An Analytic Model for Full Spectral Skydome Radiance

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Motivation: Skydome models

- outdoor scenes
- prediction of material appearance
- architecture, outdoor scenes
- alternative: skyboxes
The Preetham Model

- De facto standard analytic skydome model
- Simple closed form formulas
- Outputs tristimulus data, convertible to spectral
- Perez formula, fitted to the outputs of Nishita model
- Fitting results publicly available

\[
F(\theta, \gamma) = \left(1 + Ae^{B/\cos \theta}\right)\left(1 + Ce^D \gamma + Ecos^2\gamma\right)
\]

- luma distribution parameters
- solar angle
- zenith angle
The Preetham Model

**Inputs**
- Turbidity
- Solar-zenith angle

**Once per scene**
- \( x_z = \begin{bmatrix} T^2 & T & 1 \end{bmatrix} \)
- \[
\begin{bmatrix}
0.00166 & -0.00375 & 0.00209 & 0 \\
-0.02903 & 0.06377 & -0.03202 & 0.00394 \\
0.11693 & -0.21196 & 0.06052 & 0.25886 \\
\end{bmatrix}
\]

**Zenith luminance**
- \( x = x_z \cdot \frac{F_{A,B,C,D,E}(\theta, \gamma)}{F_{A,B,C,D,E}(0, \gamma_s)} \)

**Output**
- Zenith angles

**Luma distribution parameters**
- \( \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0.1787 & -1.4630 \\ -0.3554 & 0.4275 \\ -0.0227 & 5.3251 \\ 0.1206 & -2.5771 \\ -0.0670 & 0.3703 \end{bmatrix} \)
Issues With the Preetham Model

• Simple functions for calculating zenith luminance and luma distribution parameters
  • Only a small subset of possible atmospheric conditions can be captured
• Lack of localised aureole at higher turbidities
• Spectral data are obtained by conversion from trisitculus data
Issues with the Preetham Model
Goals for the New Model

- spectral (including UV)
- larger range of turbidities
- eliminate artifacts
- ground albedo (came later & almost for free)
Obtaining Reference Data

• Brute force Monte Carlo path tracer that incorporates
  • Mie scattering
  • Rayleigh scattering
  • Albedo & colour of Earth surface
• Delta scattering
  • Approaches the simulation as a transport problem, the same way physicists simulate e.g. gamma rays passing through containment shells of nuclear reactors
Obtaining Reference Data

- ray exits the atmosphere
- check for Sun collision
- scattering events
- Earth surface reflection
- camera
- 50km exponential density decay
Result: Raw Data
Result: Comparison
Modified formulas for a better fit

**Original Perez Formula**

\[ F(\theta, \gamma) = \left( 1 + Ae^{B/cos \theta} \right) \left( 1 + Ce^D\gamma + E\cos^2\gamma \right) \]

- Fudge factor to prevent divergence to infinity at zenith
-Suppresses luminance peak extending too far up

**Modified Formula**

\[ F(\theta, \gamma) = \left( 1 + Ae^{B/cos \theta + 0.01} \right) \left( C + De^{E\gamma} + F \cos^2\gamma + G \chi(H, \gamma) + I \cos^2 \theta \right) \]

- Anisotropic term for luminance peaks around sun

\[ \chi(g, \alpha) = \frac{1 + \cos^2 \alpha}{3} \]

\( (1 + g^2 - 2g \cdot \cos \alpha)^2 \)
inputs

once per scene

for each sample

output mean radiance radiance distribution parameters
for each sample
solar elevation albedo turbidity

\[ x_z = B_{M_r}(\eta, \alpha, T) \]
\[ x = x_z \cdot F_{A,B,...,I}(\theta, \gamma) \]
\[ A \\
\begin{bmatrix}
  B \\
  I
\end{bmatrix} = B_{M_d}(\eta, \alpha, T) \]
How the Fitting was Done

• Levenberg-Marquardt nonlinear optimization
• Yields a mean radiance and radiance distribution array
  • These are then used as control points for Bezier interpolation
• Albedo 0-1
  Turbidity 1-10
  Solar elevation 0-90°
  320nm – 720nm bands, 40nm increment
Results
Sky Colour Patterns (sunset)
UV & Fluorescence
Ground Albedo

black

white

water blue

forest green
Conclusion

• Reference implementation and data available for download
• Version 1.1 out!
• http://cgg.mff.cuni.cz/projects/SkylightModelling/