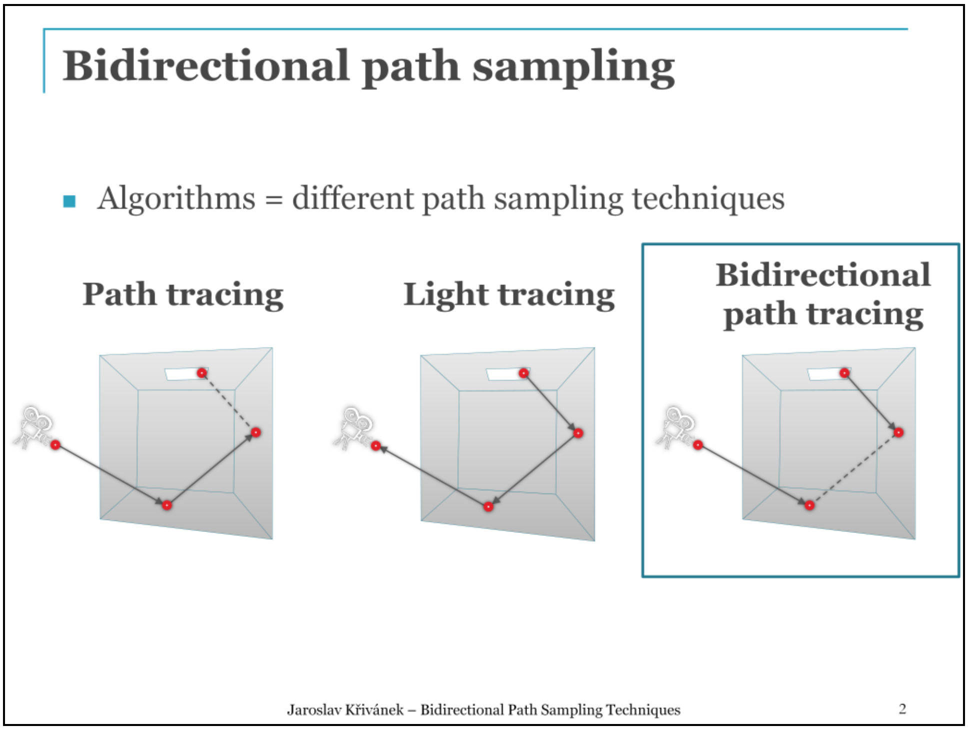
BIDIRECTIONAL PATH SAMPLING TECHNIQUES

Jaroslav Křivánek
Charles University in Prague
http://cgg.mff.cuni.cz/~jaroslav/



• In the previous part of the course, I said that from the point of view of the path integral framework, the major difference between different LTS algorithms is the employed path sampling technique.

• In this part of the course, we will focus on bidirectional path sampling techniques, that build a sub-path from the camera and from the light source and connect the two sub-paths to generate an entire path. Such sampling techniques are employed in the bidirectional path tracing algorithm.

VPL RENDERING

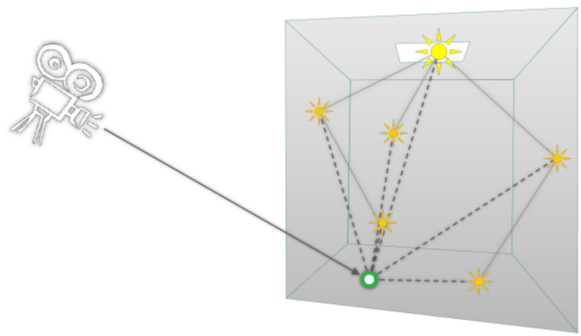
AS A PATH SAMPLING TECHNIQUE

- But before I get to the description of bidirectional path tracing, I would like to start with Virtual Point Light Rendering (a.k.a. Instant radiosity) and show the advantages of looking at this algorithm through the prism of the path integral framework.

Instant radiosity – VPL rendering

[Keller 1997]

1. **Distribute VPLs**
2. **Accumulate VPL contributions**



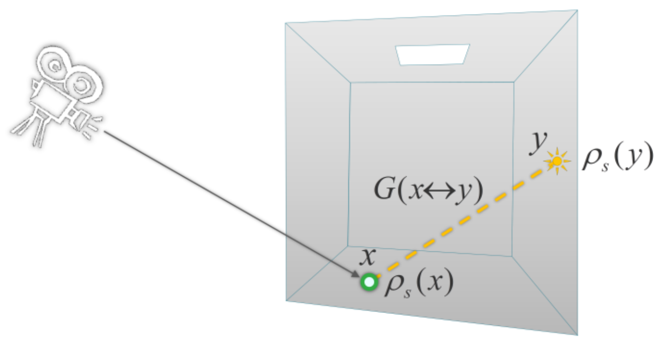
•The instant radiosity algorithm proceeds in two stages.

• In the first stage, we trace sub-paths starting from the light sources, depositing “virtual point lights” at every surface intersection.

• In the second stage, we render the image by computing the contribution from all the virtual point lights to the scene points seen through camera pixels.

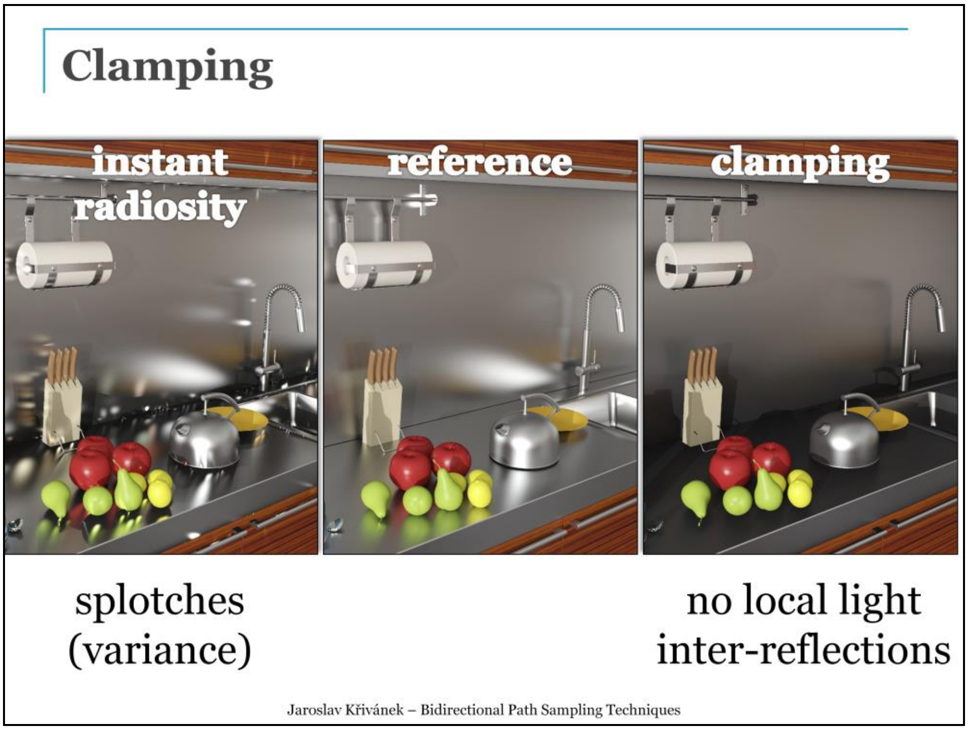
VPL contribution

- High values for spiky BRDFs
- Diverges as $\| x - y \| \to 0$



• The contribution of the VPL at *y* to a surface point at *x* is given by the product of the scattering term (BSDF) at the two vertices, the geometry term of the connecting edge, and the VPL energy.

• This expression can take extremely large values for spiky BSDFs and when the points *x* and *y* approach each other (in fact, the expression diverges – goes to infinity – in the latter case).

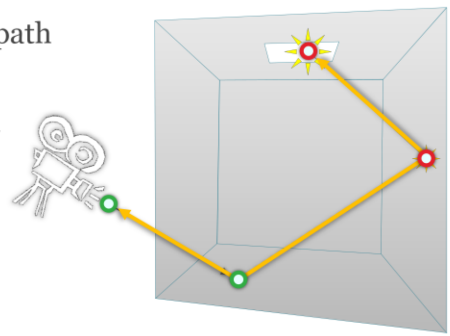


• The usual approach to deal with the singularities in VPL contribution is the so called *clamping*, where we simply limit the contribution of the VPL by some maximum allowed value.

• However, this is far from being an ideal solution because it removes a lot of energy from the scene, yielding darkening of surface and change of material appearance.

VPL rendering as a bidirectional path sampling technique

1. **Distribute VPLs**
   - = sample light sub-paths
2. **Camera ray**
   - = sample camera sub-path
3. **VPL contribution**
   - = sub-path connection



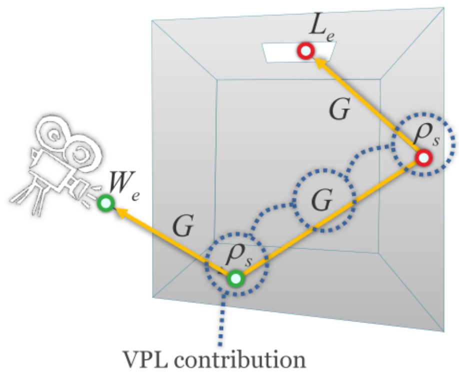
Jaroslav Křivánek – Bidirectional Path Sampling Techniques 7

- Let’s now look at the exact same process in the path integral framework.
- In the first step, we distribute the VPLs. Of course, that is nothing else than sampling sub-paths starting from a light source.
- Finding out a point visible through a pixel from the camera involves building a length-1 sub-path from the camera.
- And finally, evaluating the VPL contribution completes a full light transport path by connecting two sub-paths together.

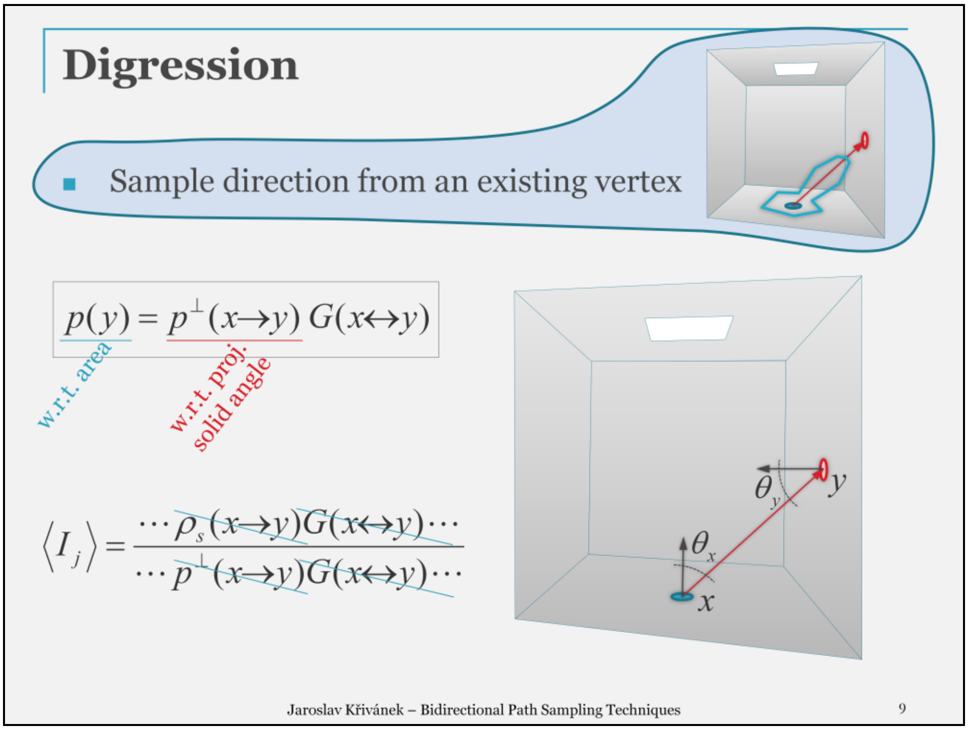
VPL rendering as a bidirectional path sampling technique

- The usual path integral **estimator**

$$\langle I_j \rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$



- Once again, we use the exact same general form the of path integral estimator, that is the value of the measurement contribution function divided by the path PDF.
- Again, the measurement contribution function is given by the product of the emission, sensor sensitivity, BSDFs at the path vertices, and the geometry terms for the path segments.
- And notice that the factor of the contribution function associated with the VPL connection edge are exactly the terms that need to be evaluated when computing the contribution of a VPL.
- So how does the path PDF looks like?



• To see that, I will allow myself a little digression back to a slide I showed a couple of minutes ago.

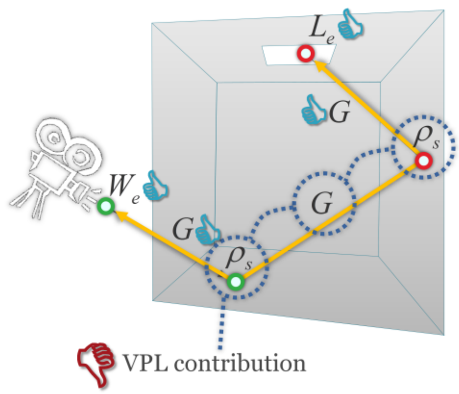
• On this slide I was showing that every time we sample a vertex, *y* in this example, by picking a random direction from another vertex, *x* here, and shooting a ray, we automatically importance sample the geometry term along the edge.

• However, the geometry term for edges constructed by connection are not importance sampled.

VPL rendering as a bidirectional path sampling technique

- **The usual estimator**

$$\langle I_j \rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$



• So we see here that the geometry terms for the segments on the light and camera sub-path are importance sampled.

• Also, the radiance emission and sensor sensitivity are importance sampled because we usually pick the initial path vertex and initial direction proportional to these quantities.

• But notice that none of the quantities associated to the connecting edge are importance sampled – Indeed, we just blindly connect two vertices. And that's exactly where all the problems with VPLs are coming from.

VPL rendering summary

- **VPL rendering** corresponds to a **bidirectional path sampling technique**
- Splotches = noise = variance
  - Due to **bad path sampling**
  - Correlation

Jaroslav Křivánek – Bidirectional Path Sampling Techniques 11

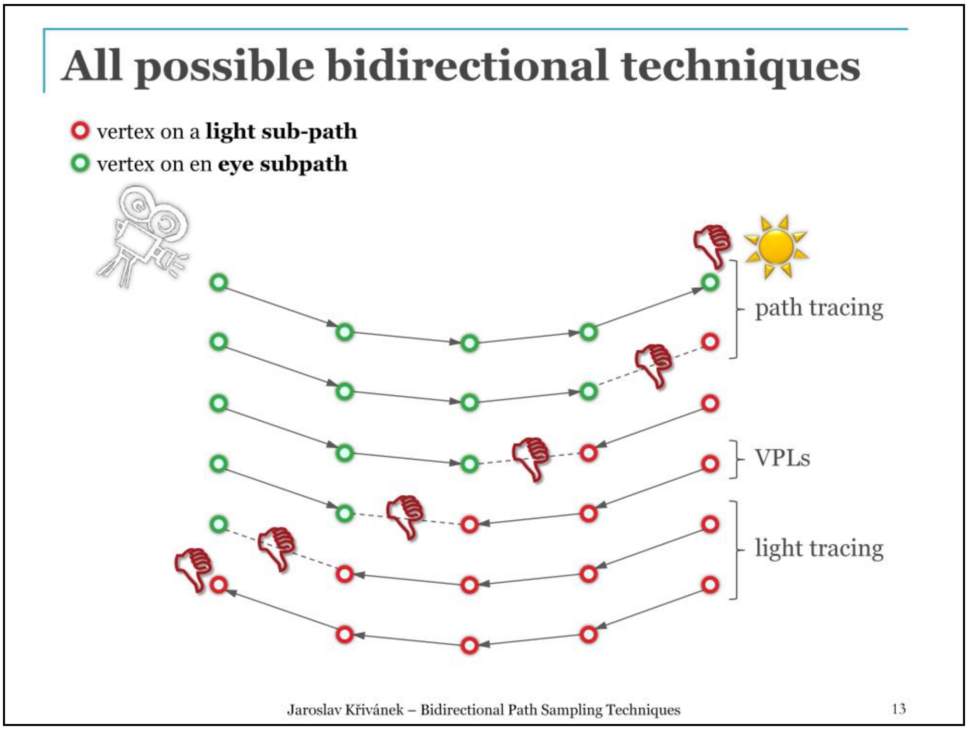
• To summarize, VPL rendering is easily interpreted in the path integral framework as a bidirectional path sampling technique.

• This view allows us to clearly identify that the splotches typical for VPL rendering are in fact just a demonstration of the variance caused by bad path sampling.

• Also notice that the splotches are in fact conceptually the exact same thing as noise in path tracing: both are just a visual manifestation of the variance of the underlying estimators. The reason we obtain splotches in VPL rendering is the inter-pixel correlation due to the VPL reuse across different pixels.

COMBINING PATH SAMPLING TECHNIQUES

• Let’s now use the experience form the VPL rendering example to motivate the bidirectional path tracing algorithm.



• This slide schematically shows all the possible bidirectional techniques that we can obtain by starting a path either on light source or on the camera and applying only the basic three operations of local path sampling for an example path of length 4.

• The first two cases correspond to what a regular path tracer usually does (randomly hitting the light sources and explicit light source connections.)

• The fourth correspond to VPL sampling.

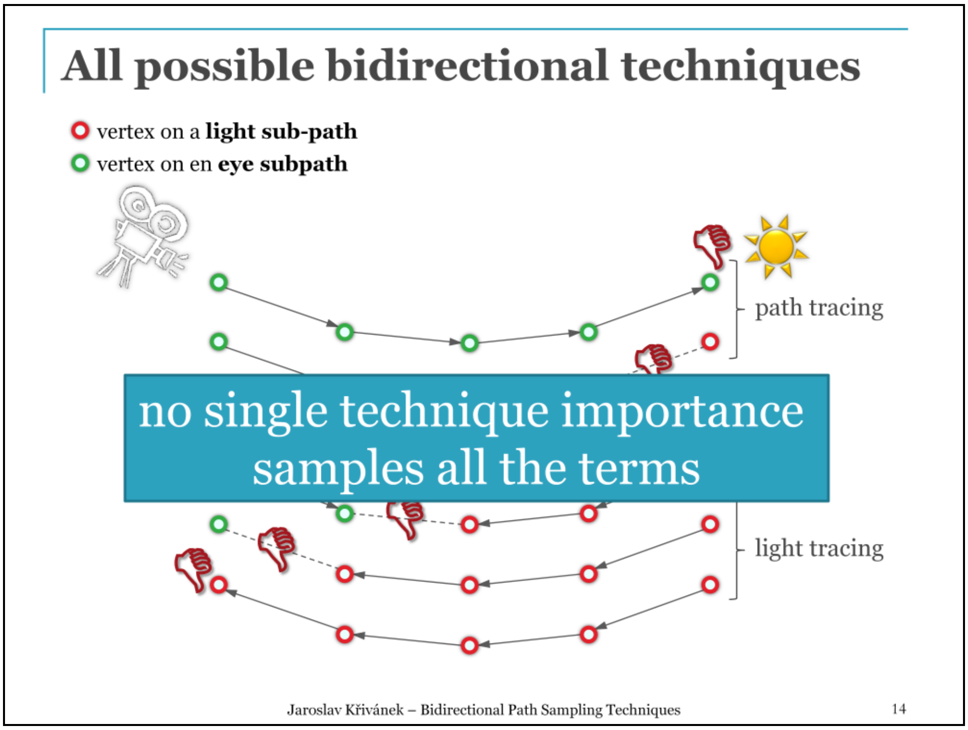
• And the last two are complementary to the first two and therefore correspond to light tracing.

• Each sampling technique importance samples a different subset of terms of the measurement contribution function.

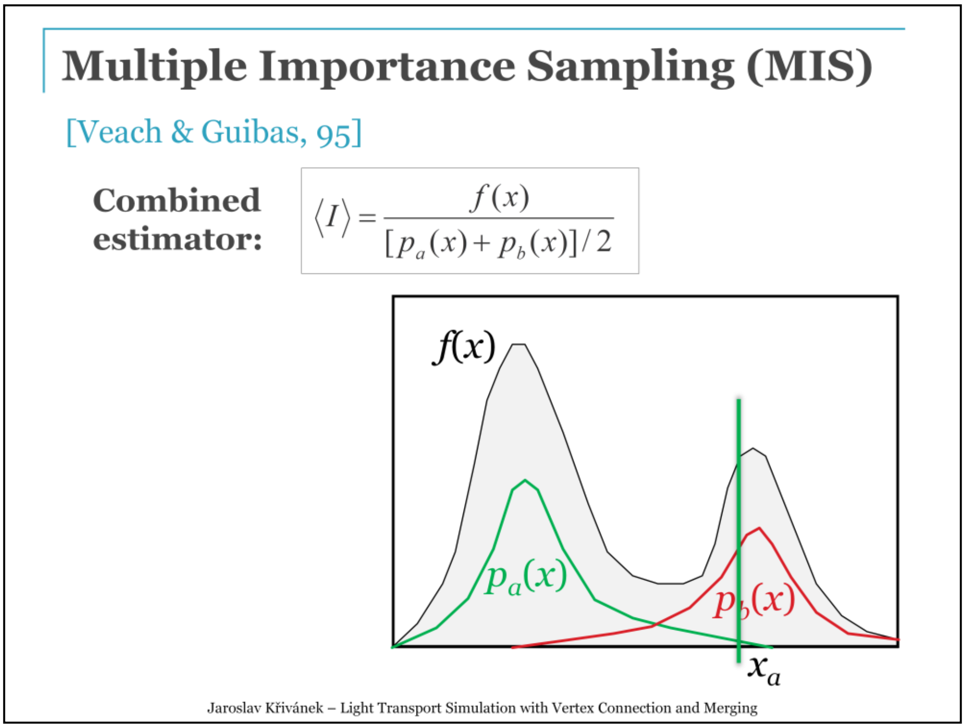
• However, in each of these techniques, there are some terms of the measurement contribution function that are not importance sampled.

• The purely unidirectional techniques (top and bottom) do not importance sample the light emission and sensor sensitivity, respectively. Indeed, for example the technique at the top relies on randomly hitting a light source, without incorporating any information about the location of light sources in the scene.

• All the bidirectional techniques, that is, those that involve connection of two sub-paths, are unable to importance sample the terms associated with the connection edge, exactly as in the case of VPLs.



• But none of the techniques shown here is able to importance sample all of the terms of the measurement contribution function.



• This slide shows this situation in a simpler setting.

• We have a complex multimodal integrand *f*(*x*) that we want to numerically integrate using a MC method with importance sampling.

• Unfortunately, we do not have a PDF that would mimic the integrand in the entire domain.

• Instead, we can draw the sample from two different PDFs, p_a and p_b each of which is a good match for the integrand under different conditions – i.e. in different part of the domain.

• However, the estimators corresponding to these two PDFs have extremely high variance – shown on the slide.

• We can use Multiple Importance Sampling (MIS) to combine the sampling techniques corresponding to the two PDFs into a single, robust, combined technique.

• The MIS procedure is extremely simple: it randomly picks one distribution to sample from (p_a or p_b , say with fifty-fifty chance) and then takes the sample from the selected distribution.

• This essentially corresponds to sampling from a weighted average of the two distributions, which is reflected in the form of the estimator, shown on the slide.

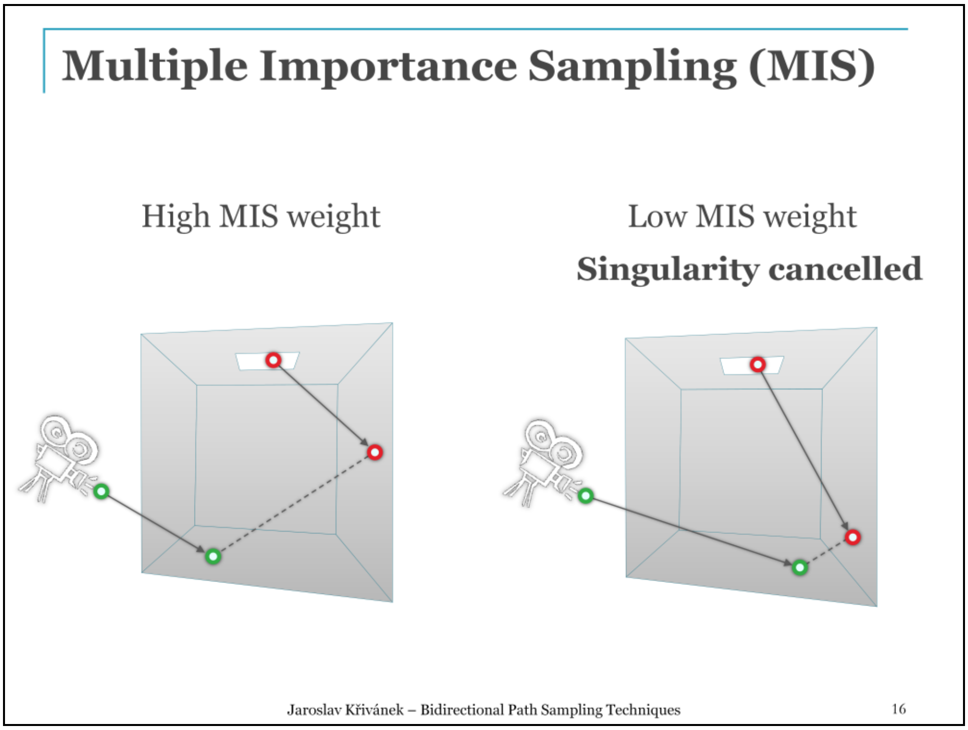
• This estimator is really powerful at suppressing outlier samples such as those that you would obtain by picking *x*_from the tail of p_a, where *f*(*x*) might still be large.

• Without having p_b at our disposal, we would be dividing the large *f*(*x*) by the small p_a

(x), producing an outlier.

• However, the combined technique has a much higher chance of producing this particular x (because it can sample it also from p_b), so the combined estimator divides $f(x)$ by $[p_a(x) + p_b(x)] / 2$, which yields a much more reasonable sample value.

• I want to note that what I'm showing here is called the "balance heuristic" and is a part of a wider theory on weighted combinations of estimators proposed by Veach and Guibas.



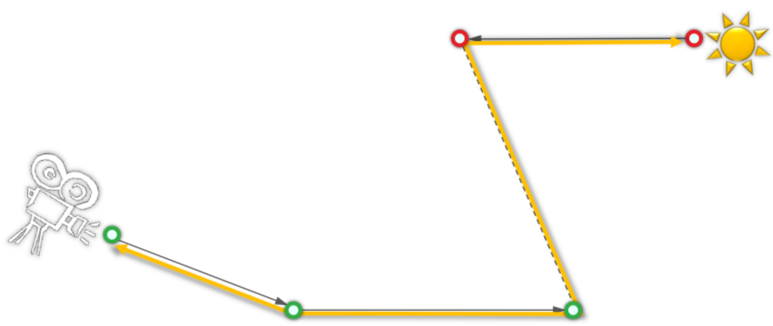
• Applying this to path sampling, this technique will weight down a lot the contribution of paths with short connecting edges, suppressing the high variance observed in the VPL methods.

Bidirectional path tracing

- Use **all** of the above sampling techniques
- Combine using **Multiple Importance Sampling**

• The main idea of bidirectional path tracing is to use all of the sampling techniques above and combine them using multiple importance sampling.

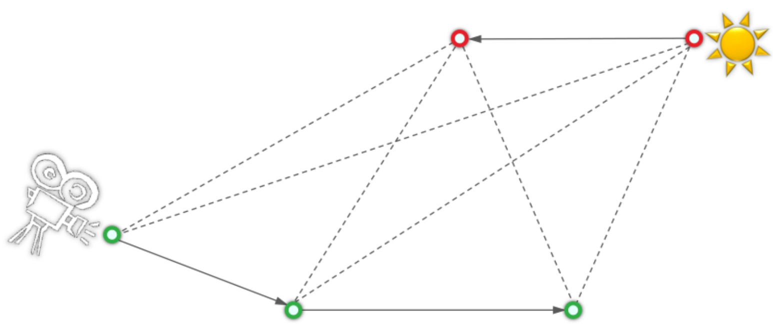
Naive BPT implementation



Jaroslav Křivánek – Bidirectional Path Sampling Techniques 18

• The usual description of bidirectional path tracing, where two independent sub-paths are generated first, and the each vertex from the first is connected to each vertex of the second, is really just an implementation detail. It does improve the efficiency thanks to reuse of the sub-paths, but does not contribute to the robustness of BPT in any way.

BPT Implementation in practice



• The usual description of bidirectional path tracing, where two independent sub-paths are generated first, and the each vertex from the first is connected to each vertex of the second, is really just an implementation detail. It does improve the efficiency thanks to reuse of the sub-paths, but does not contribute to the robustness of BPT in any way.

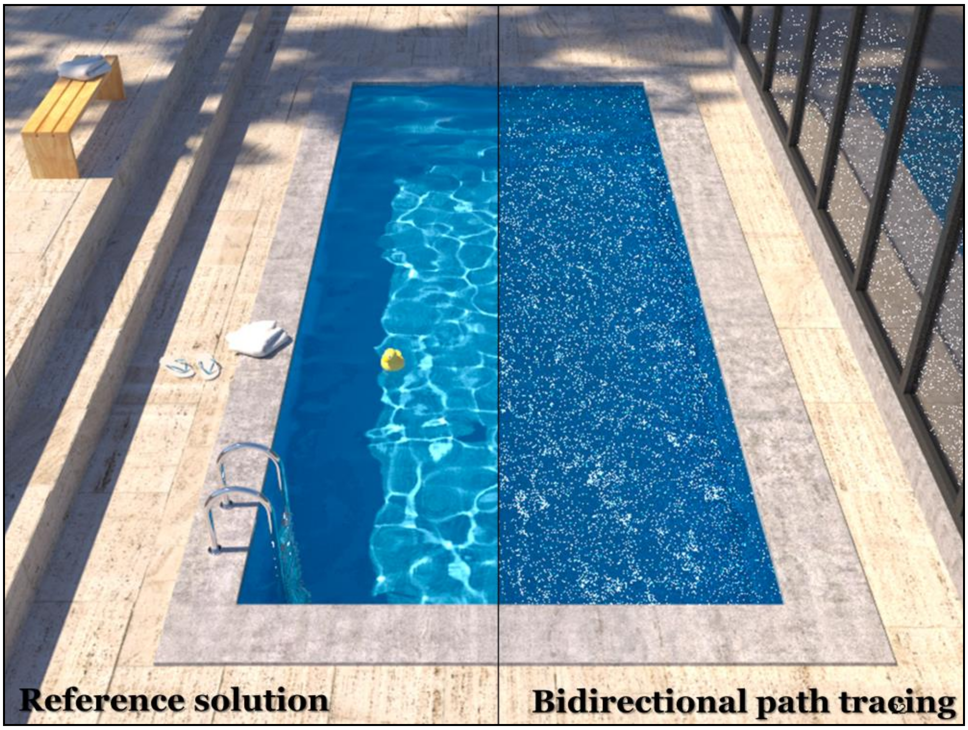
Results

BPT, 25 samples per pixel

PT, 56 samples per pixel

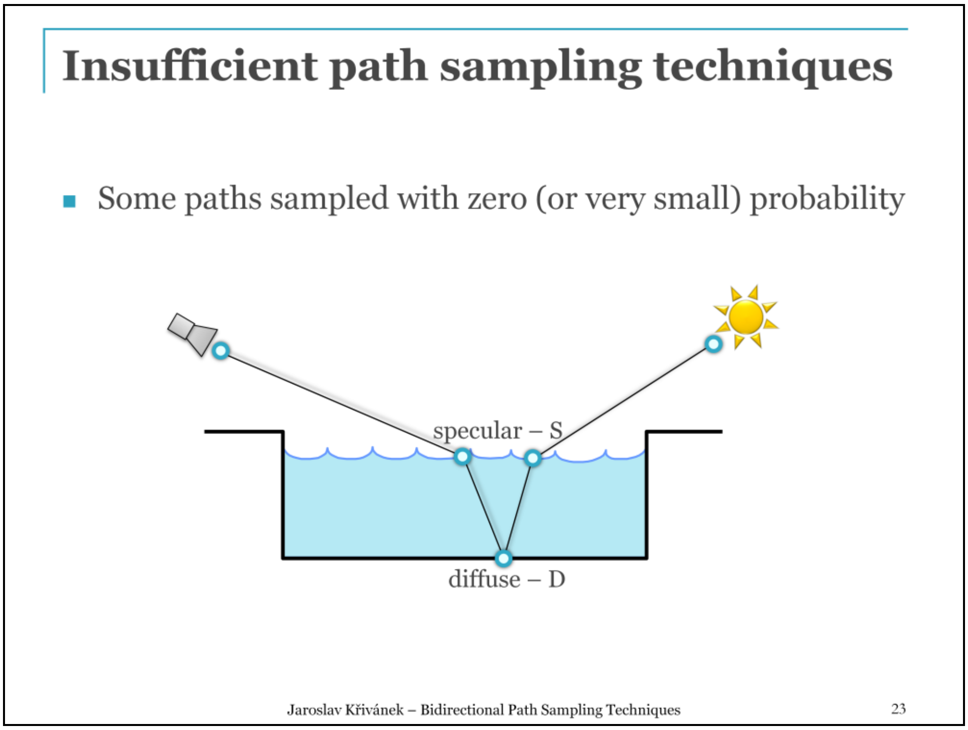
Images: Eric Veach

LIMITATIONS OF LOCAL PATH SAMPLING



• Bidirectional path tracing is much more robust than path tracing, light tracing, or VPL rendering.

• However, it still struggles with some lighting effects, the most common of which are the specular-diffuse-specular (SDS) paths corresponding to reflected caustics – in this example the caustics at the pool bottom.



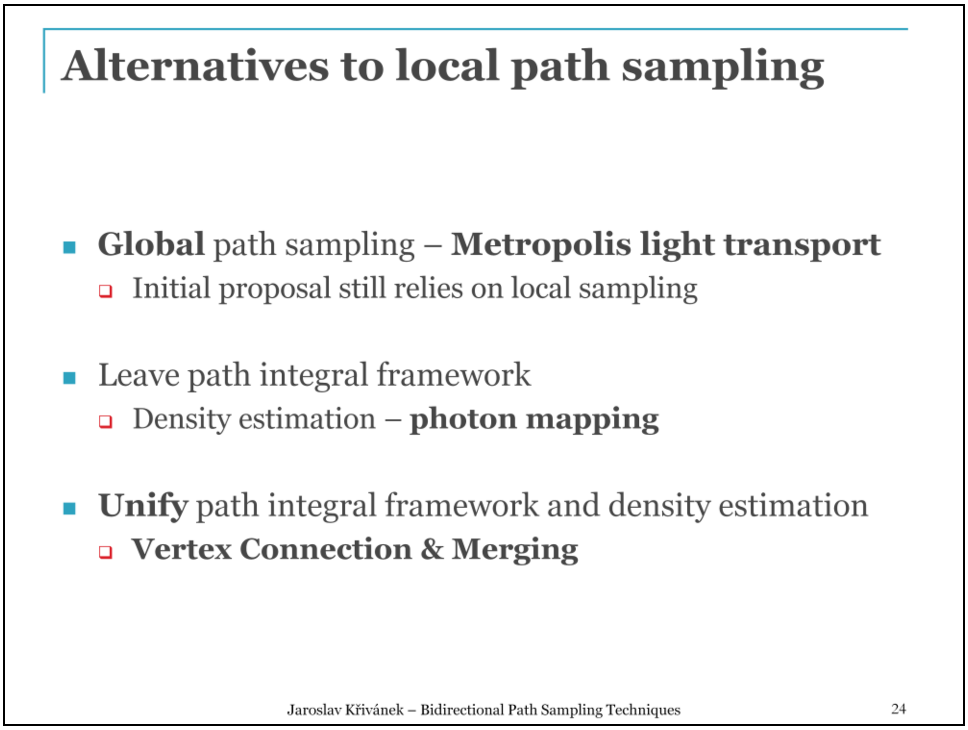
• As usual, the culprit is inappropriate path sampling. The problem is that none of the path sampling techniques used in bidirectional path tracing is efficient at sampling the SDS effects, so their combination cannot sample those effects either.

• To see this, consider the example on the slide. We have a pool (diffuse – D) filler with water (specular – S), a pinhole camera, and a small light source.

• No path connections are possible because of the two specular vertices. Unidirectional sampling from the light source is not possible either because of the pinhole camera.

• So we are left with one single (unidirectional) path sampling technique that starts from the camera, and hopes to randomly strike the light source. It is not hard to see that the smaller the source, the lower the probability of hitting the source and the higher the estimator variance.

• In the limit, for point sources and pinhole camera, the SDS effects cannot be sampled by local path sampling at all.



• To sample SDS effects efficiently, we need to look for alternatives to local path sampling.

• One such alternative are global path sampling techniques that sample a path as a whole, as opposed to incrementally vertex-by-vertex. This is for example the case of Metropolis light transport and other Markov Chain Monte Carlo techniques. The issue is that the Markov chain of path mutations needs to be initialized with some path, and this path has to be generated somehow – and we're back to local path sampling.

• We could also look for an entirely different solution of light transport, outside the classic path integral formulation. A popular example of this approach is photon mapping, which relies on density estimation.

• However, photon mapping is fairly inefficient at some lighting effects (such as diffuse inter-reflections) which are very efficiently handled by the path sampling techniques in BPT.

• So the vertex connection and merging algorithm reformulates photon mapping in the path integral framework, which enables a robust combination of photon mapping and BPT using Multiple Importance Sampling.

NEARLY THERE…

“Path integral” – A historical remark

- This course [Veach and Guibas 1995], [Veach 1997]
  - Easily derived form the rendering equation [Veach 1997]
- Feynman path integral formulation of quantum mechanics [Feynman and Hibbs 65]
- Homogeneous materials [Tessendorf 89, 91, 92]
- Rendering [Premože et al. 03, 04]

• The term “path integral formulation” has a different meaning for different people.

• What I’ve presented in this course has been derived by Veach and Guibas by fairly straightforward manipulation of the Neumann series solution of the rendering equation.

• More widely known is Feynman’s path integral formulation, used to solve problems in quantum mechanics. In this formulation the transport paths are general curves, so the path space that we considered here (polylines) is just a small sub-space (of measure zero) of the path space in Feynman’s formulation.

•Tessendorf used Feynman’s path integral formulation to derive the solution of light transport in strongly forward scattering media.

•This solution has been used for rendering by Premože and colleagues.

Summary

- **VPL rendering**
  - One bidirectional path sampling technique
  - Not robust
- **Bidirectional path tracing**
  - Combines many bidirectional techniques
  - More robust
  - Bad at reflected caustics

Acknowledgements

- **Czech Science Foundation**
  - grant no. P202-13-26189S

- Images
  - **Ondra “Keymaster” Karlík**
  - **Eric Tabellion**
  - **Marcos Fajardo**

THANK YOU!

Time for questions…

Tutorial: Path Integral Methods for Light Transport Simulation

Jaroslav Křivánek – Bidirectional Path Sampling Techniques