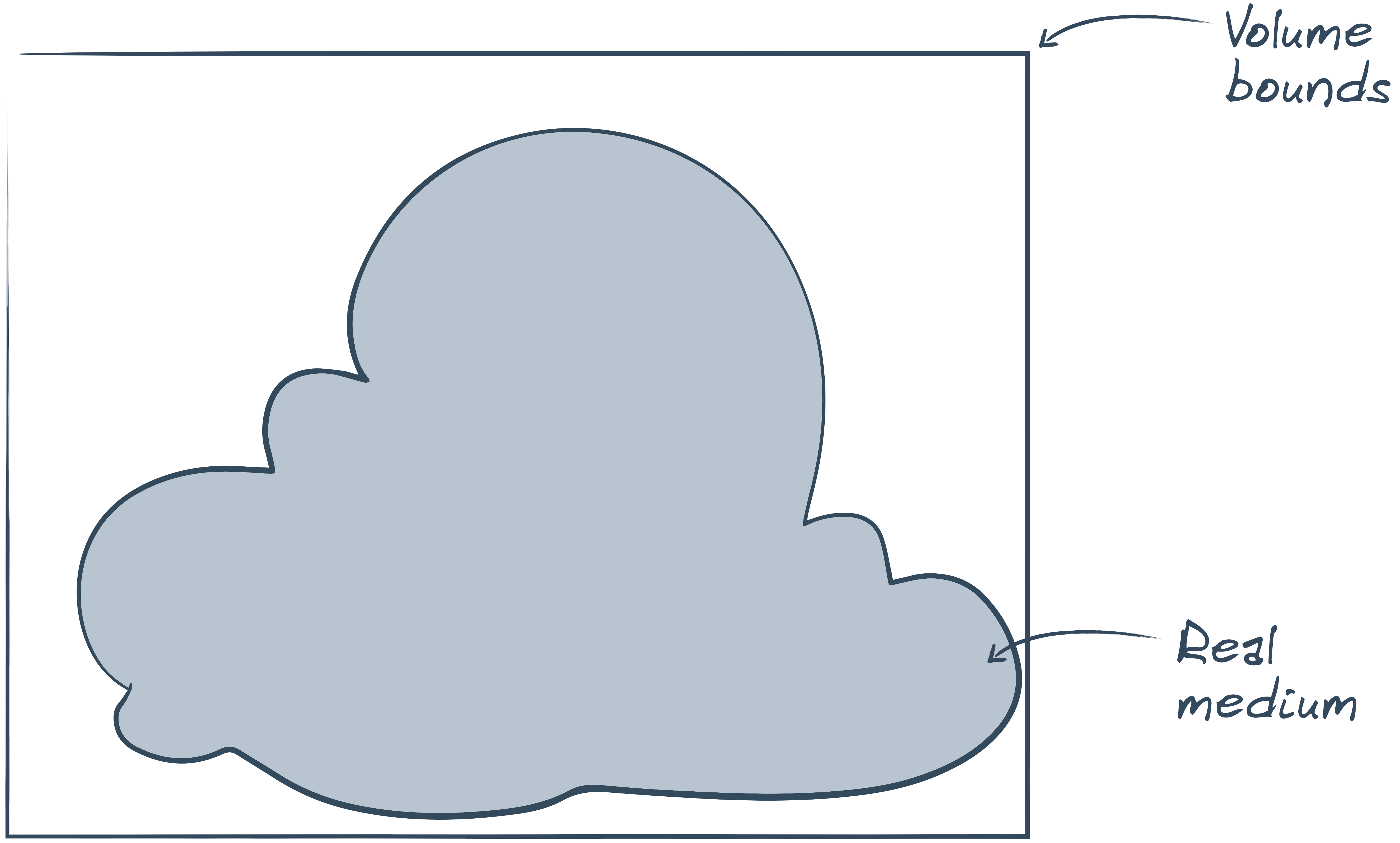


PHYSICAL INTERPRETATION

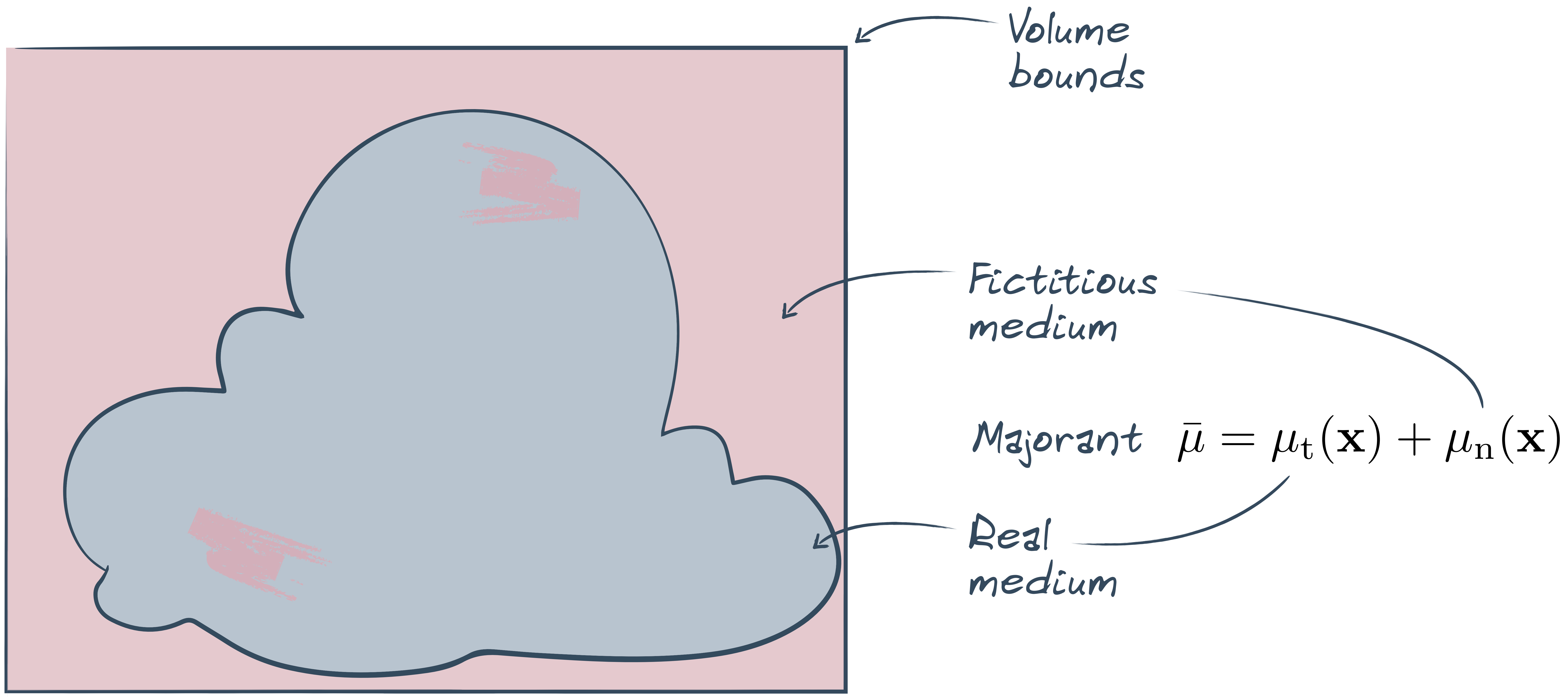
HOMOGENIZATION



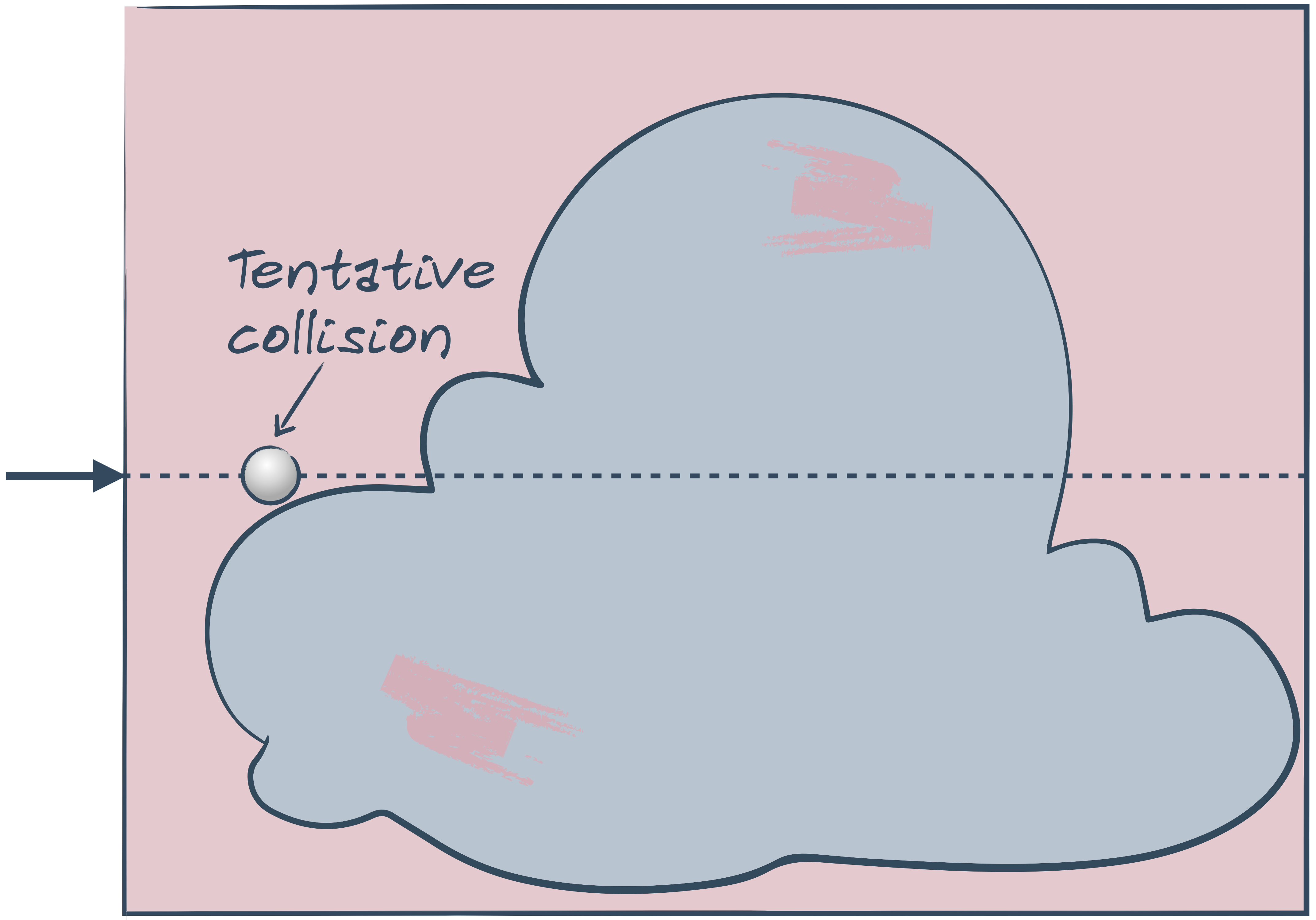
STOCHASTIC SAMPLING

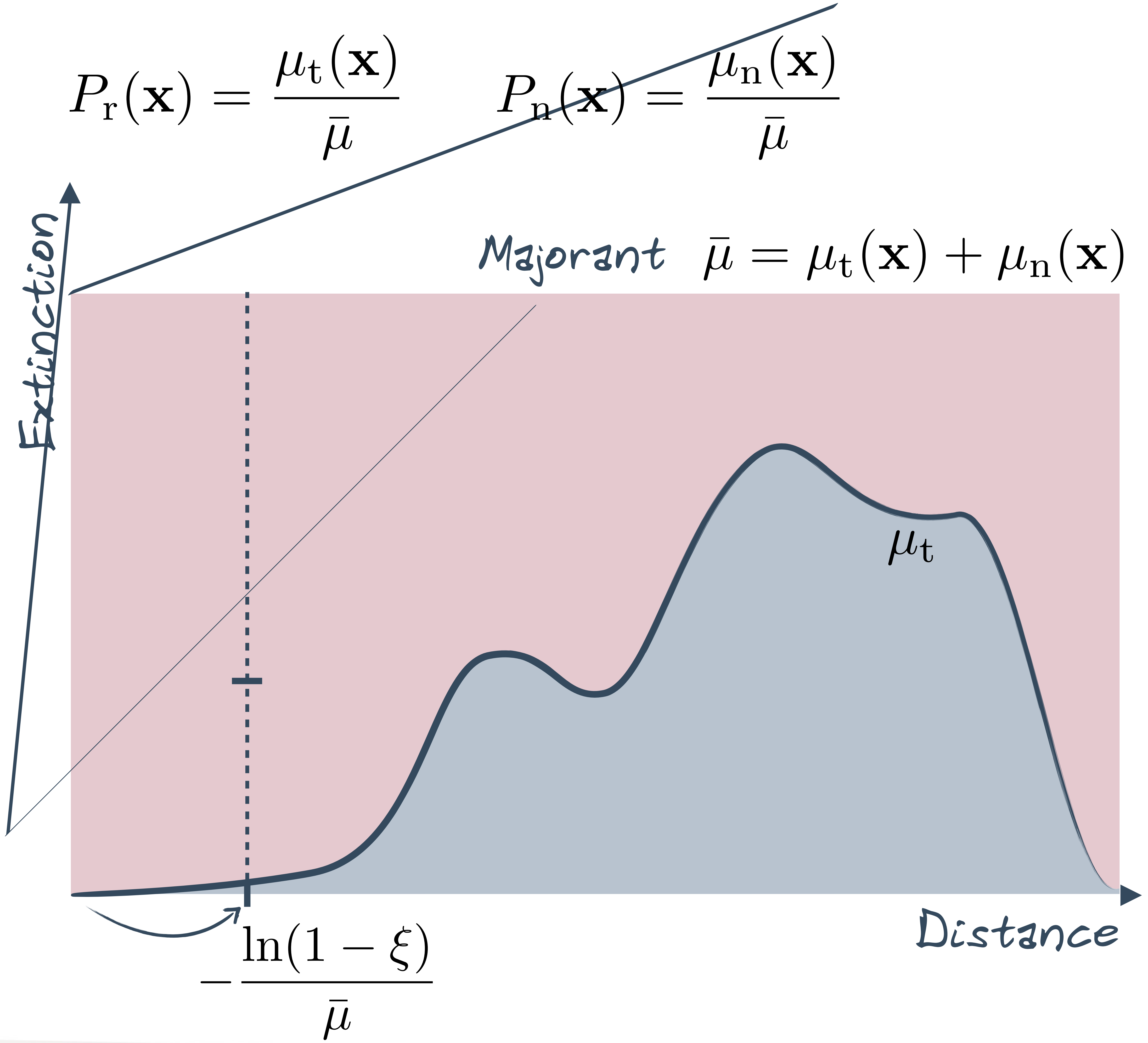


STOCHASTIC SAMPLING



STOCHASTIC SAMPLING





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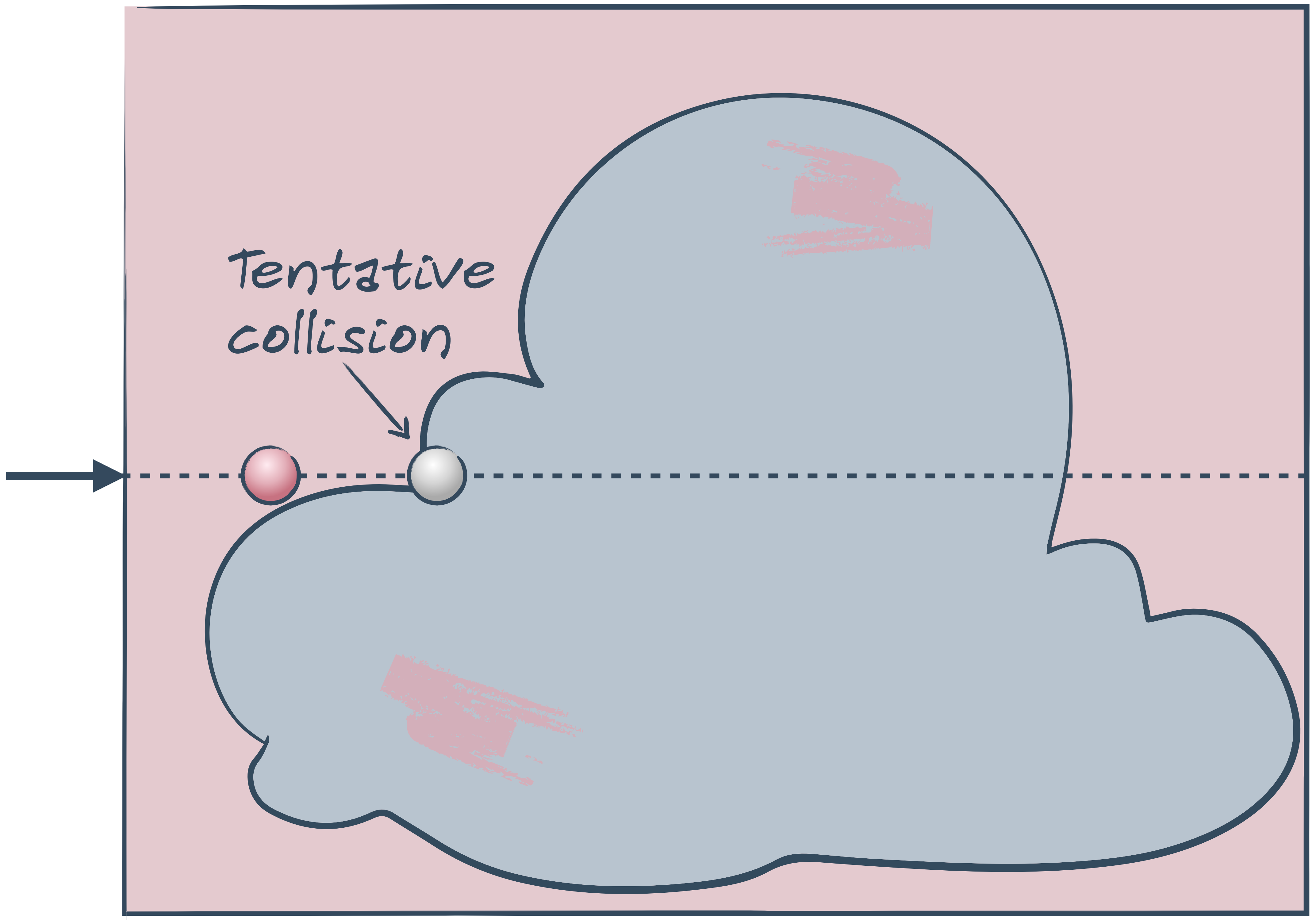
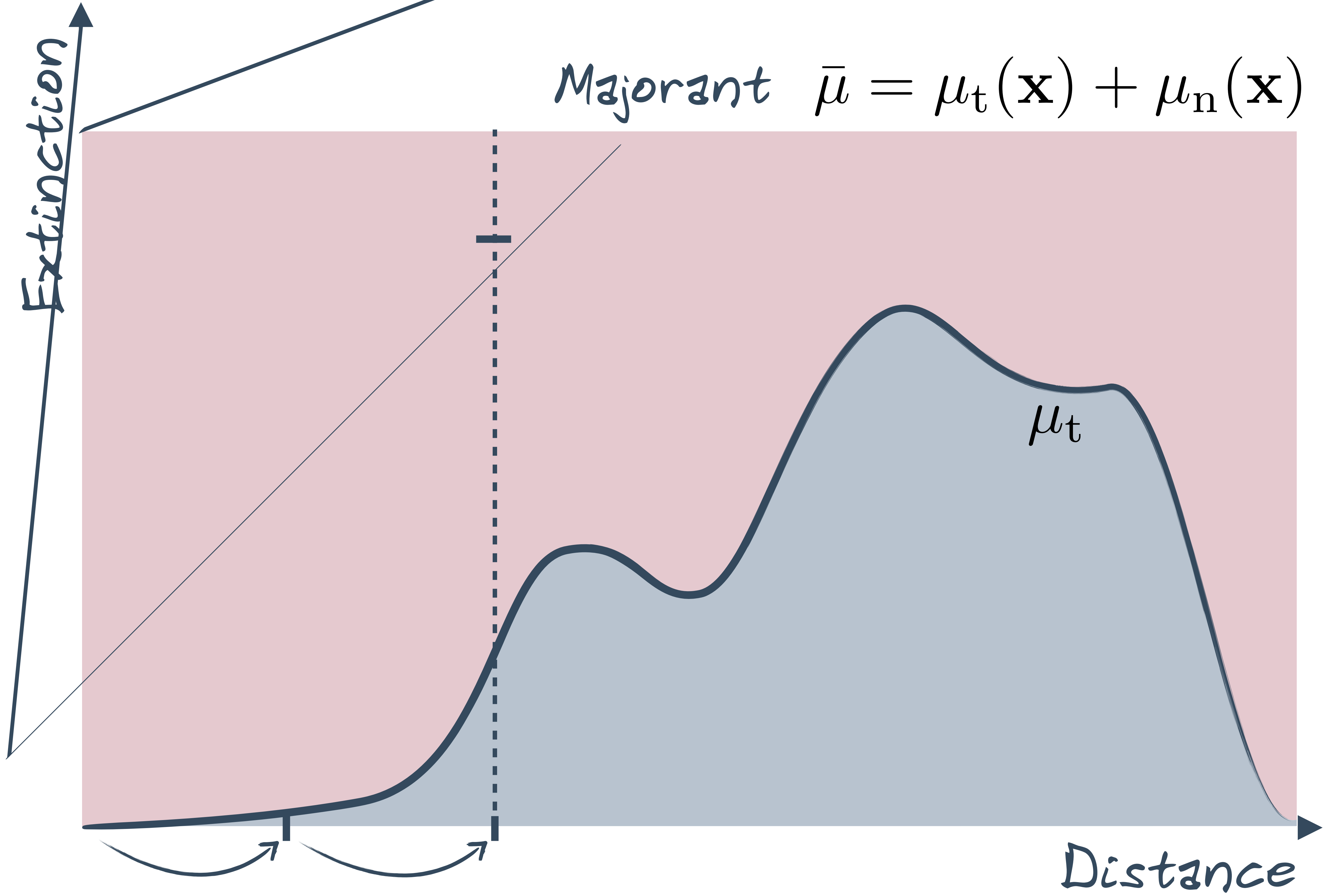


$$P_{\mathrm{r}}(\mathbf{x}) = \frac{\mu_{\mathrm{t}}(\mathbf{x})}{\bar{\mu}} \qquad P_{\mathrm{n}}(\mathbf{x}) = \frac{\mu_{\mathrm{n}}(\mathbf{x})}{\bar{\mu}}$$



STOCHASTIC SAMPLING

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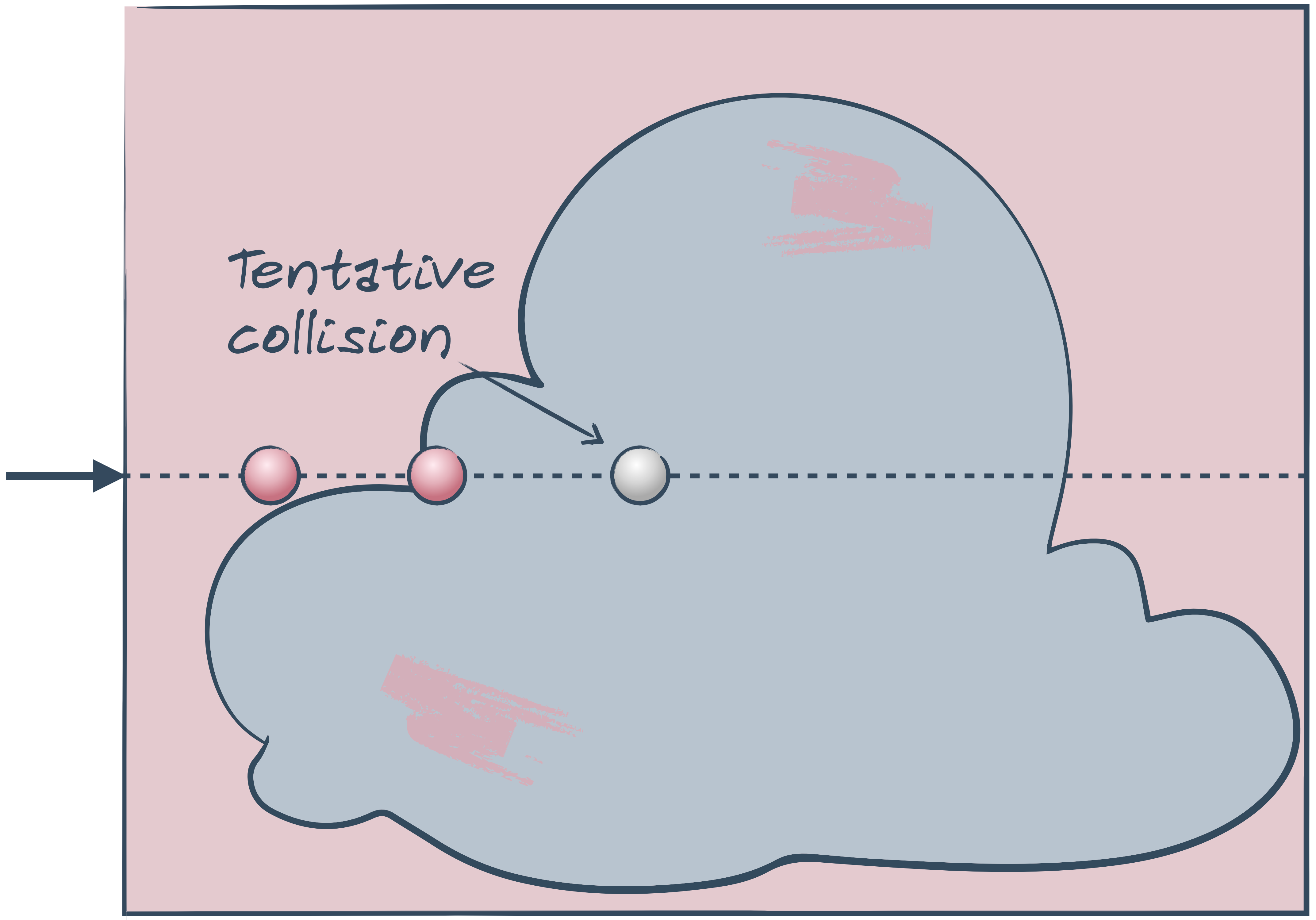
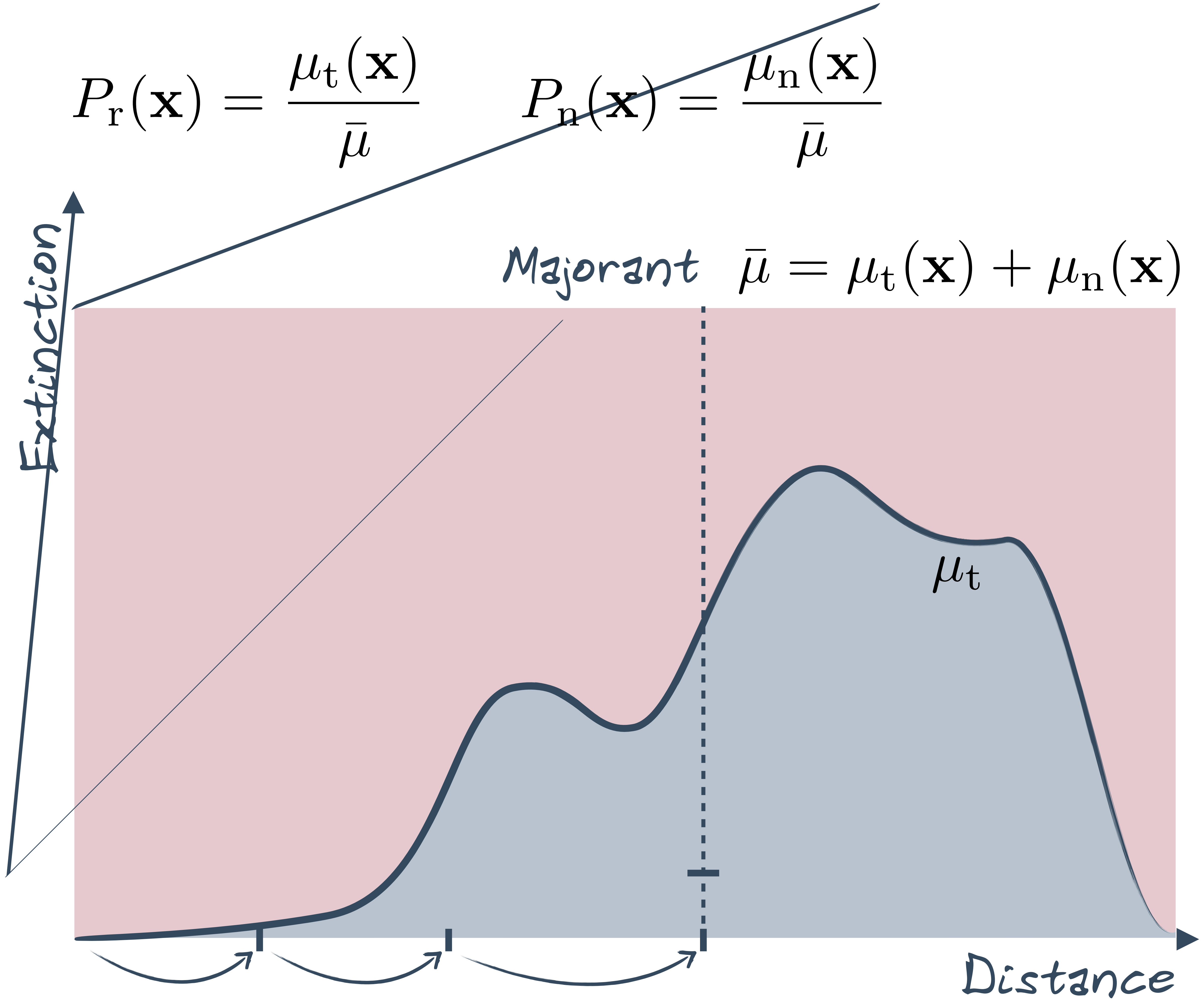
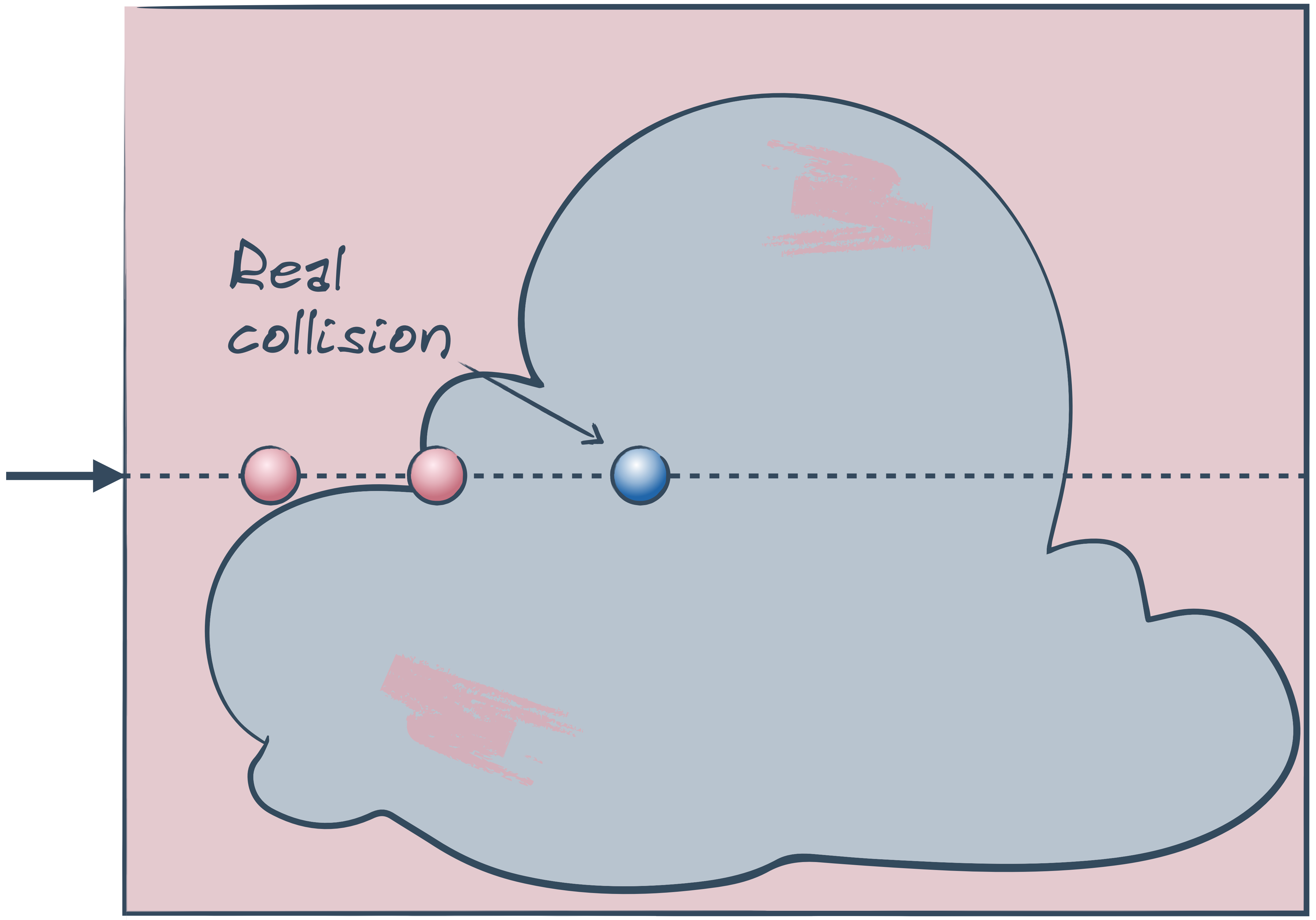
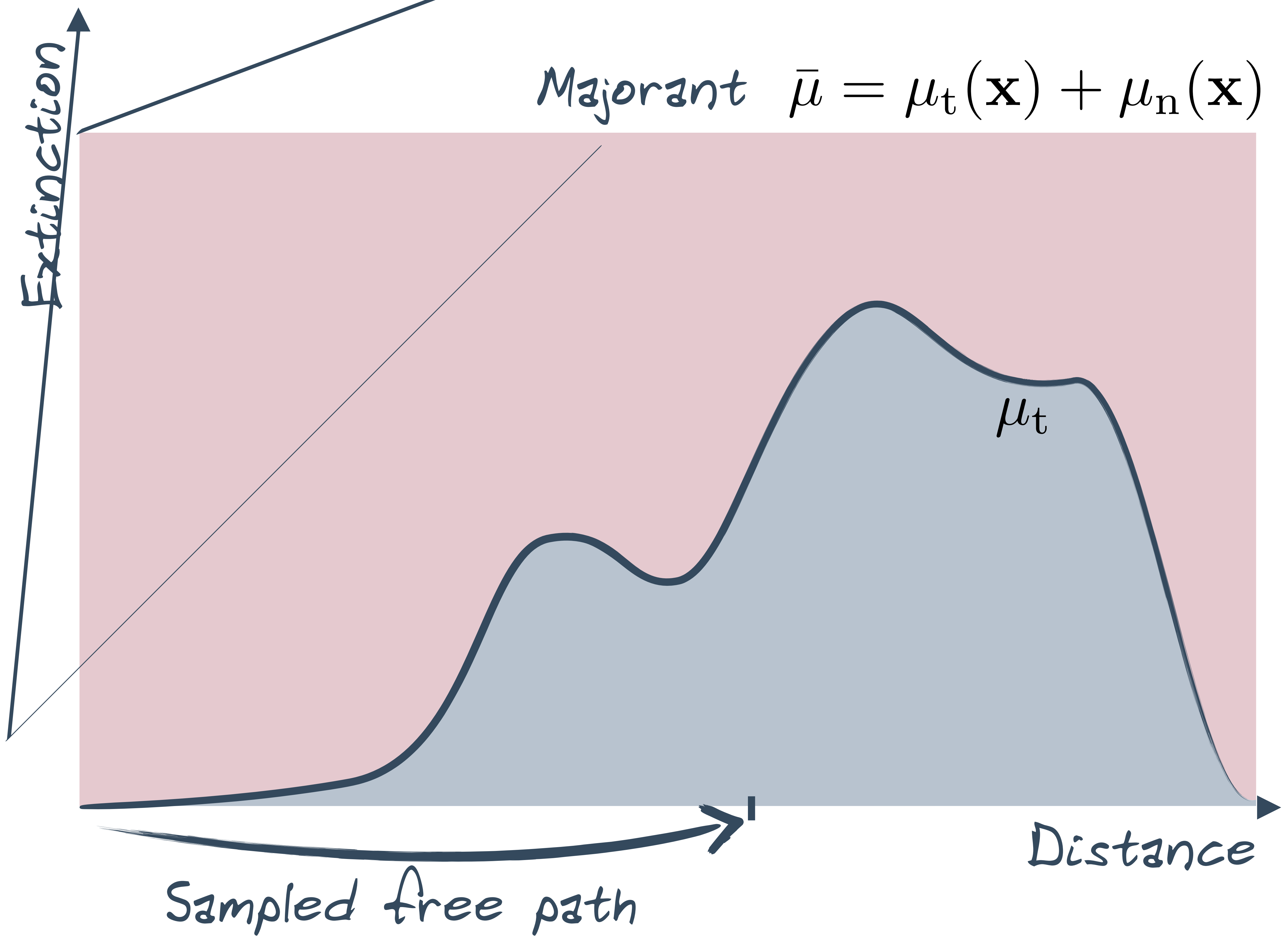




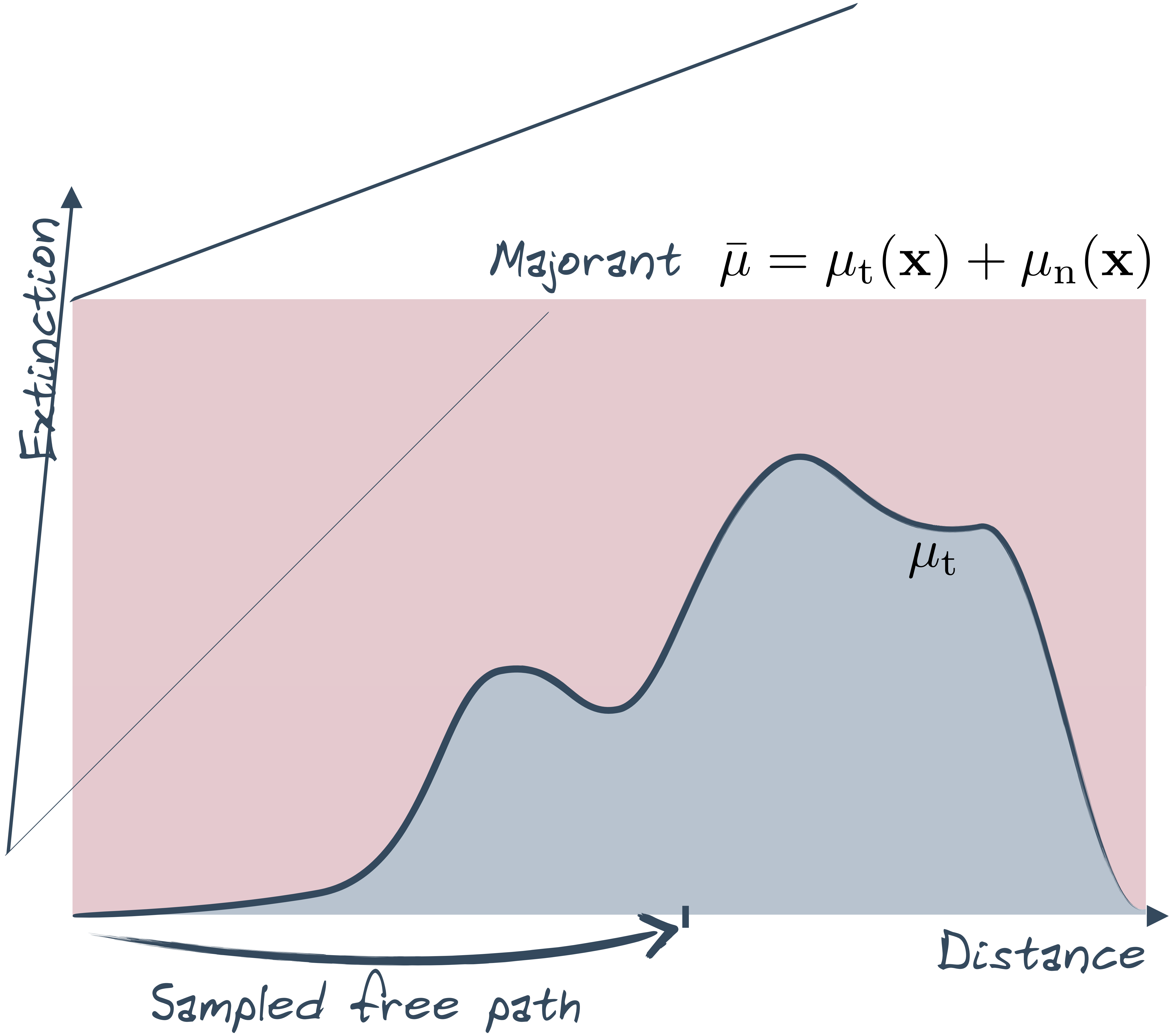
STOCHASTIC SAMPLING



$$P_\mathrm{r}(\mathbf{x}) = \frac{\mu_\mathrm{t}(\mathbf{x})}{\bar{\mu}} \qquad P_\mathrm{n}(\mathbf{x}) = \frac{\mu_\mathrm{n}(\mathbf{x})}{\bar{\mu}}$$

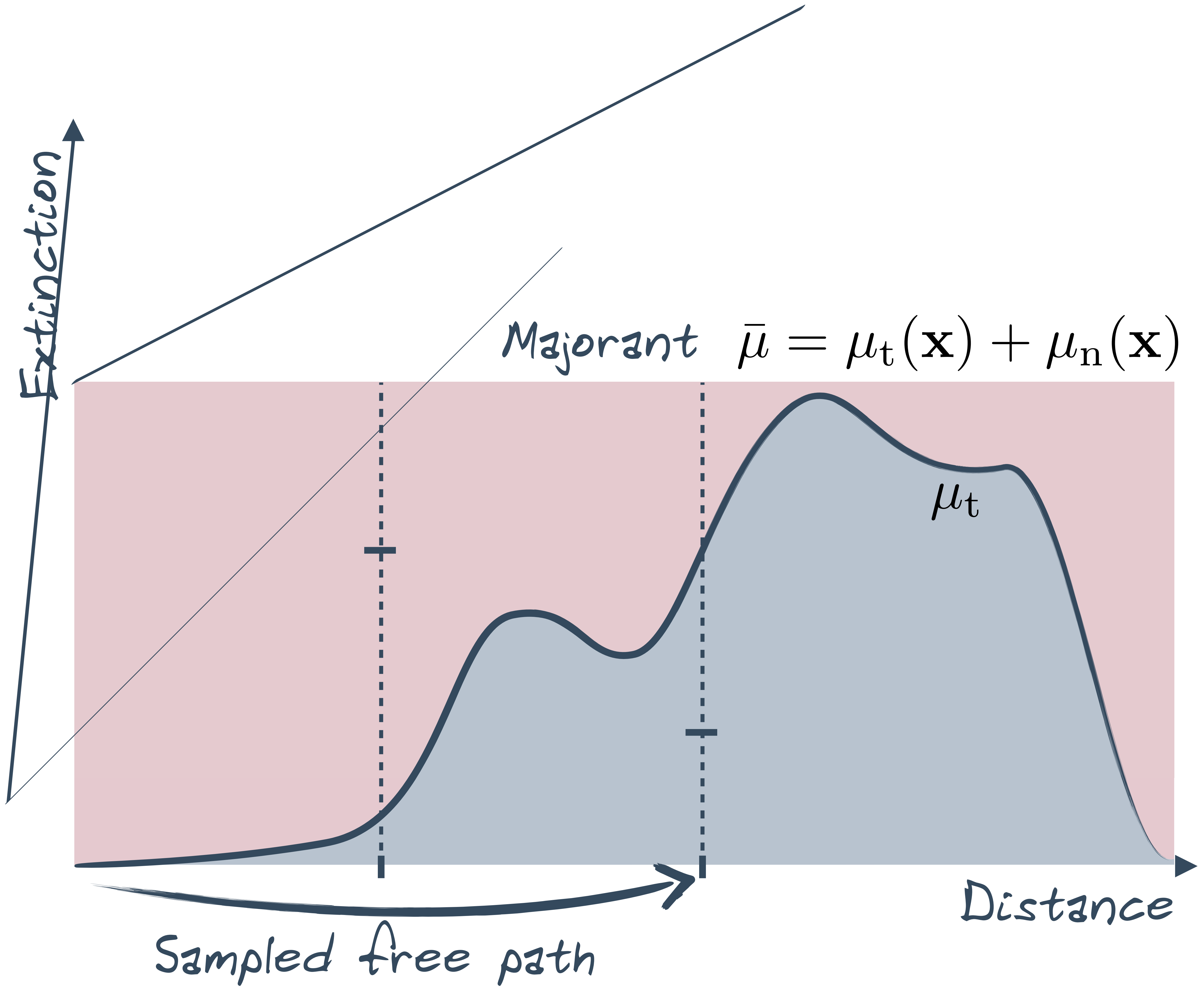


IMPACT OF MAJORANT



IMPACT OF MAJORANT

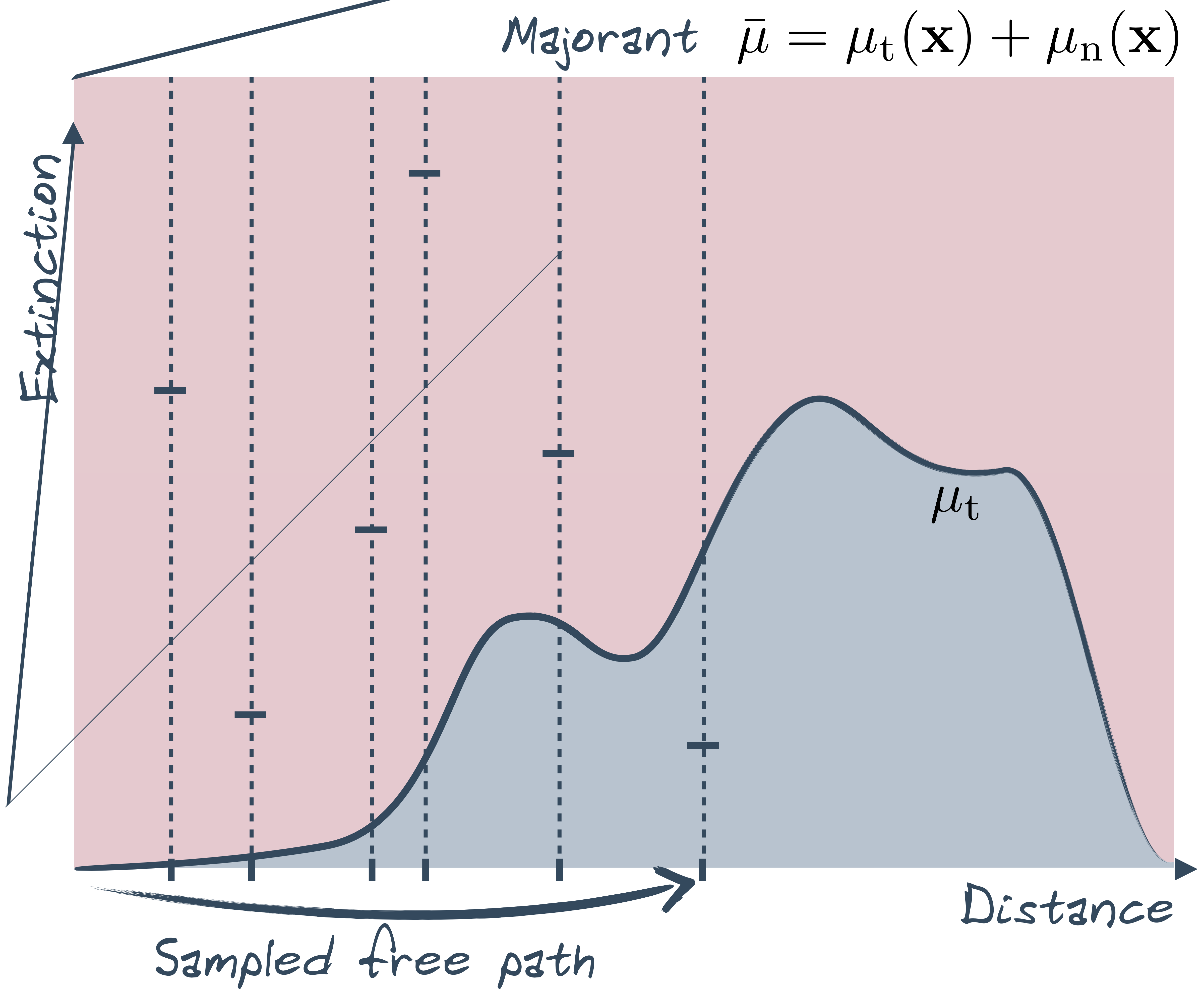
Tight majorant = GOOD
(few rejected collisions)



IMPACT OF MAJORANT

Loose majorant = BAD

(many expensive rejected collisions)



DELTA TRACKING

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PHYSICALLY-BASED interpretation

- Correctness motivated by intuitive arguments:
  Butcher and Messel [1958, 1960],
  Zerby et al. [1961], Bertini [1963],
  Woodcock et al. [1965], Skullerud [1968],
  …

MATHEMATICAL formalism

- Integral formulation: Galtier et al. [2013]

DELTA TRACKING

WEIGHTED (DELTA) TRACKING

DECOMPOSITION TRACKING

MATHEMATICAL FORMALIZATION

CHANGE OF RADIANCE due to null collisions

$$-\mu_{\mathrm{n}}(\mathbf{x})L(\mathbf{x},\omega) + \mu_{\mathrm{n}}(\mathbf{x}) \int_{\mathcal{S}^2} \delta(\omega - \bar{\omega})L(\mathbf{x},\bar{\omega})\,\mathrm{d}\bar{\omega} = 0$$

Losses — Gains ("in-scattering")

Cancel each other

based on Galtier et al. [2013]

MATHEMATICAL FORMALIZATION

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CHANGE OF RADIANCE due to null collisions

$$-\mu_{\mathrm{n}}(\mathbf{x})L(\mathbf{x},\omega) + \mu_{\mathrm{n}}(\mathbf{x}) \int_{\mathcal{S}^2} \delta(\omega - \bar{\omega}) L(\mathbf{x},\bar{\omega})\,\mathrm{d}\bar{\omega} = 0$$

INTEGRAL RTE with null collisions

$$L(\mathbf{x},\omega) = \int_0^\infty T_{\bar{\mu}}(y) \left[\mu_{\mathrm{a}}(\mathbf{y}) L_{\mathrm{e}}(\mathbf{y},\omega) + \mu_{\mathrm{s}}(\mathbf{y}) L_{\mathrm{s}}(\mathbf{y},\omega) + \mu_{\mathrm{n}}(\mathbf{y}) L(\mathbf{y},\omega) \right] \mathrm{d}y$$

Transmittance through the combined (real+fictitious) medium

Null-collided radiance

based on Galtier et al. [2013]

MATHEMATICAL FORMALIZATION

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INTEGRAL RTE with null collisions

$$L(\mathbf{x},\omega) = \int_0^\infty T_{\bar{\mu}}(y)\Big[\mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega) + \mu_\mathrm{n}(\mathbf{y})L(\mathbf{y},\omega)\Big]\mathrm{d}y$$

based on Galtier et al. [2013]

MATHEMATICAL FORMALIZATION

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)}\left[\mu_{\mathrm{a}}(\mathbf{y})L_{\mathrm{e}}(\mathbf{y},\omega)+\mu_{\mathrm{s}}(\mathbf{y})L_{\mathrm{s}}(\mathbf{y},\omega)+\mu_{\mathrm{n}}(\mathbf{y})L(\mathbf{y},\omega)\right]$$

Probabilistic evaluation using Russian roulette

$$\langle f(x)\rangle_P = \begin{cases}\frac{f(x)}{P(x)} & \text{if accepted}\\ 0 & \text{otherwise}\end{cases}$$

based on Galtier et al. [2013]

MATHEMATICAL FORMALIZATION

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RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)}\left[\left\langle \mu_{\mathrm{a}}(\mathbf{y})L_{\mathrm{e}}(\mathbf{y},\omega)\right\rangle_{P_{\mathrm{a}}} + \left\langle \mu_{\mathrm{s}}(\mathbf{y})L_{\mathrm{s}}(\mathbf{y},\omega)\right\rangle_{P_{\mathrm{s}}} + \left\langle \mu_{\mathrm{n}}(\mathbf{y})L(\mathbf{y},\omega)\right\rangle_{P_{\mathrm{n}}}\right]$$

Probabilistic evaluation
using Russian roulette

$$\langle f(x)\rangle_P = \begin{cases} \frac{f(x)}{P(x)} & \text{if accepted} \\ 0 & \text{otherwise} \end{cases}$$

based on Galtier et al. [2013]

MATHEMATICAL FORMALIZATION

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_{\mathrm{a}}(\mathbf{y}) L_{\mathrm{e}}(\mathbf{y},\omega)\rangle_{P_{\mathrm{a}}} + \langle \mu_{\mathrm{s}}(\mathbf{y}) L_{\mathrm{s}}(\mathbf{y},\omega)\rangle_{P_{\mathrm{s}}} + \langle \mu_{\mathrm{n}}(\mathbf{y}) L(\mathbf{y},\omega)\rangle_{P_{\mathrm{n}}}\right]$$

Represents an entire family of (weighted) trackers that all solve RTE!

Delta tracking is just one specific instance.

...see EG STAR or
Galtier et al. [2013]
for complete derivation

WEIGHTED (DELTA) TRACKING

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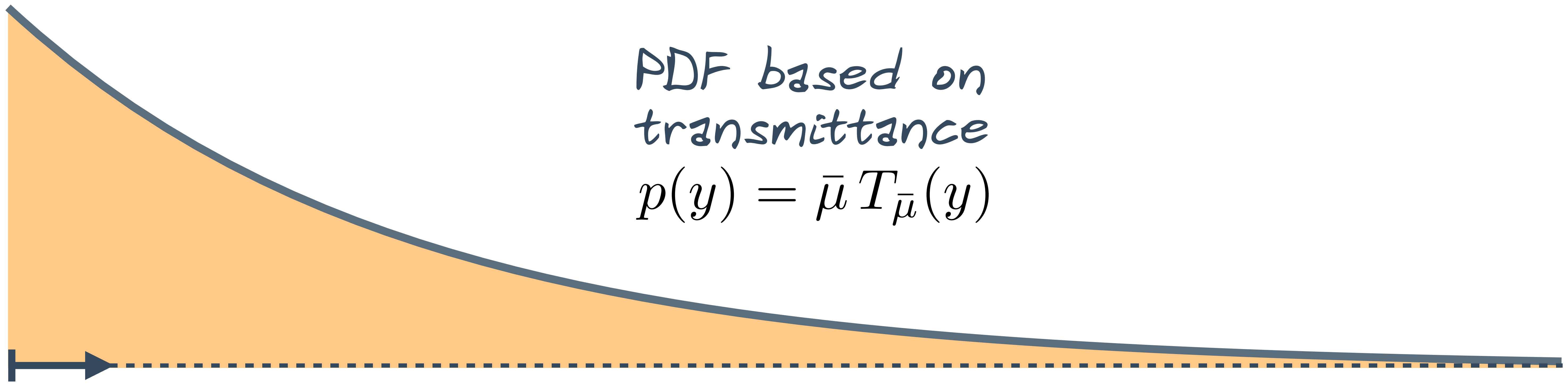
RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y},\omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y},\omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y},\omega) \rangle_{P_n} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$



PDF based on transmittance

$p(y) = \bar{\mu}\, T_{\bar{\mu}}(y)$

*some parameters dropped for brevity

WEIGHTED (DELTA) TRACKING

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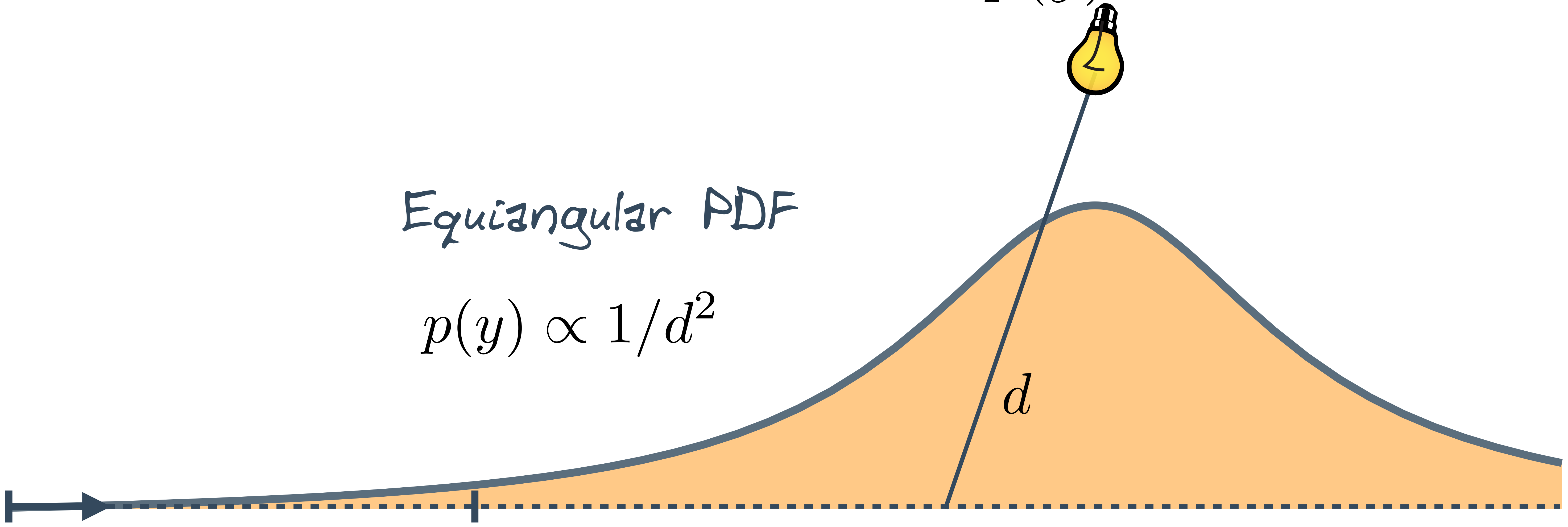
RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\left\langle \mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega)\right\rangle_{P_\mathrm{a}} + \left\langle \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\right\rangle_{P_\mathrm{s}} + \left\langle \mu_\mathrm{n}(\mathbf{y})L(\mathbf{y},\omega)\right\rangle_{P_\mathrm{n}}\right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$



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WEIGHTED (DELTA) TRACKING

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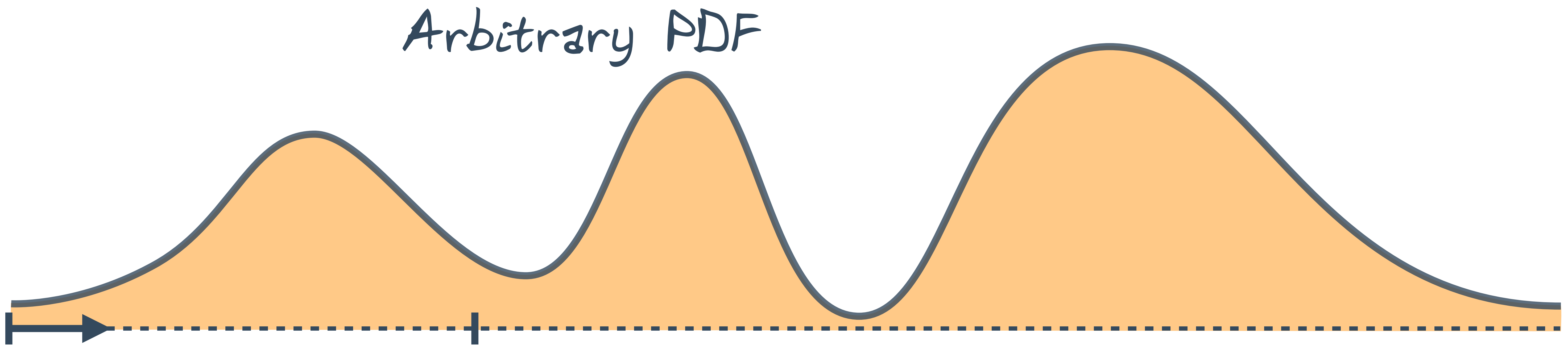
RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_{\mathrm{a}}(\mathbf{y}) L_{\mathrm{e}}(\mathbf{y},\omega) \rangle_{P_{\mathrm{a}}} + \langle \mu_{\mathrm{s}}(\mathbf{y}) L_{\mathrm{s}}(\mathbf{y},\omega) \rangle_{P_{\mathrm{s}}} + \langle \mu_{\mathrm{n}}(\mathbf{y}) L(\mathbf{y},\omega) \rangle_{P_{\mathrm{n}}} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$



*some parameters dropped for brevity

WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

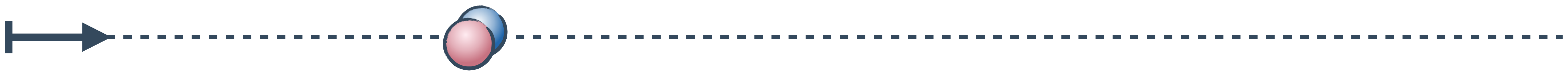
$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\left\langle \mu_{\mathrm{a}}(\mathbf{y}) L_{\mathrm{e}}(\mathbf{y},\omega) \right\rangle_{P_{\mathrm{a}}} + \left\langle \mu_{\mathrm{s}}(\mathbf{y}) L_{\mathrm{s}}(\mathbf{y},\omega) \right\rangle_{P_{\mathrm{s}}} + \left\langle \mu_{\mathrm{n}}(\mathbf{y}) L(\mathbf{y},\omega) \right\rangle_{P_{\mathrm{n}}} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$

INTERACTION PROBABILITIES $P_{\mathrm{a}}, P_{\mathrm{s}}, P_{\mathrm{n}}$

Delta tracking

$$P_{\mathrm{a}} = \frac{\mu_{\mathrm{a}}}{\bar{\mu}} \qquad P_{\mathrm{s}} = \frac{\mu_{\mathrm{s}}}{\bar{\mu}} \qquad P_{\mathrm{n}} = \frac{\mu_{\mathrm{n}}}{\bar{\mu}}$$



*some parameters dropped for brevity

WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

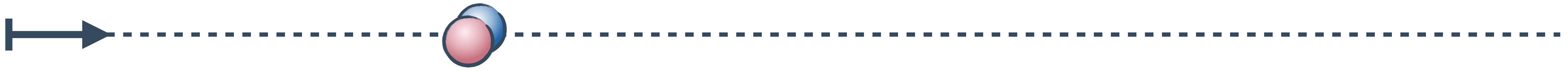
$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)}\left[\left\langle \mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega)\right\rangle_{P_\mathrm{a}} + \left\langle \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\right\rangle_{P_\mathrm{s}} + \left\langle \mu_\mathrm{n}(\mathbf{y})L(\mathbf{y},\omega)\right\rangle_{P_\mathrm{n}}\right]$$

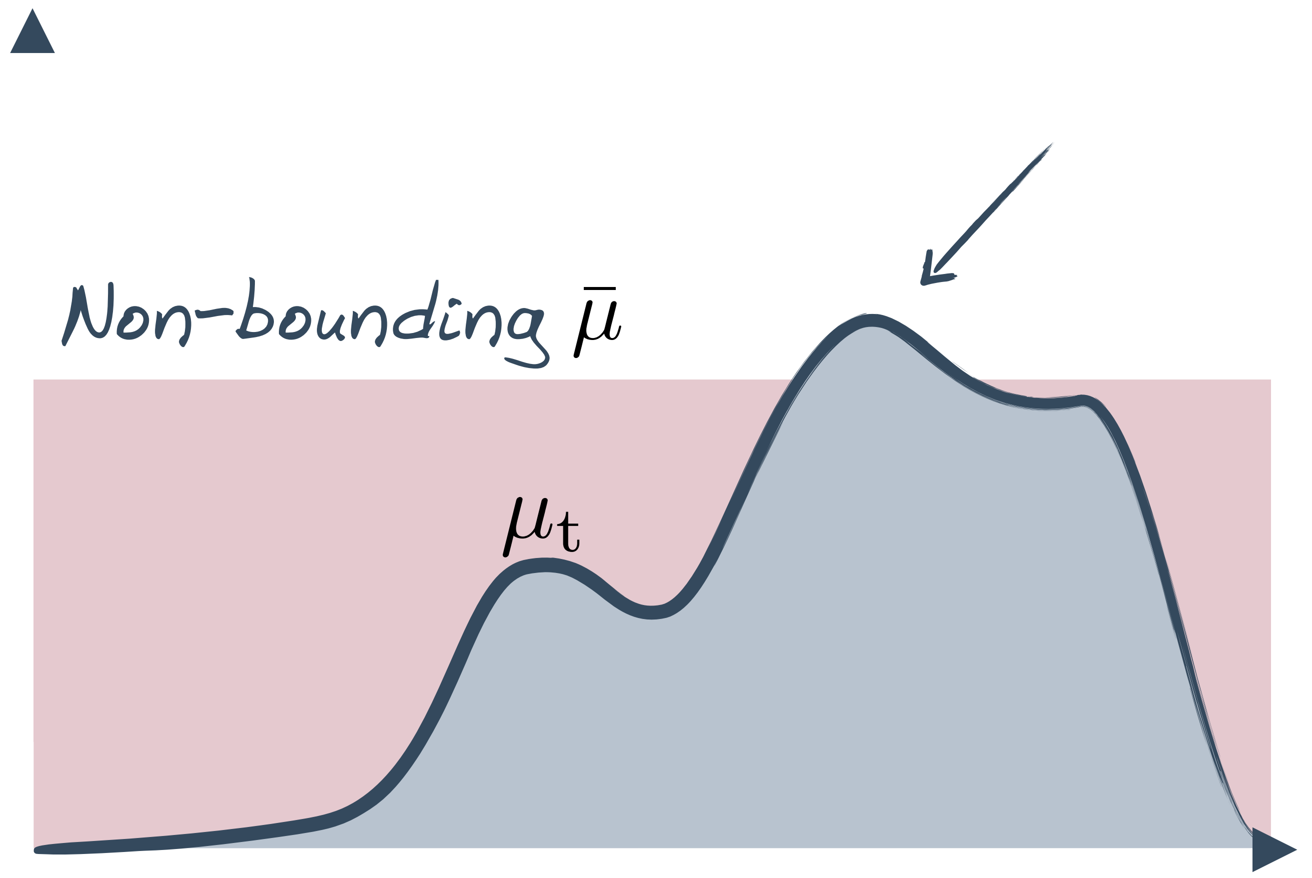
Degrees of freedom: **DISTANCE SAMPLING** $p(y)$

INTERACTION PROBABILITIES $P_\mathrm{a}, P_\mathrm{s}, P_\mathrm{n}$

Weighted tracking that handles

$$P_\mathrm{a} = \frac{\mu_\mathrm{a}}{\mu_\mathrm{t} + |\mu_\mathrm{n}|} \qquad P_\mathrm{s} = \frac{\mu_\mathrm{s}}{\mu_\mathrm{t} + |\mu_\mathrm{n}|} \qquad P_\mathrm{n} = \frac{|\mu_\mathrm{n}|}{\mu_\mathrm{t} + |\mu_\mathrm{n}|}$$





*some parameters dropped for brevity

WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

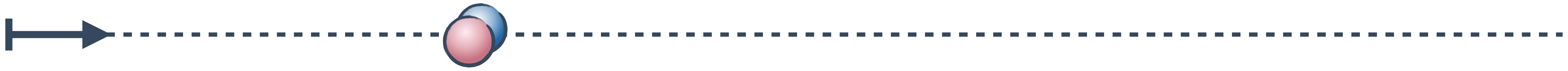
$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\left\langle \mu_{\mathrm{a}}(\mathbf{y}) L_{\mathrm{e}}(\mathbf{y},\omega)\right\rangle_{P_{\mathrm{a}}} + \left\langle \mu_{\mathrm{s}}(\mathbf{y}) L_{\mathrm{s}}(\mathbf{y},\omega)\right\rangle_{P_{\mathrm{s}}} + \left\langle \mu_{\mathrm{n}}(\mathbf{y}) L(\mathbf{y},\omega)\right\rangle_{P_{\mathrm{n}}} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$

INTERACTION PROBABILITIES $P_{\mathrm{a}}, P_{\mathrm{s}}, P_{\mathrm{n}}$

Disabled absorption/emission sampling

$$P_{\mathrm{a}} = 0 \qquad P_{\mathrm{s}} = \frac{\mu_{\mathrm{s}}}{\mu_{\mathrm{s}} + |\mu_{\mathrm{n}}|} \qquad P_{\mathrm{n}} = \frac{|\mu_{\mathrm{n}}|}{\mu_{\mathrm{s}} + |\mu_{\mathrm{n}}|}$$



*some parameters dropped for brevity

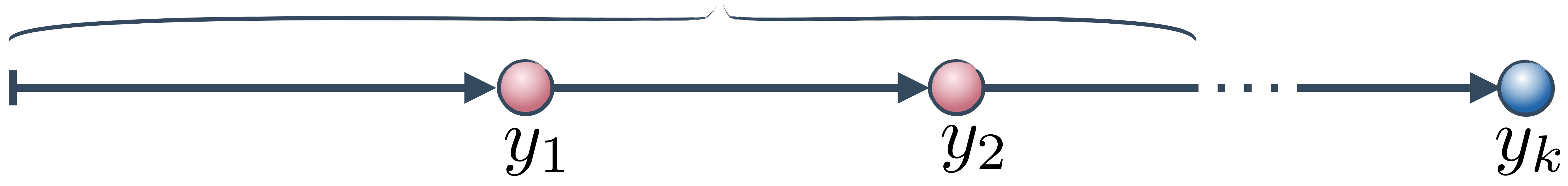
WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\left\langle \mu_\mathrm{a}(\mathbf{y}) L_\mathrm{e}(\mathbf{y},\omega)\right\rangle_{P_\mathrm{a}} + \left\langle \mu_\mathrm{s}(\mathbf{y}) L_\mathrm{s}(\mathbf{y},\omega)\right\rangle_{P_\mathrm{s}} + \left\langle \mu_\mathrm{n}(\mathbf{y}) L(\mathbf{y},\omega)\right\rangle_{P_\mathrm{n}}\right]$$

WEIGHT due to multiple null collisions:

$$\prod_{i=1}^{k-1} \frac{T_{\bar{\mu}}(y_i)}{p(y_i)} \frac{\mu_\mathrm{n}(y_i)}{P_\mathrm{n}(y_i)}$$



*some parameters dropped for brevity

WEIGHTED (DELTA) TRACKING

- Integral framework for null-collision algorithms
  [Galtier et al. 2013]
- Handling of non-bounding "majorants"
  [Cramer 1978, Galtier et al. 2013, Eymet et al. 2013, Novák et al. 2014, Szirmay-Kalos et al. 2017, Kutz et al. 2017, Szirmay-Kalos et al. 2018]
- Improved transmittance estimation
  [Cramer 1978, Novák et al. 2014—Ratio tracking]
- Sample splitting
  [Eymet et al. 2013], [Szirmay-Kalos et al. 2017—Single vs. Double particle model]
- Spectral tracking
  [Kutz et al. 2017]

WEIGHTED (DELTA) TRACKING

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SUMMARY

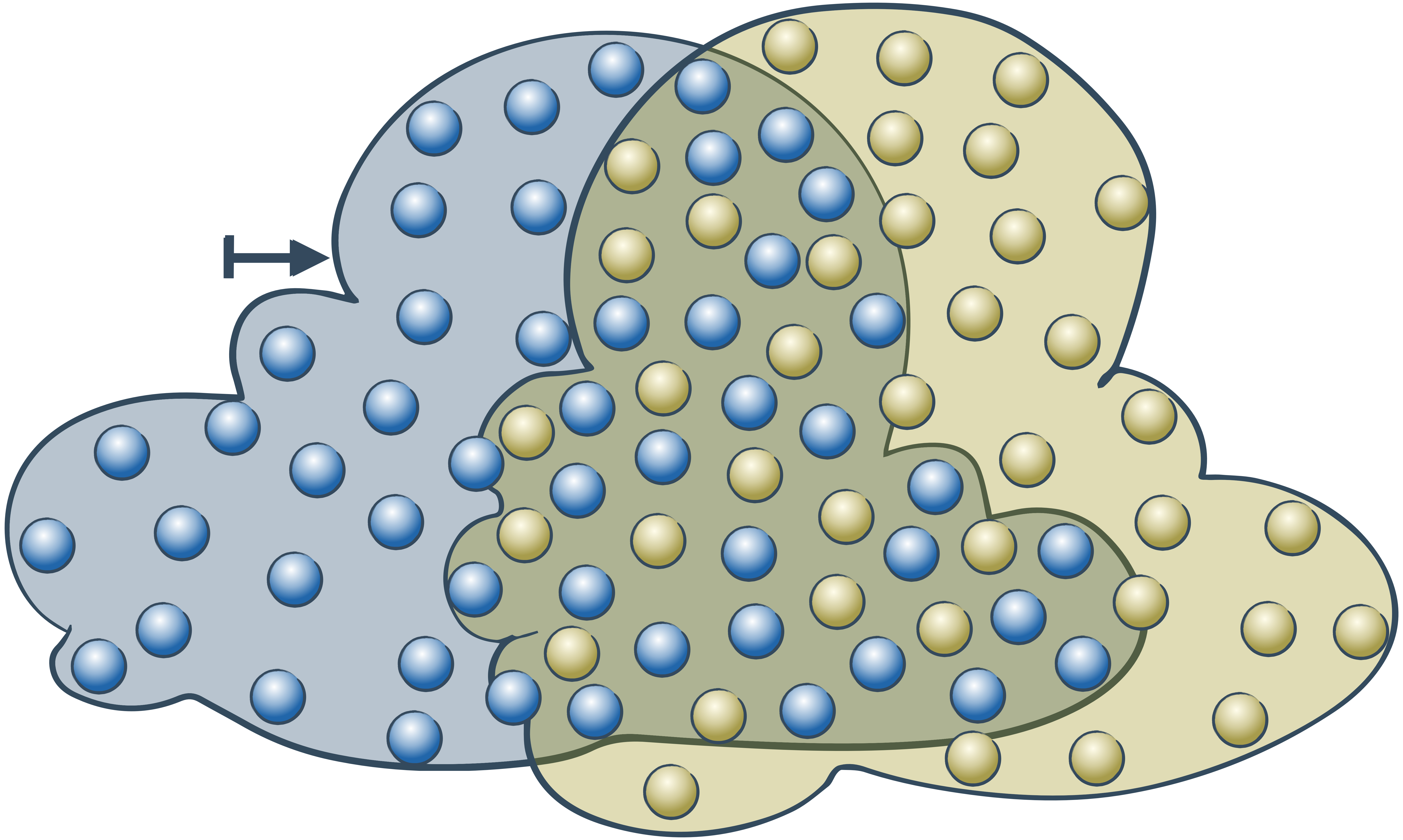
- Non-analog tracker
- Distance distribution differs from free-path distribution, but…
  distribution of **WEIGHTED** distance samples is **IDENTICAL** to free-path distribution
- Allows handling non-bounding “majorants”
- Enables reducing variance by adjusting:
  - distance sampling of tentative collisions
  - collision probabilities

DELTA TRACKING

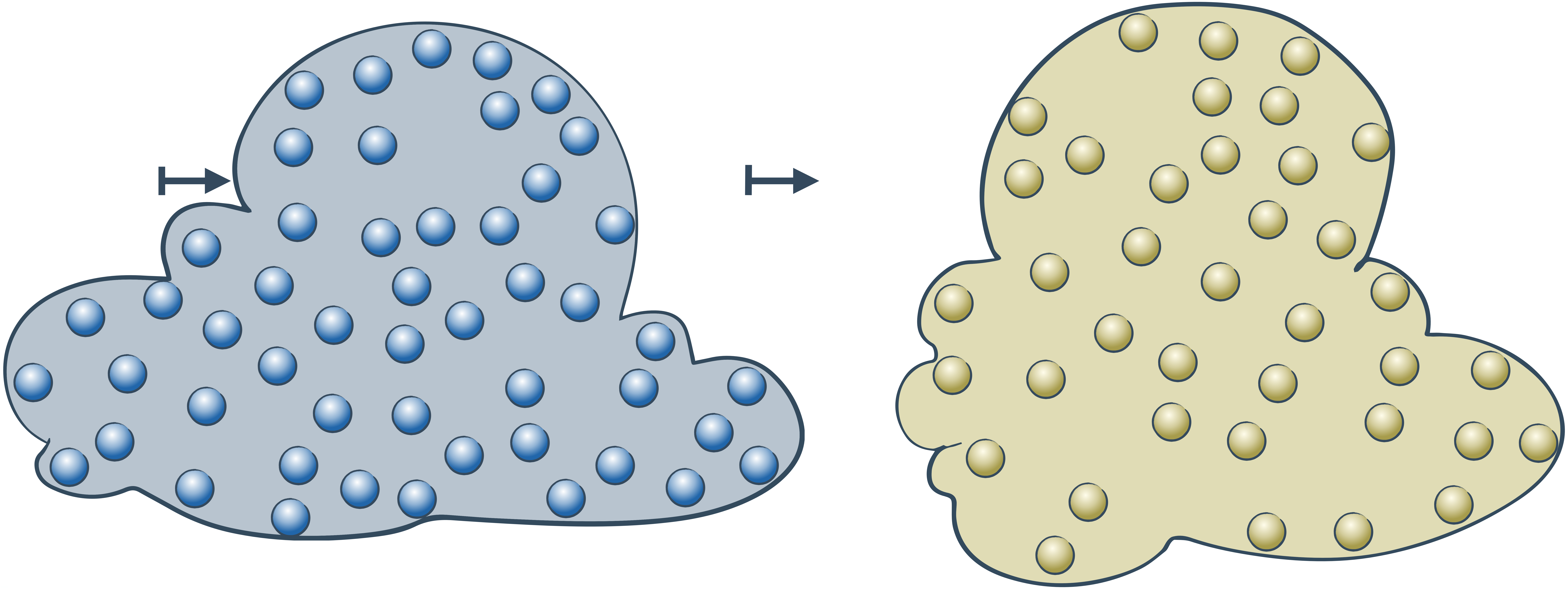
WEIGHTED (DELTA) TRACKING

DECOMPOSITION TRACKING

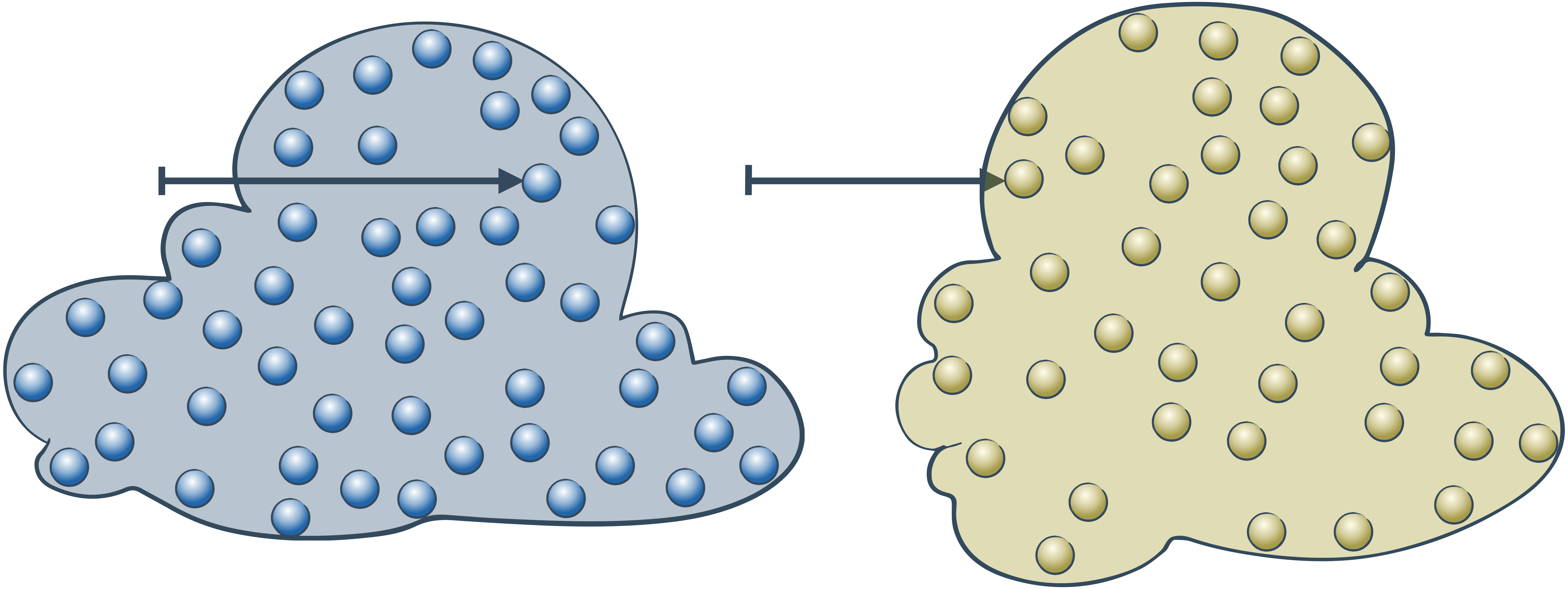
OVERLAPPING VOLUMES



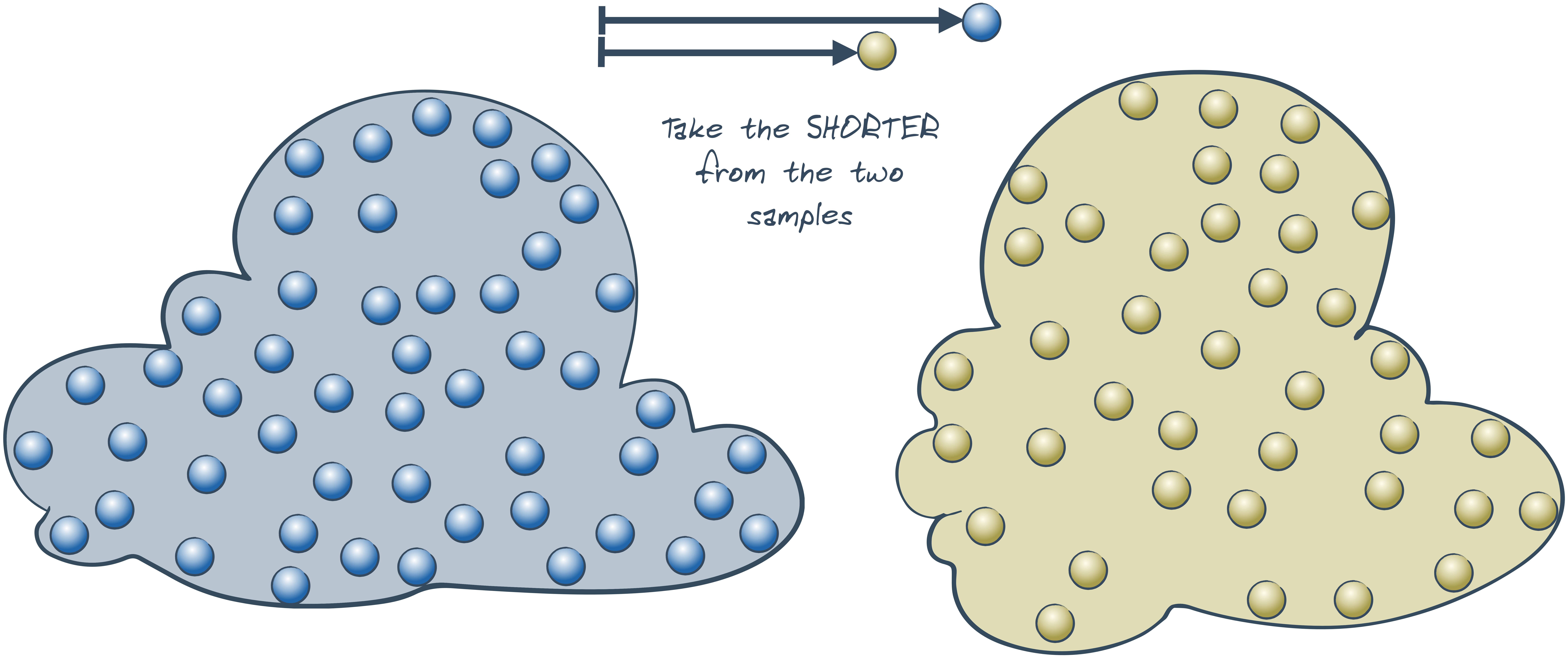
OVERLAPPING VOLUMES



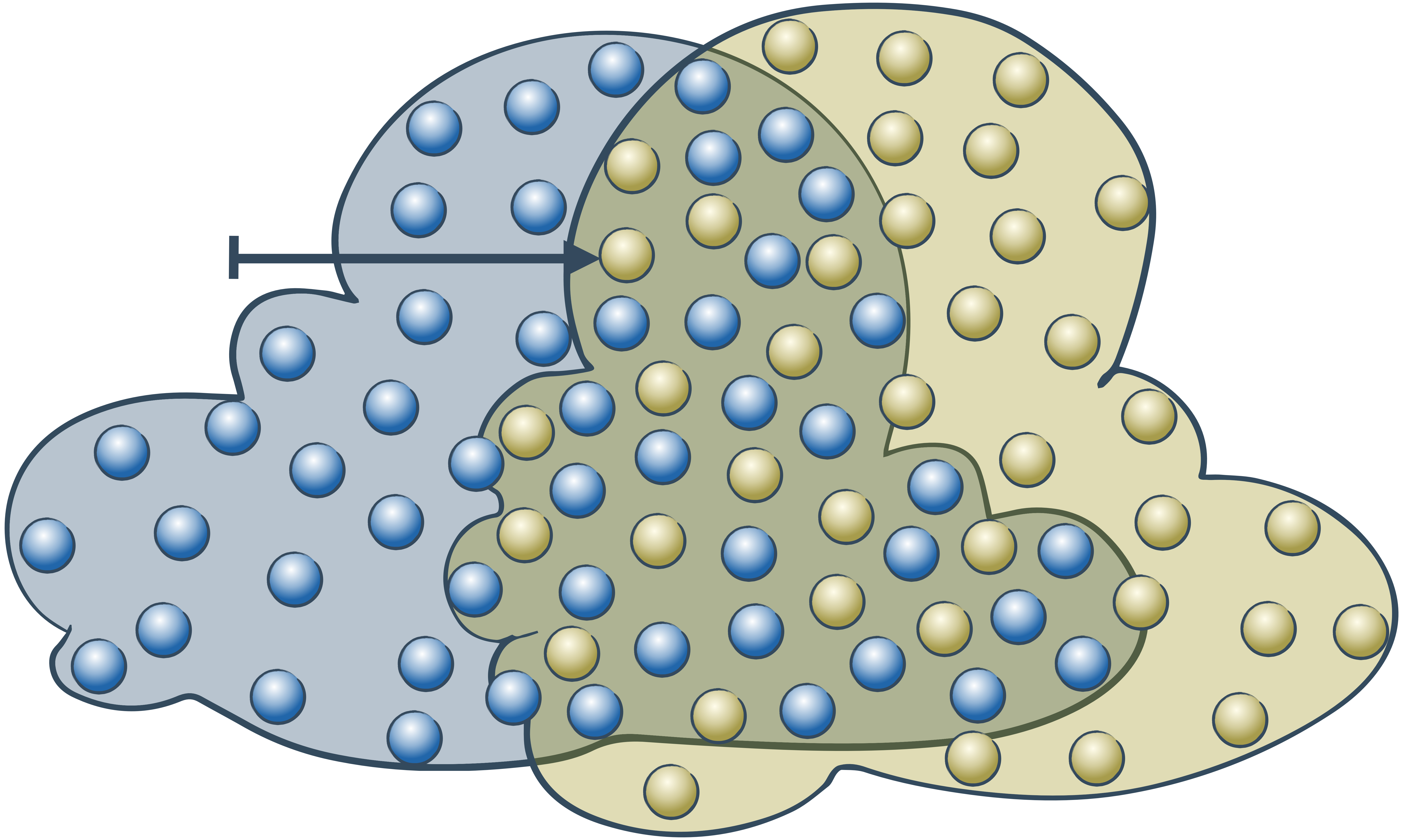
OVERLAPPING VOLUMES



OVERLAPPING VOLUMES



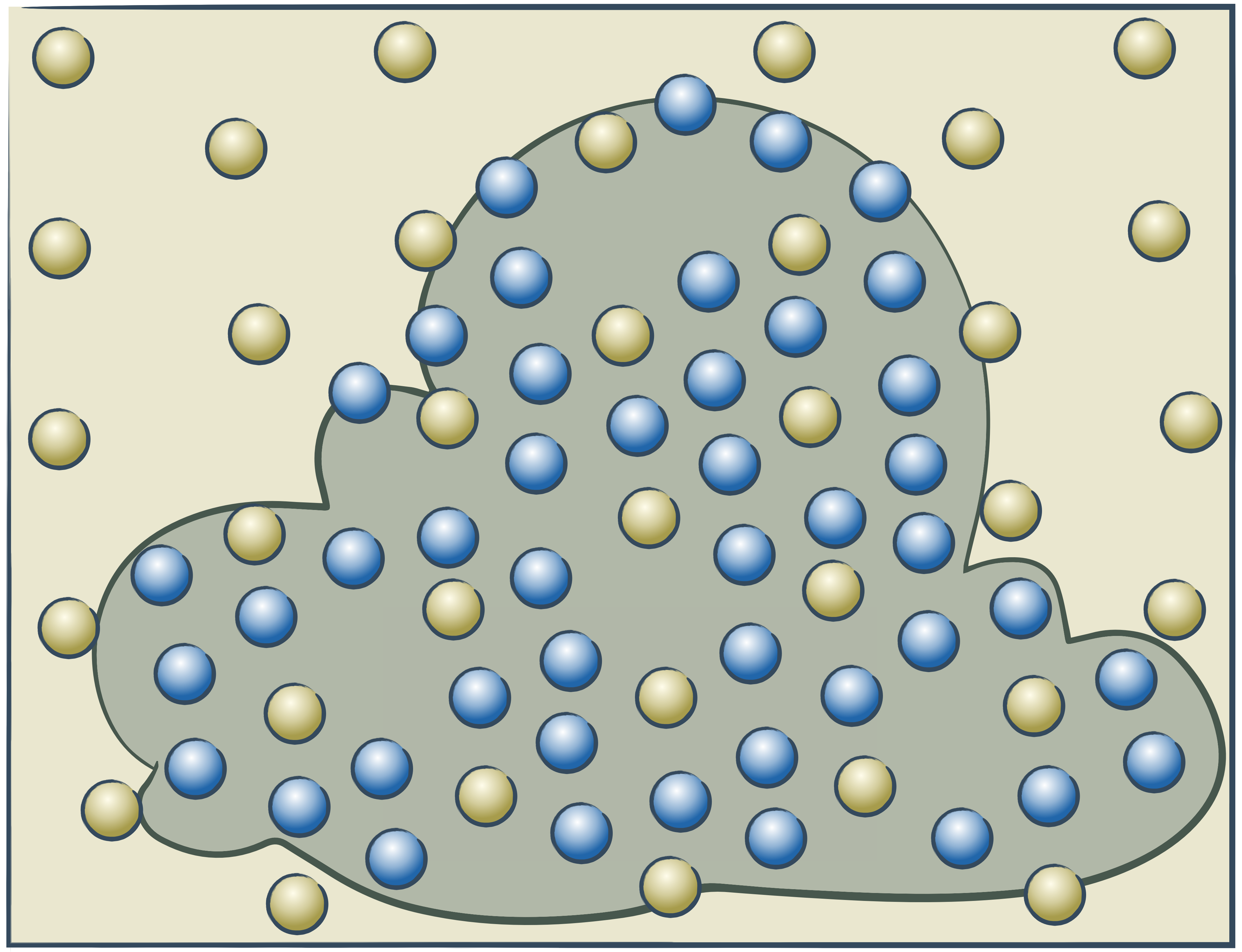
OVERLAPPING VOLUMES



DECOMPOSITION TRACKING

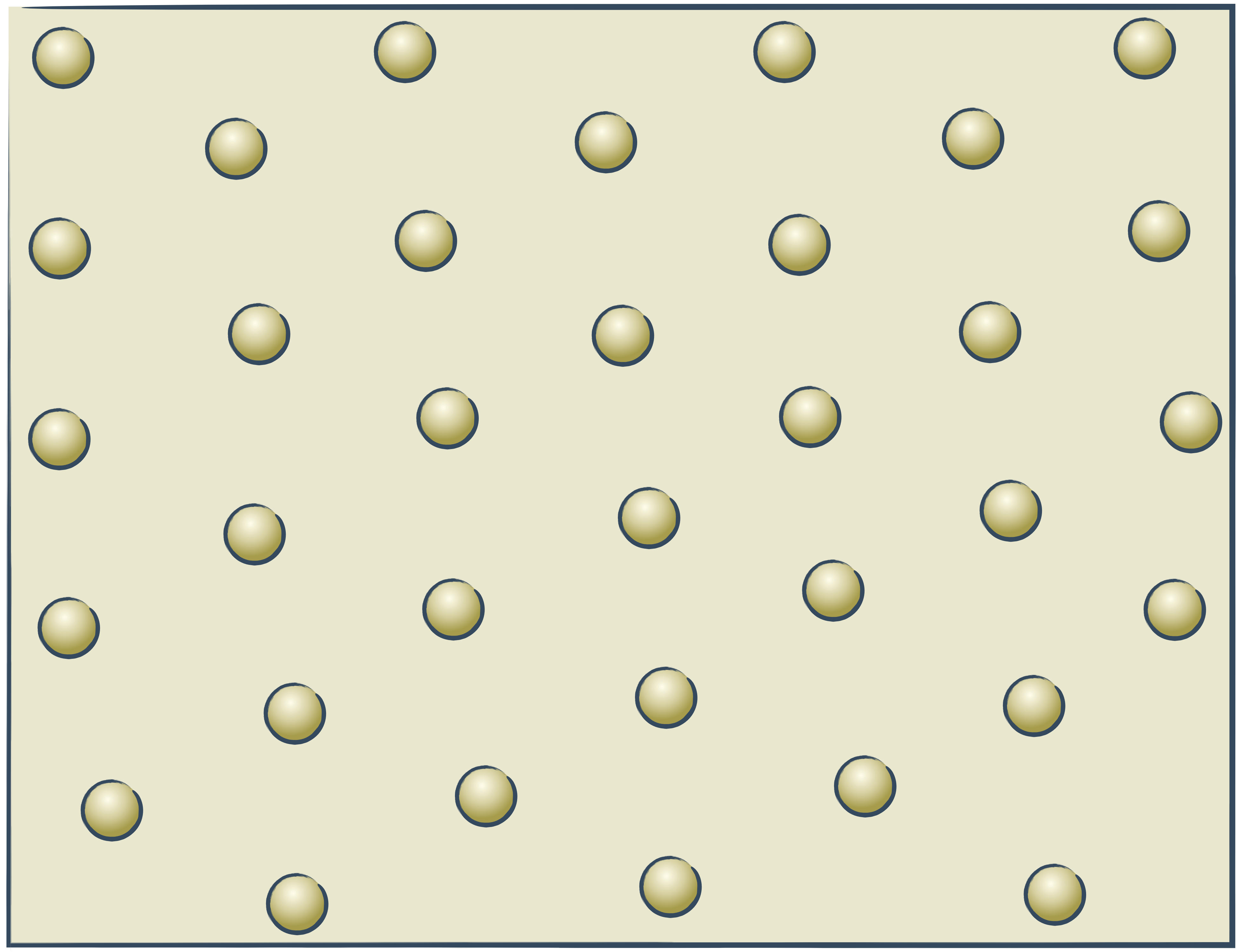
Accelerate free-path sampling by reducing expensive extinction evaluations

- [Kutz et al. 2017]

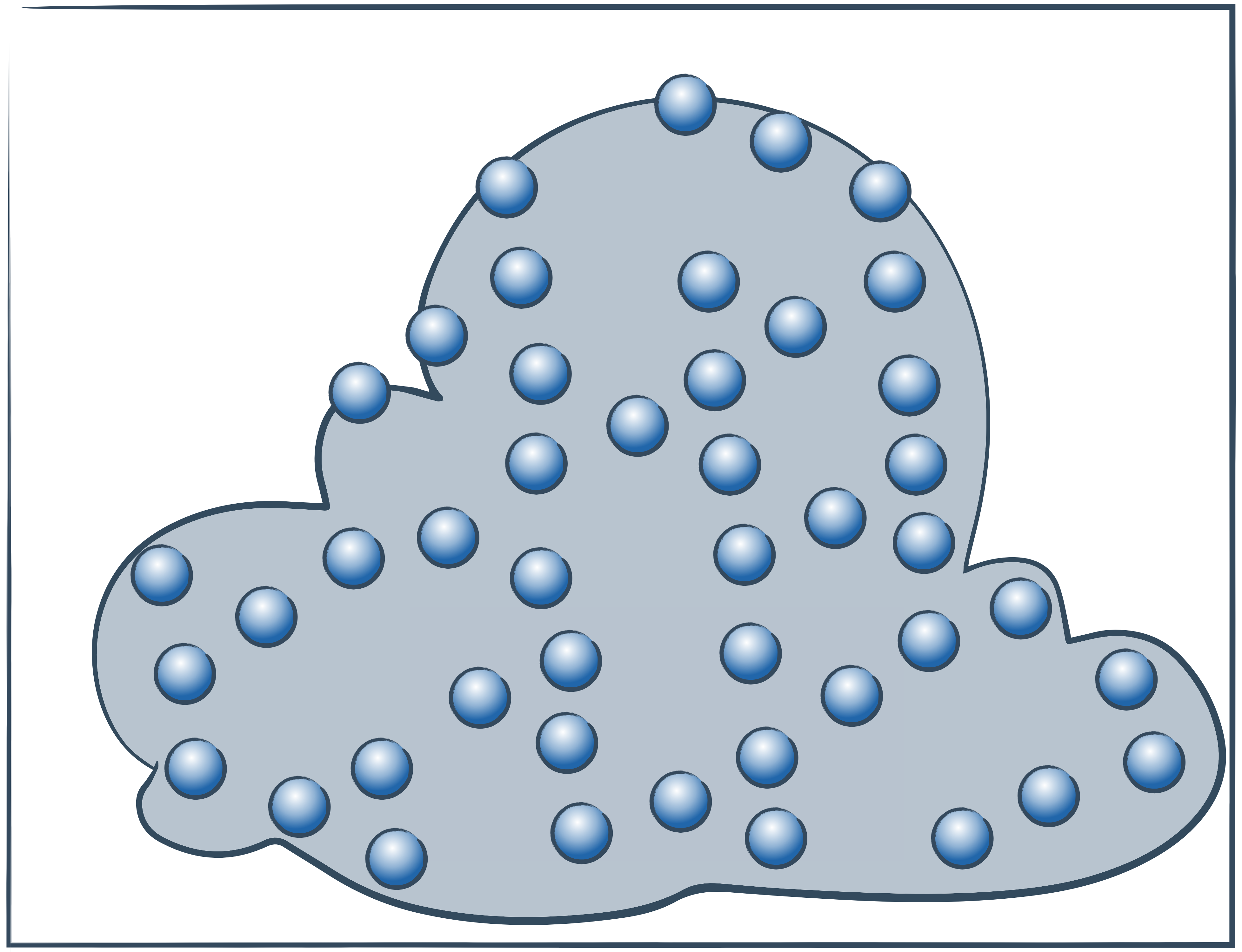


DECOMPOSITION TRACKING

(Piecewise-) HOMOGENEOUS component



HETEROGENEOUS component



DECOMPOSITION TRACKING

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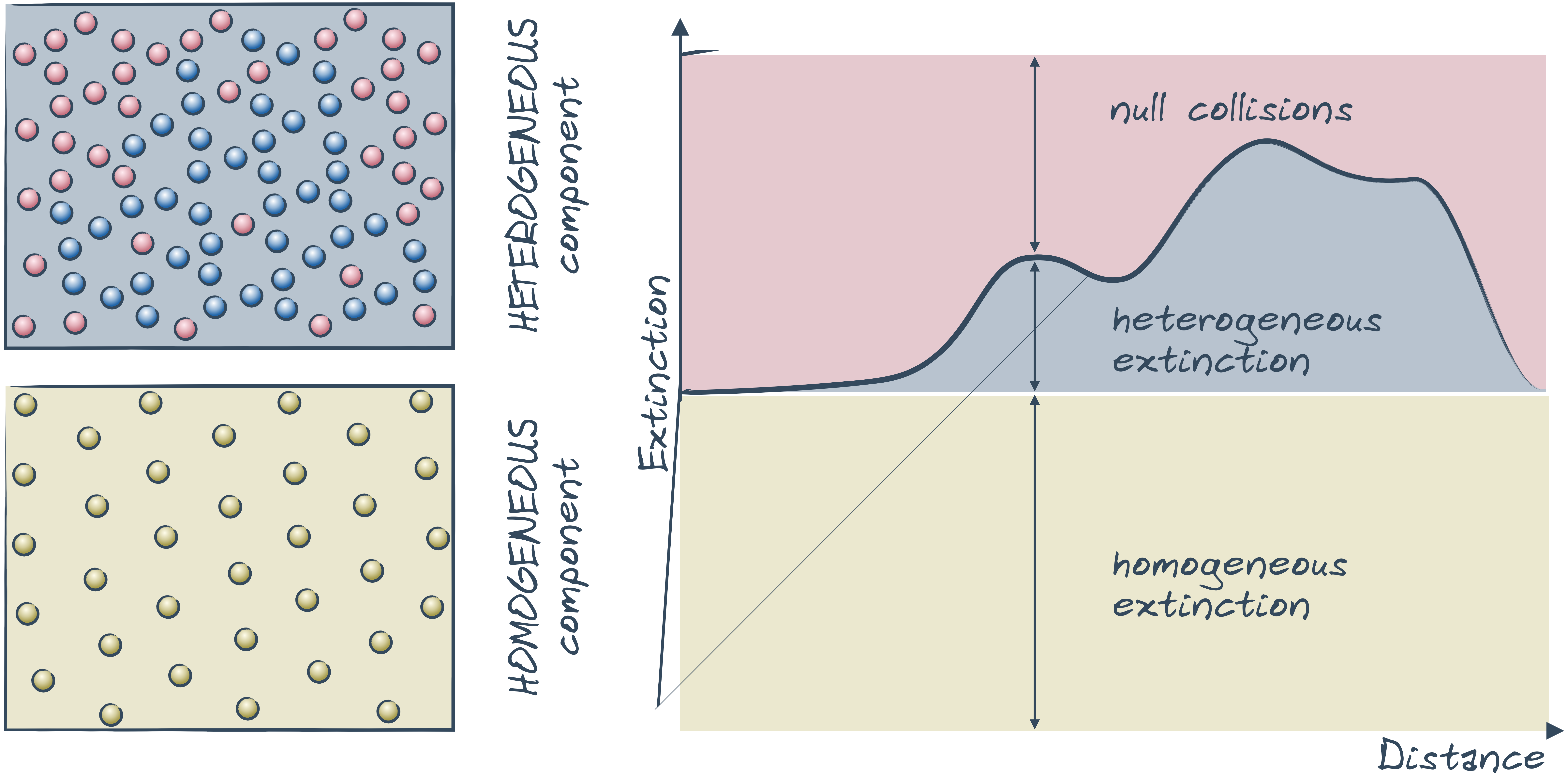
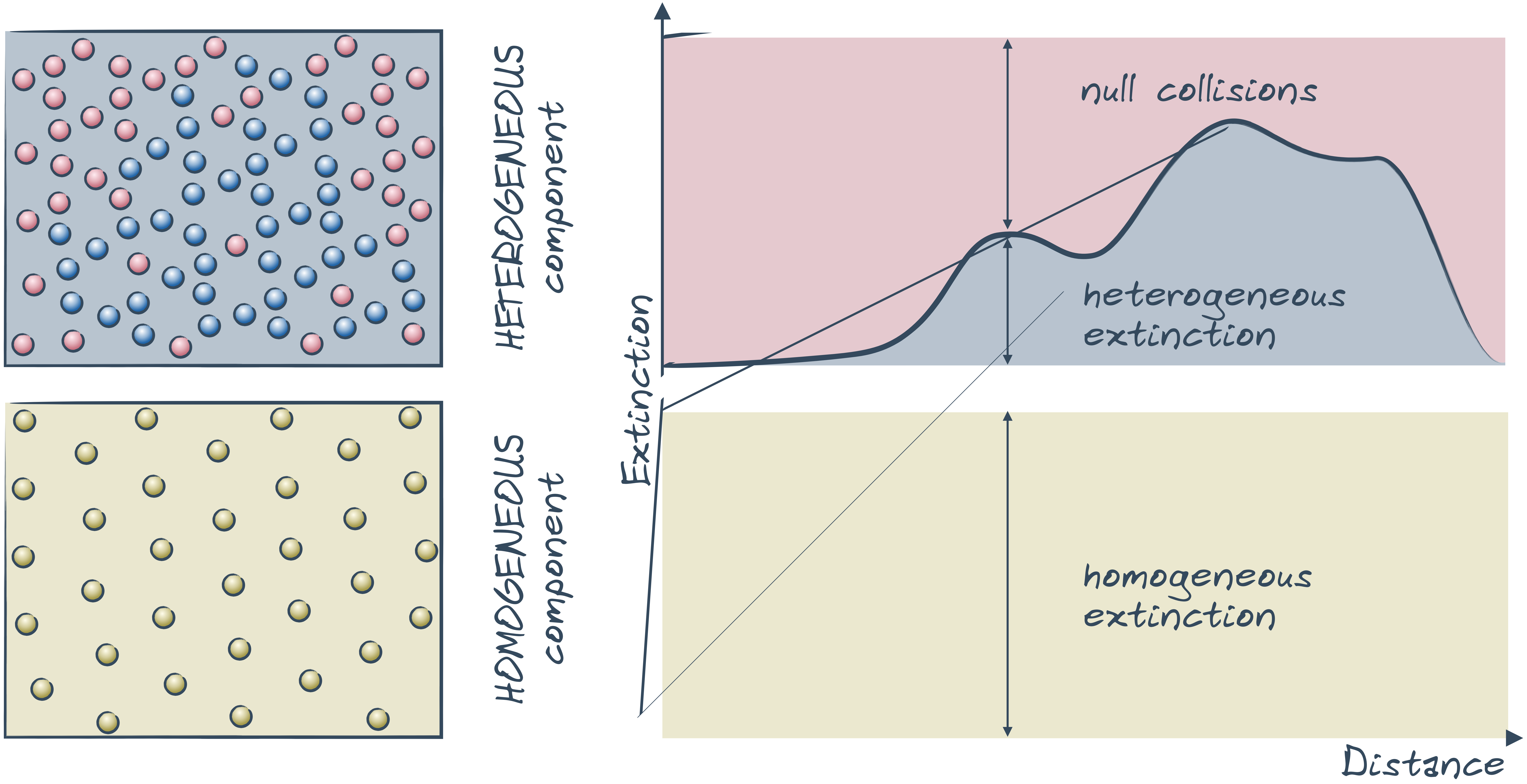
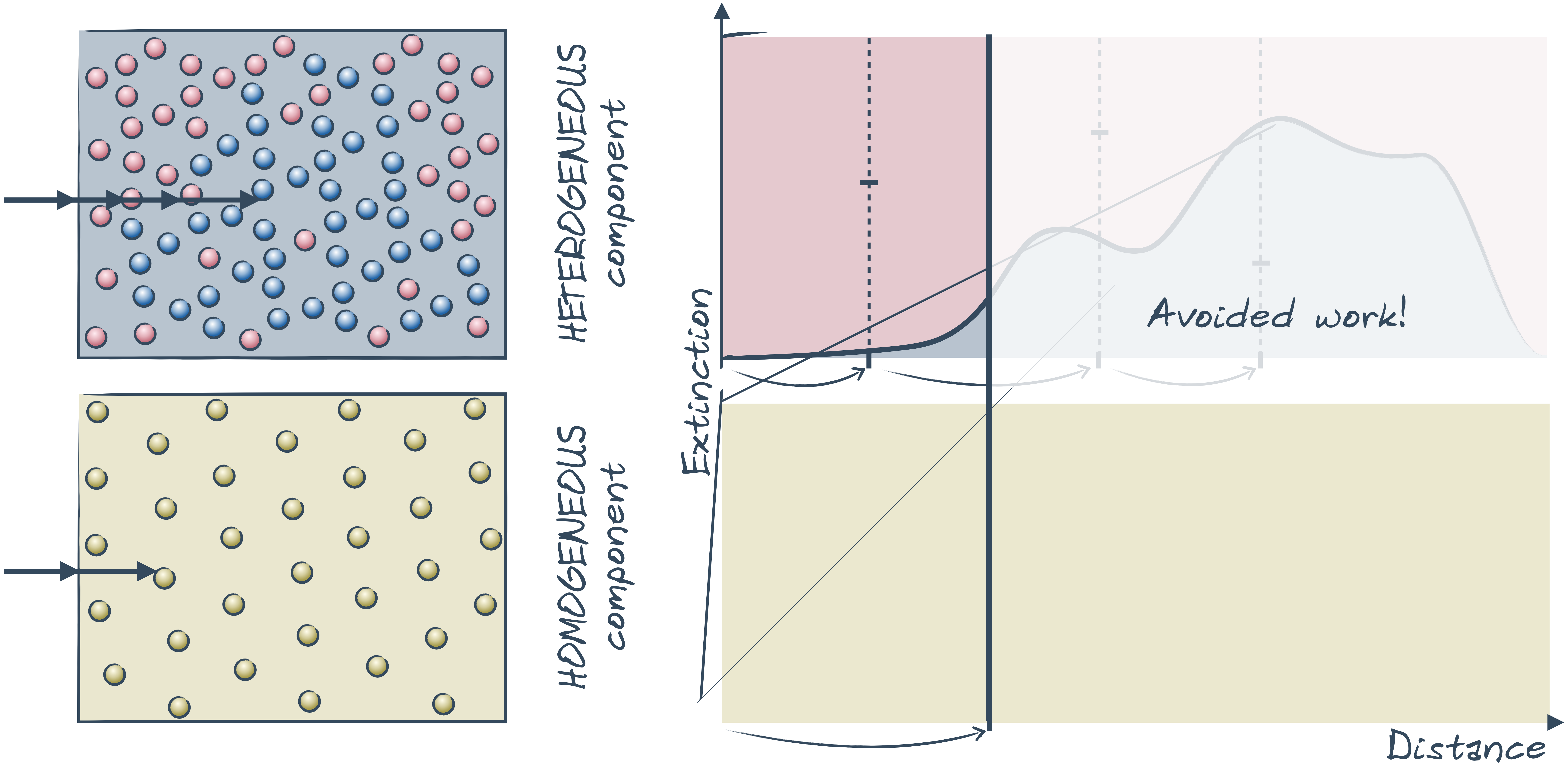


DECOMPOSITION TRACKING



DECOMPOSITION TRACKING



DECOMPOSITION TRACKING

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Decomposition tracking:

1) Decompose into control and residual

2) Sample control component

Repeat

3) Sample tentative free path in residual component

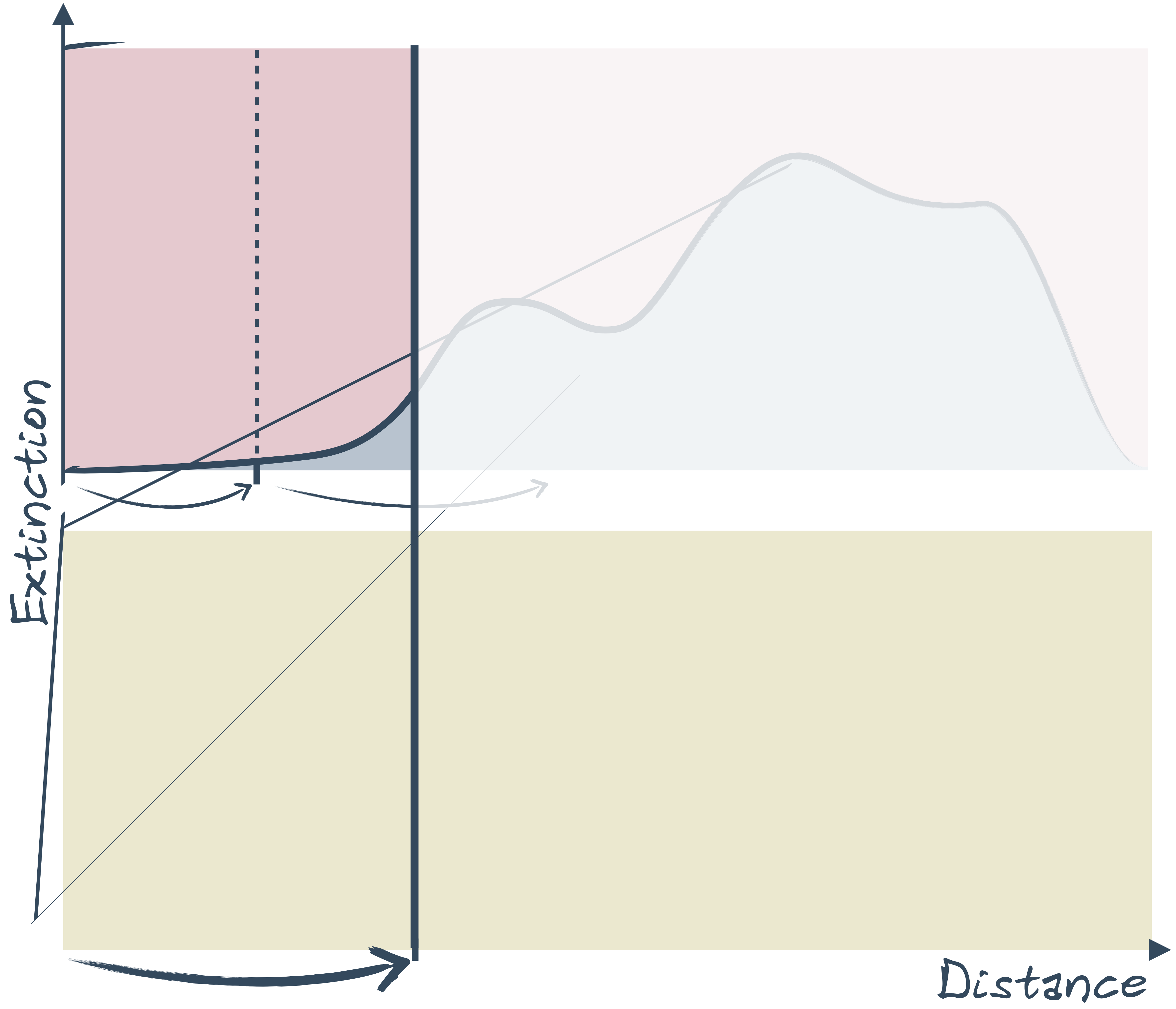
4) If beyond control sample

5) Return control sample

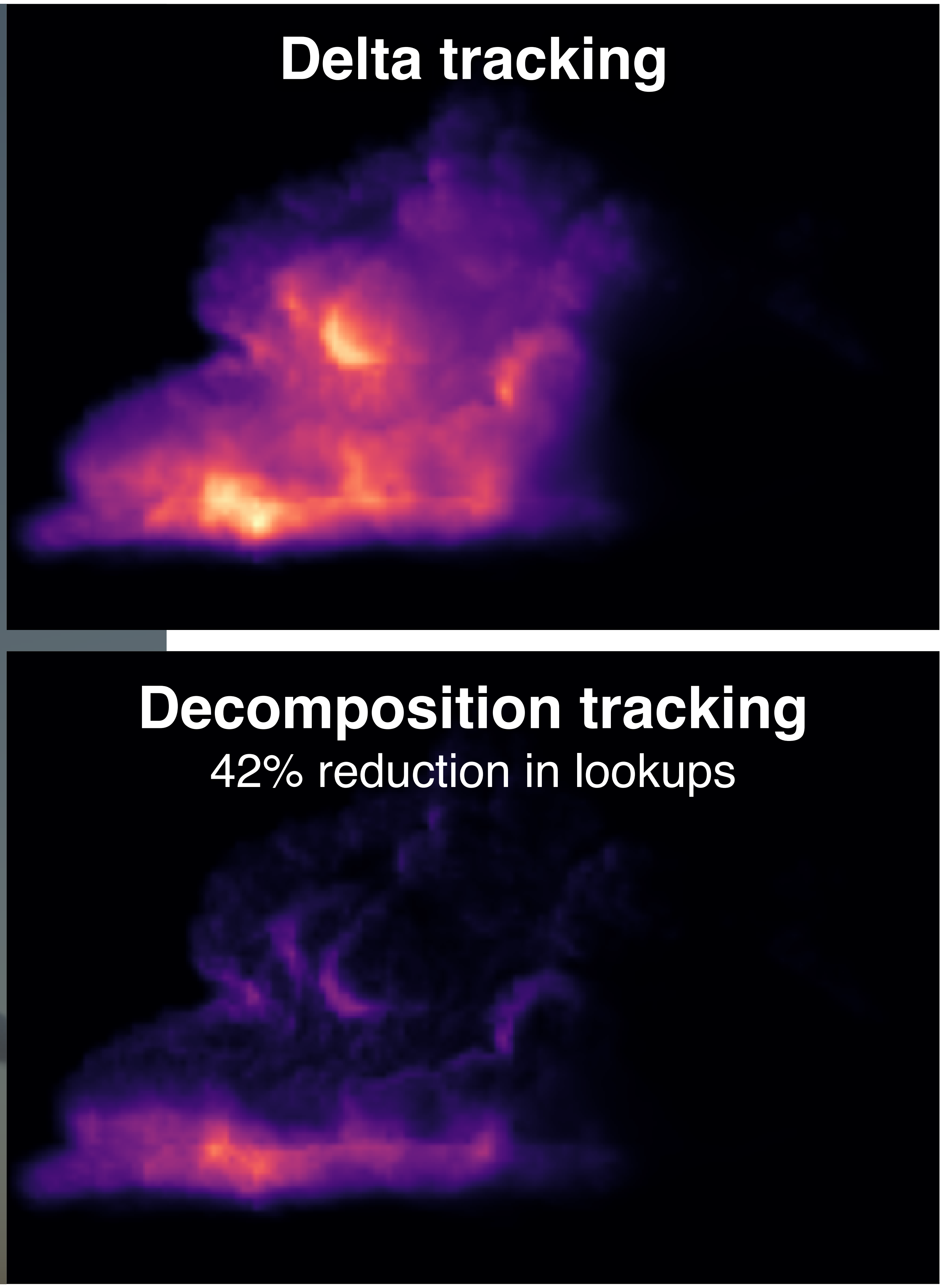
6) Probabilistically classify collision

Until collision classified as real

7) Return residual sample



DECOMPOSITION TRACKING



Kutz et al. [2017]

DECOMPOSITION TRACKING

HOMOGENEOUS and **RESIDUAL HETEROGENEOUS** components

- Reduces evaluations of spatially varying collision coefficients
- Requires a space-partitioning data structure (e.g. octree) to be practical
- Can be combine with weighted tracking to handle arbitrary decompositions

MORE DISTANCE SAMPLING…

- Equiangular sampling
  [Kulla and Fajardo 2012]
- Joint-importance sampling
  [Georgiev et al. 2013]

Discussed by Iliyan later

- Tabulation approaches
  [Kulla and Fajardo 2012, Novák et al. 2012, Georgiev et al. 2013, Novák et al. 2014]

…END OF THIS PART