Combining path integral estimators and particle density estimators in light transport simulation

Jaroslav Křivánek

Charles University in Prague

Computer Graphics Charles University

Contributors

- **I. Georgiev**, **J. Křivánek**, **T. Davidovič**, **P. Slusallek**
  SIGGRAPH Asia 2012.

- **T. Hachisuka**, **J. Pantaleoni**, and **H. W. Jensen**
  SIGGRAPH Asia 2012.

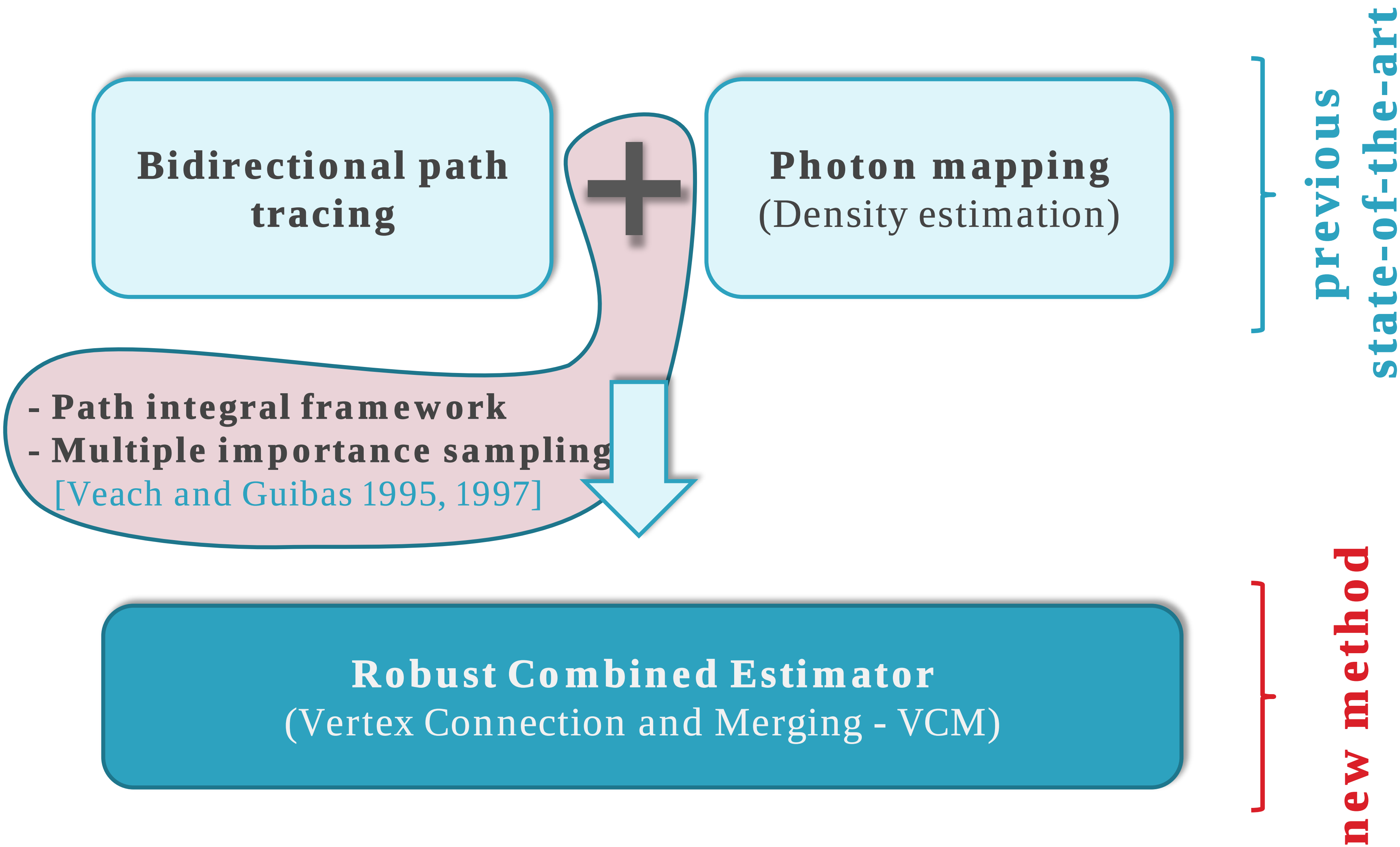
- **J. Křivánek**, **I. Georgiev**, **T. Hachisuka**, **P. Vévoda**, **M. Šik**, **D. Nowrouzezahrai**, **W. Jarosz**
  SIGGRAPH 2014.


Bidirectional path tracing (30 min)

Photon mapping (Density estimation) (30 min)


Vertex connection and merging (30 min)

Approach

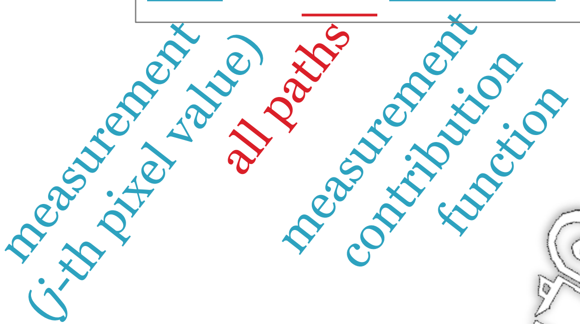

Bidirectional path tracing
Photon mapping
(Density estimation)
- Path integral framework
- Multiple importance sampling
[Veach and Guibas 1995, 1997]
Robust Combined Estimator
(Vertex Connection and Merging - VCM)
previous state-of-the-art
new method

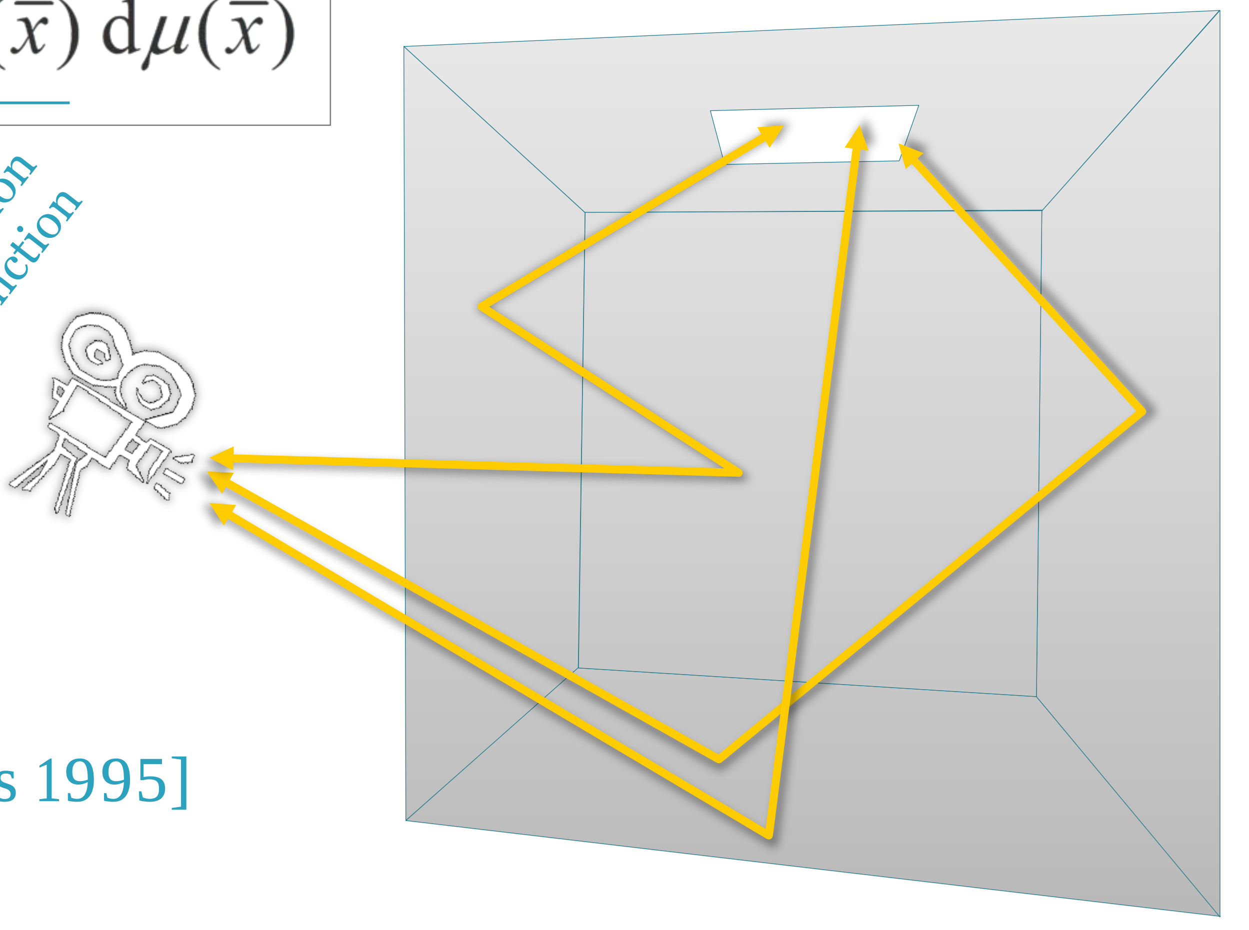
PATH INTEGRAL ESTIMATORS & BIDIRECTIONAL PATH TRACING

Path integral formulation

Path integral

$$I_j = \int_\Omega f_j(\bar{x})\, \mathrm{d}\mu(\bar{x})$$





[Veach and Guibas 1995]
[Veach 1997]

MC evaluation of the path integral

Path integral

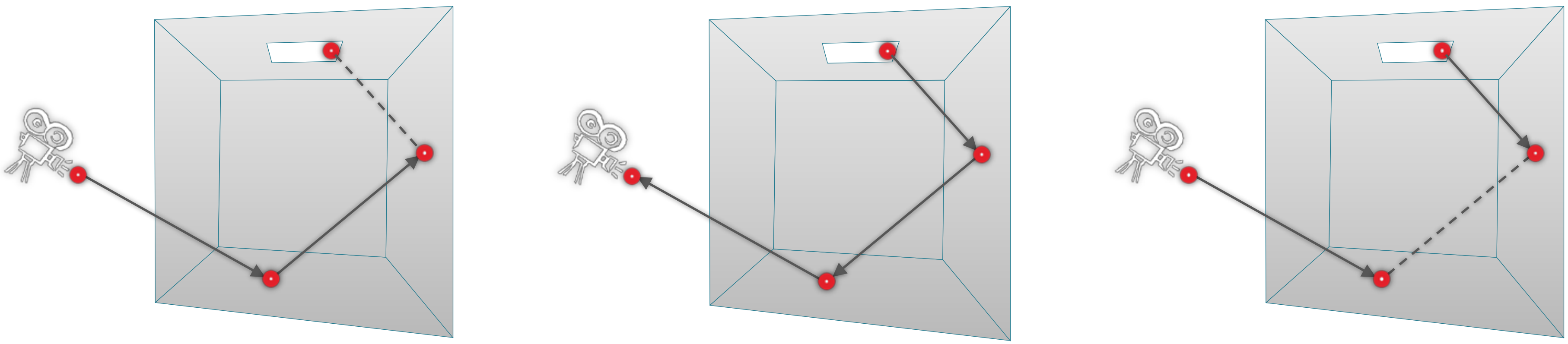
$$I_j = \int_\Omega f_j(\bar{x})\,\mathrm{d}\mu(\bar{x})$$

MC estimator

$$\langle I_j \rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$

- Sample path $\bar{x}$ from some distribution with PDF $p(\bar{x})$
- Evaluate the probability density $p(\bar{x})$
- Evaluate the integrand $f_j(\bar{x})$

Many possible sampling techniques



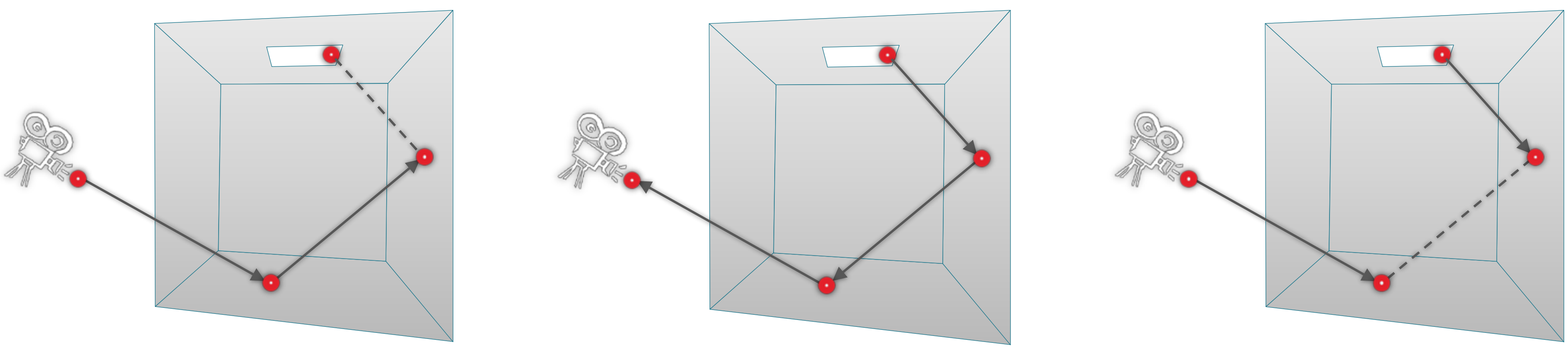
no single technique works well by itself

Bidirectional path tracing

- Use **all** of the above sampling techiques
- Combine using **Multiple Importance Sampling**

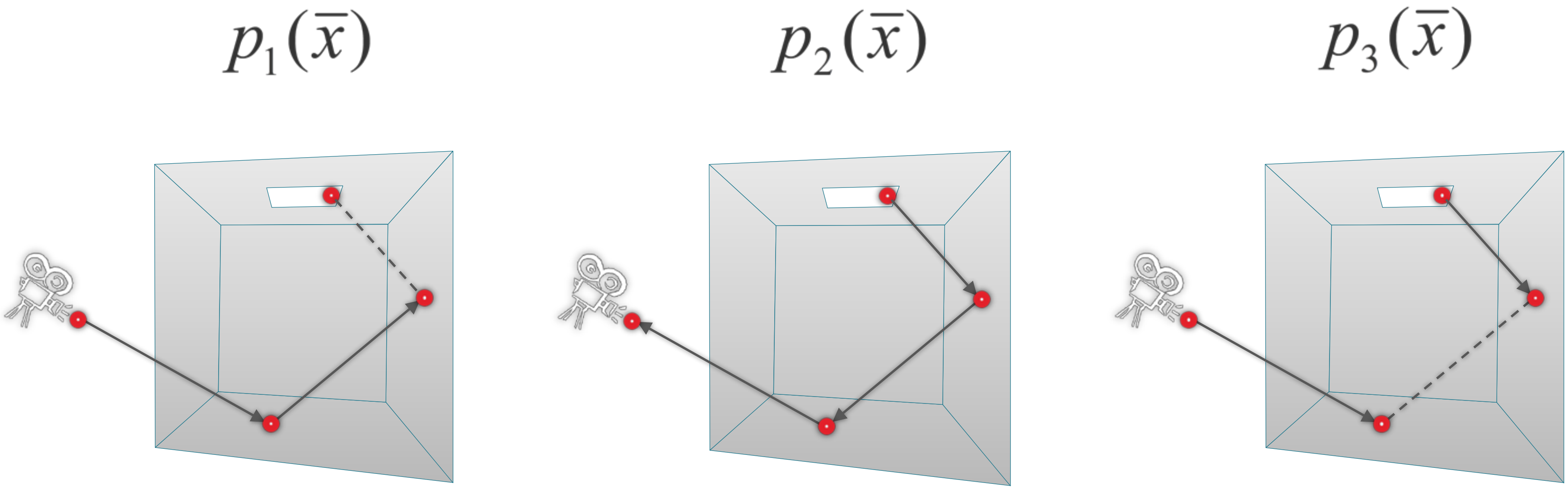
Multiple Importance Sampling

$p_1(\bar{x})$ $p_2(\bar{x})$ $p_3(\bar{x})$



$$w_i(\bar{x}) \propto p_i(\bar{x})$$

Multiple Importance Sampling



$$w_i(\bar{x}) = \frac{p_i(\bar{x})}{p_1(\bar{x}) + p_2(\bar{x}) + p_3(\bar{x})}$$

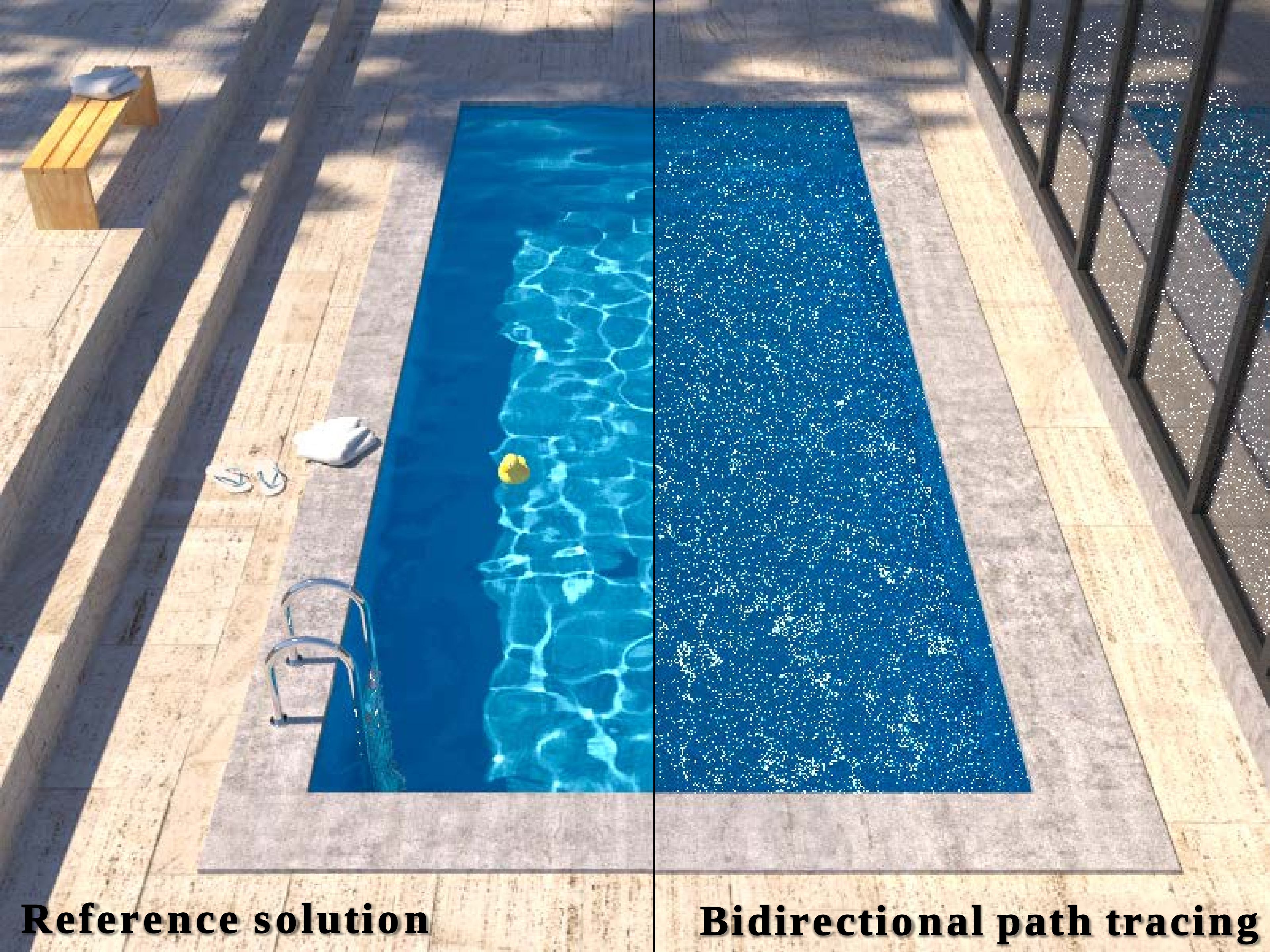
Bidirectional path tracing

BPT, 25 samples per pixel

PT, 56 samples per pixel

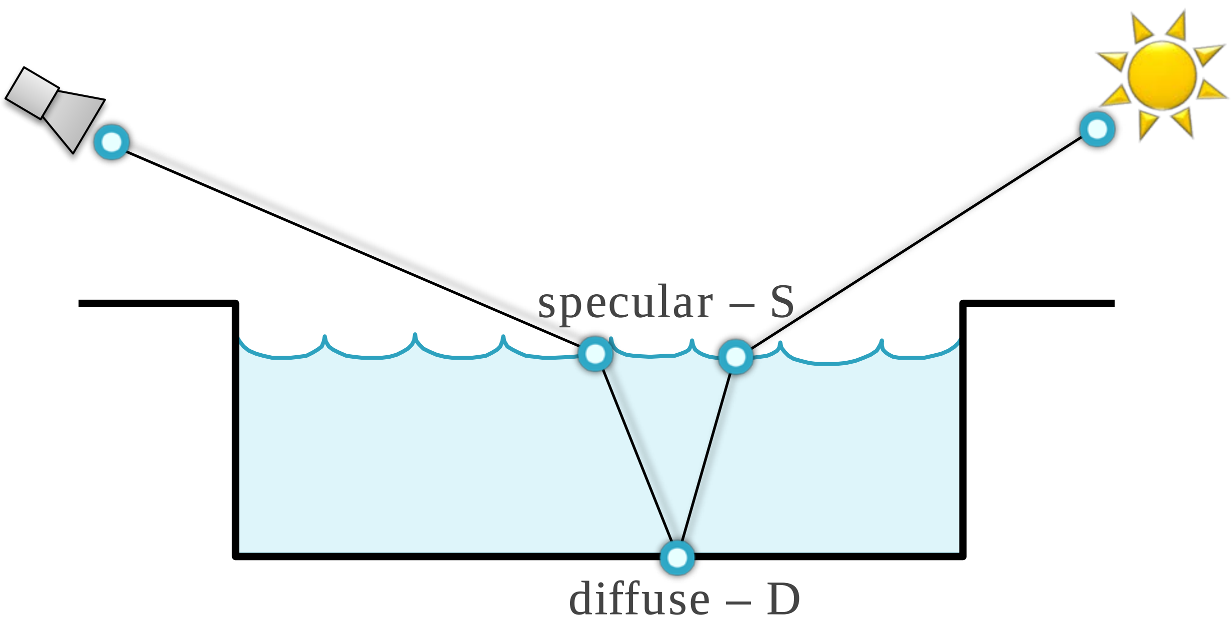
Images: Eric Veach

LIMITATIONS OF BIDIRECTIONAL PATH TRACING


Reference solution
Bidirectional path tracing

Insufficient path sampling techniques

- **SDS paths** sampled with zero (or very small) pdf

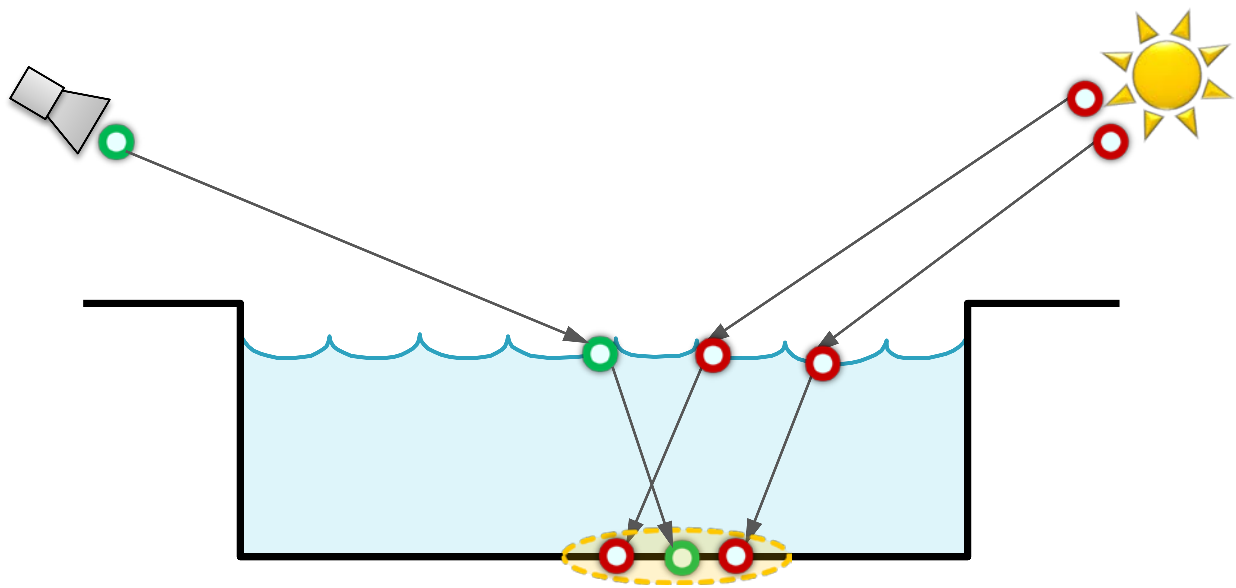


PHOTON MAPPING

(DENSITY ESTIMATION)

Photon mapping (Density estimation)

1. Many fwd walks + store collisions ("photon map")
2. Radiance estimate: (Kernel) **density estimation**



COMBINING

BIDIRECTIONAL PATH TRACING
&
PHOTON MAPPING

Overview

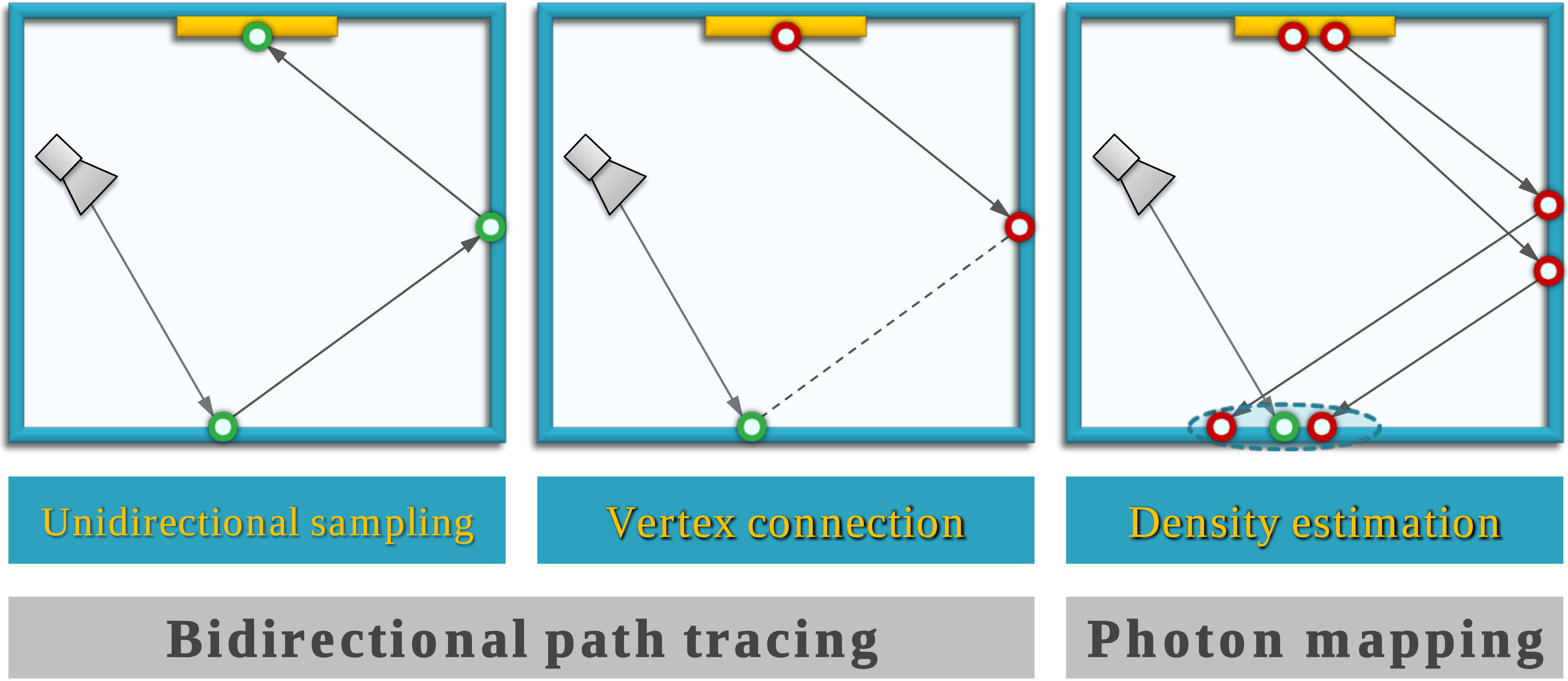
- ☹ **Problem**: different mathematical frameworks
  - ❑ **BPT**: Monte Carlo estimator of a path integral
  - ❑ **PM**: Density estimation

☝ **Key contribution:** Reformulate photon mapping in the path integral framework

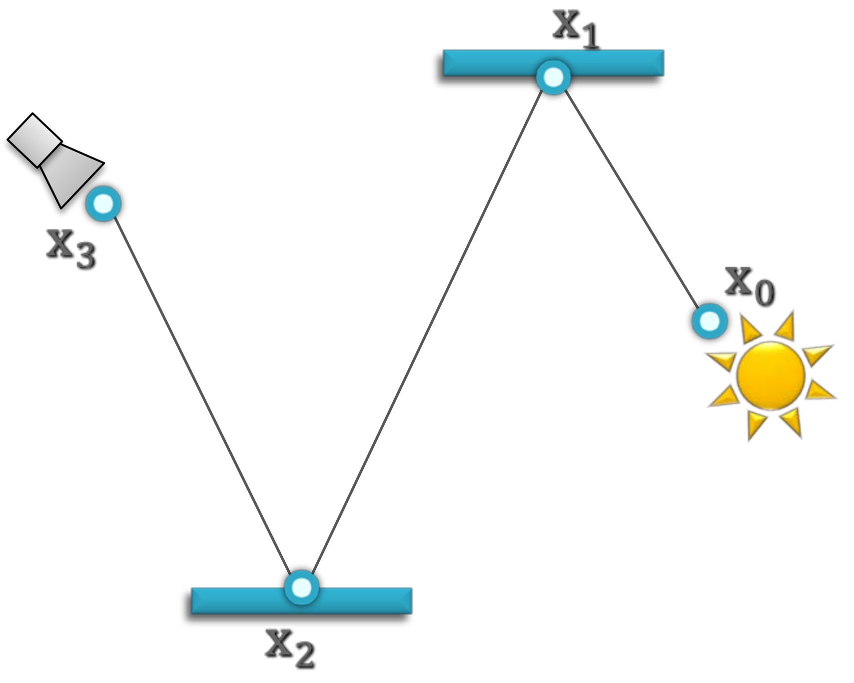
1) Formalize as path sampling technique
2) Derive path probability density

✓ Combination of BPT and PM into a **robust** algorithm

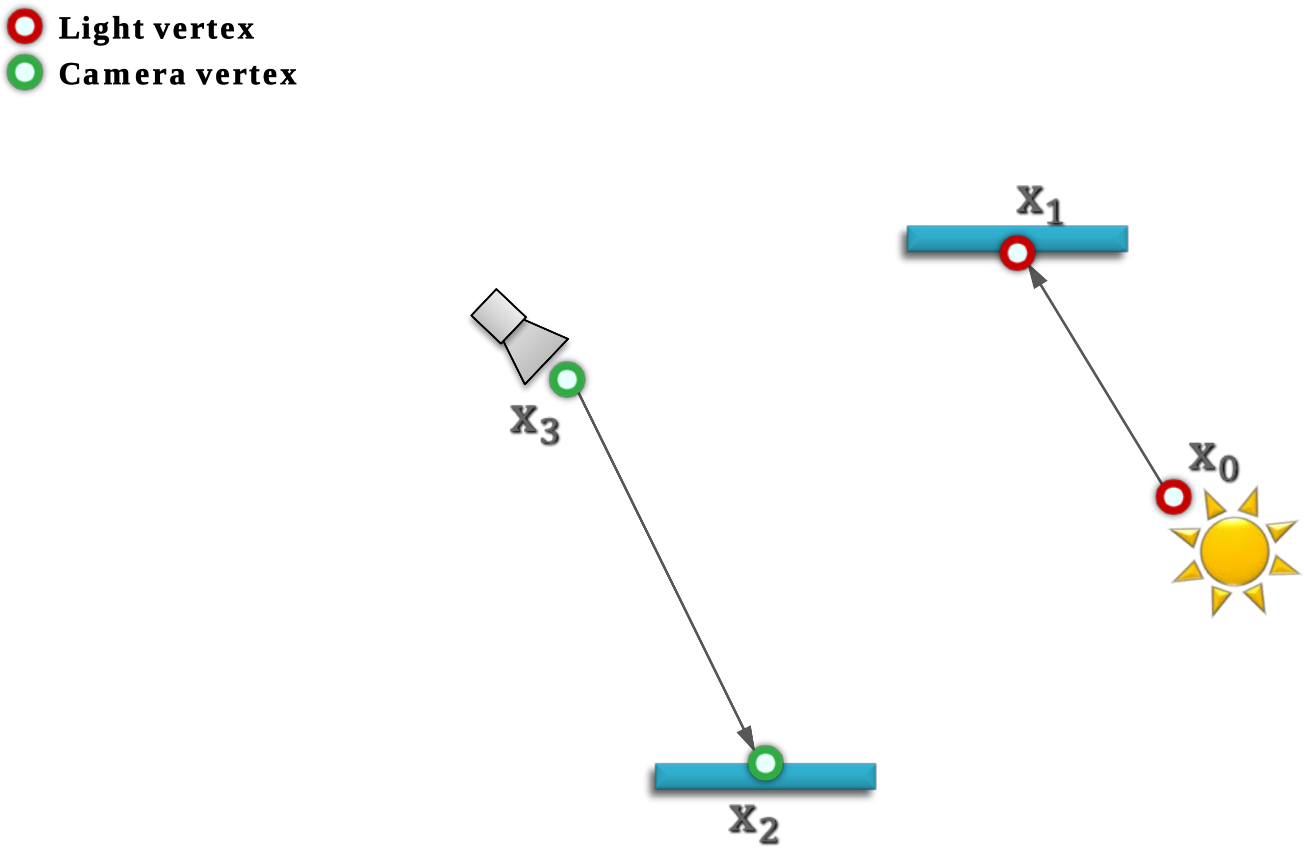
BPT vs PM


Unidirectional sampling
Vertex connection
Density estimation
Bidirectional path tracing
Photon mapping

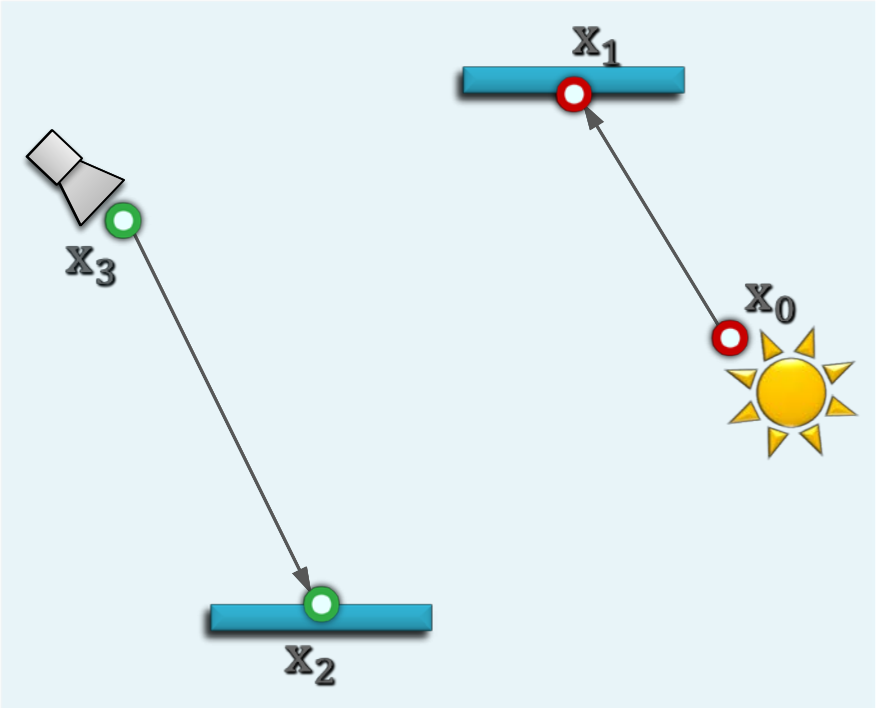
Bidirectional MC path sampling



Bidirectional MC path sampling

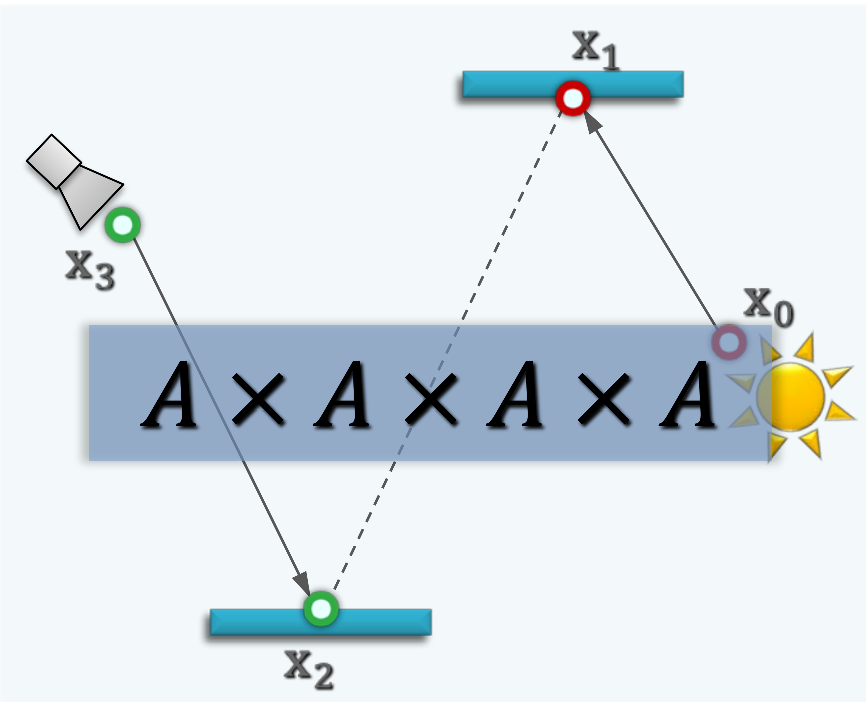


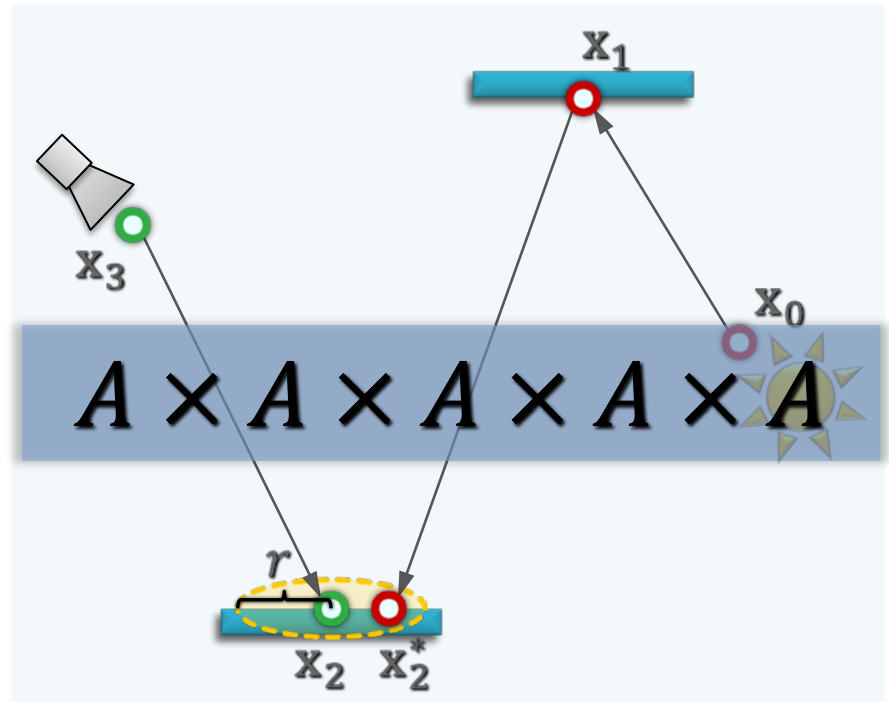
Bidirectional MC path sampling



Bidirectional path tracing | Photon mapping

BPT and PM paths – different spaces





Bidirectional path tracing

$$p(\bar{\mathrm{x}}) = p(\mathrm{x}_0)p(\mathrm{x}_0 \to \mathrm{x}_1)\, p(\mathrm{x}_3)p(\mathrm{x}_3 \to \mathrm{x}_2)$$

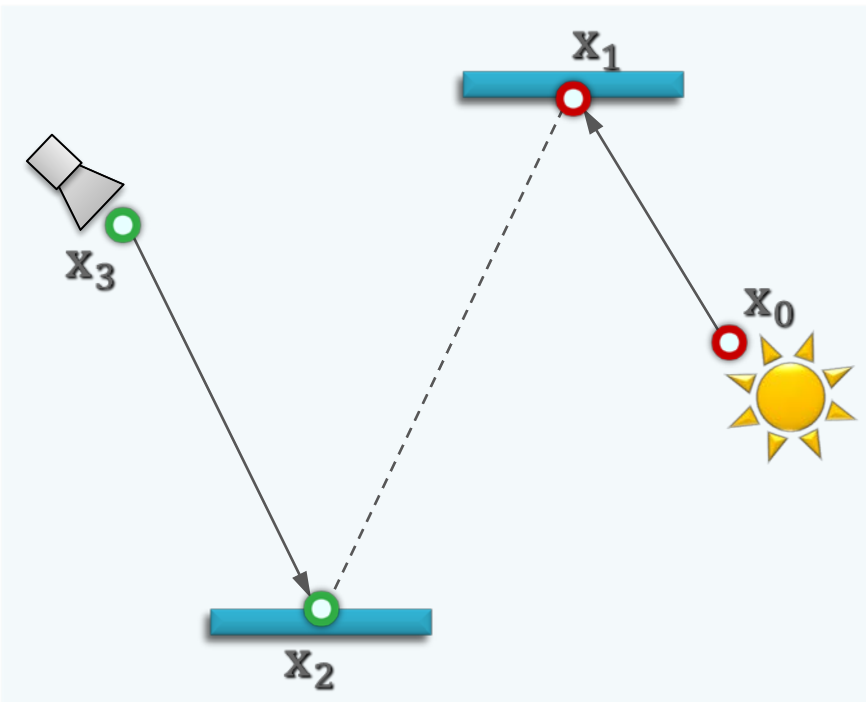
Photon mapping

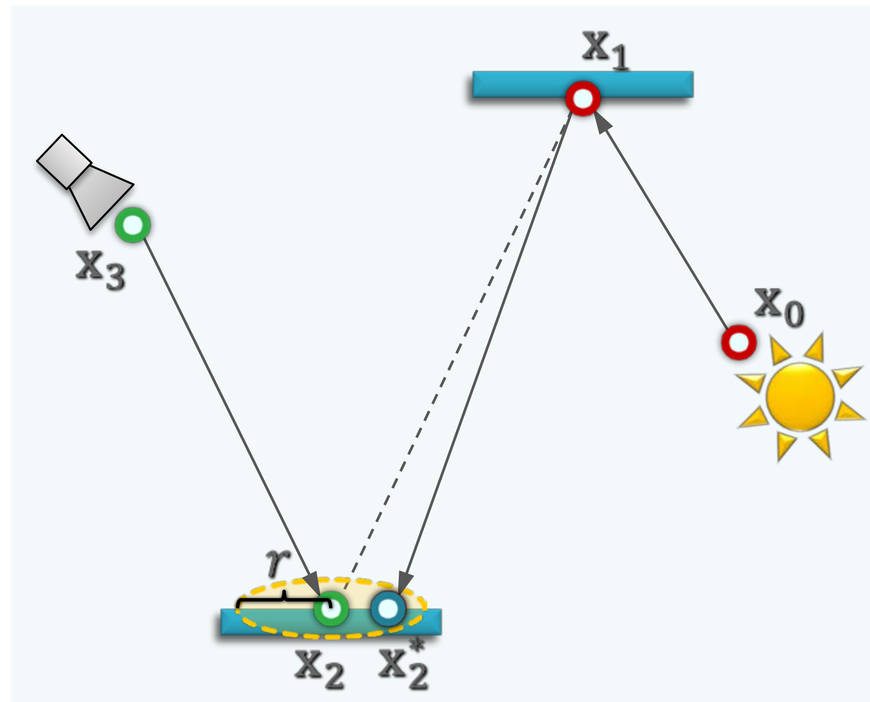
$$p(\bar{\mathrm{x}}) = p(\mathrm{x}_0)p(\mathrm{x}_0 \to \mathrm{x}_1)\, p(\mathrm{x}_1 \to \mathrm{x}_2^*)\, p(\mathrm{x}_3)p(\mathrm{x}_3 \to \mathrm{x}_2)$$

Vertex Connection and Merging

Light vertex
Camera vertex

$$A \times A \times A \times A$$





Bidirectional path tracing / **Vertex connection**

Photon mapping / **Vertex merging**

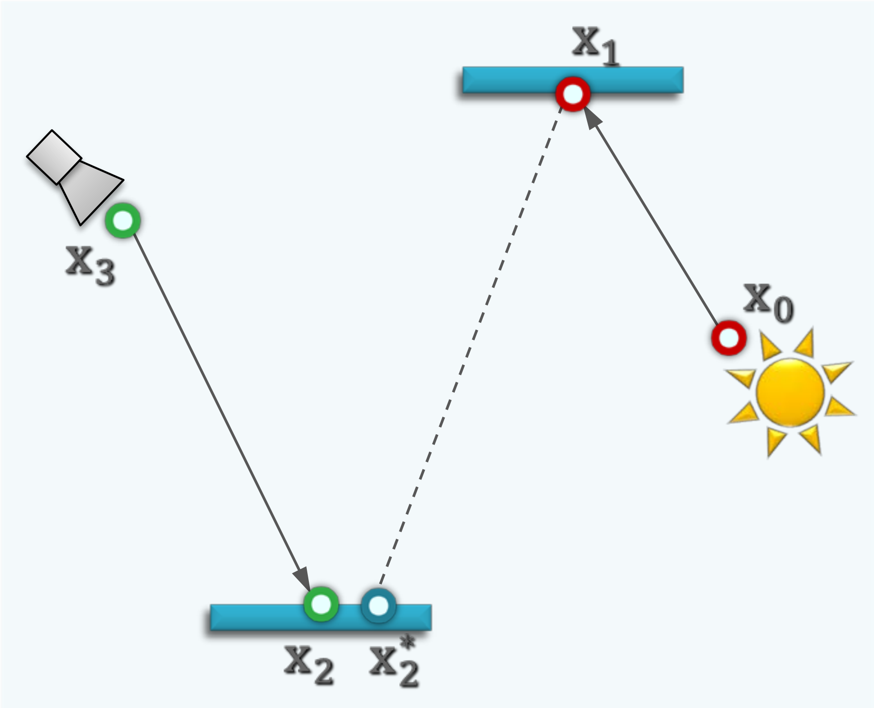
$$p_{VC}(\bar{x}) = p(x_0)p(x_0 \to x_1) \, p(x_3)p(x_3 \to x_2)$$

$$p_{VM}(\bar{x}) = p(x_0)p(x_0 \to x_1) \, P(\|x_2 - x_2^*\| < r) \, p(x_3)p(x_3 \to x_2)$$

Extended path space [Hachisuka et al.]

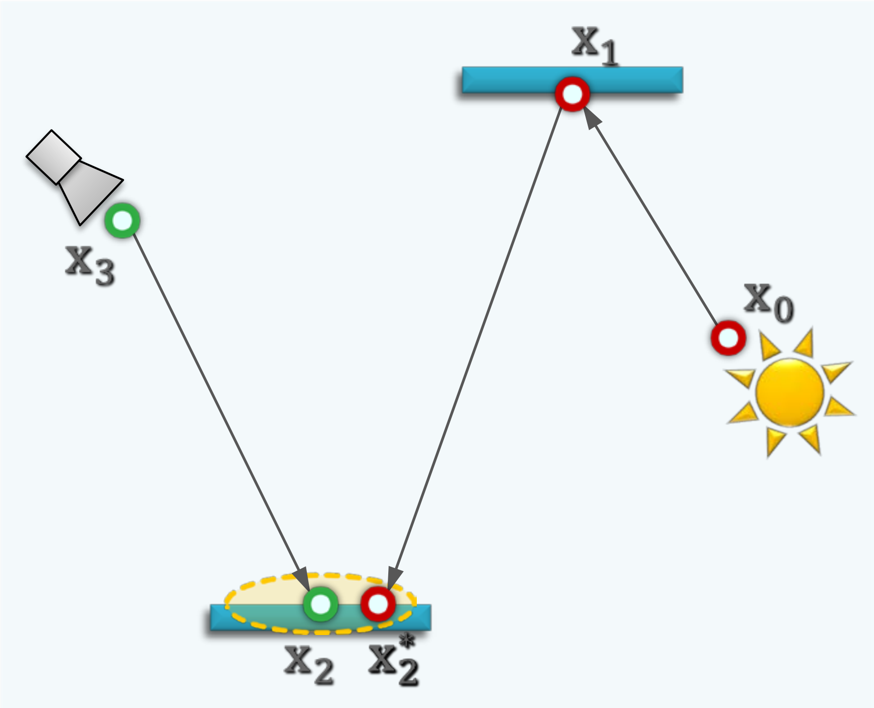
$$A \times A \times A \times A \times A$$

- Light vertex
- Camera vertex



Bidirectional path tracing

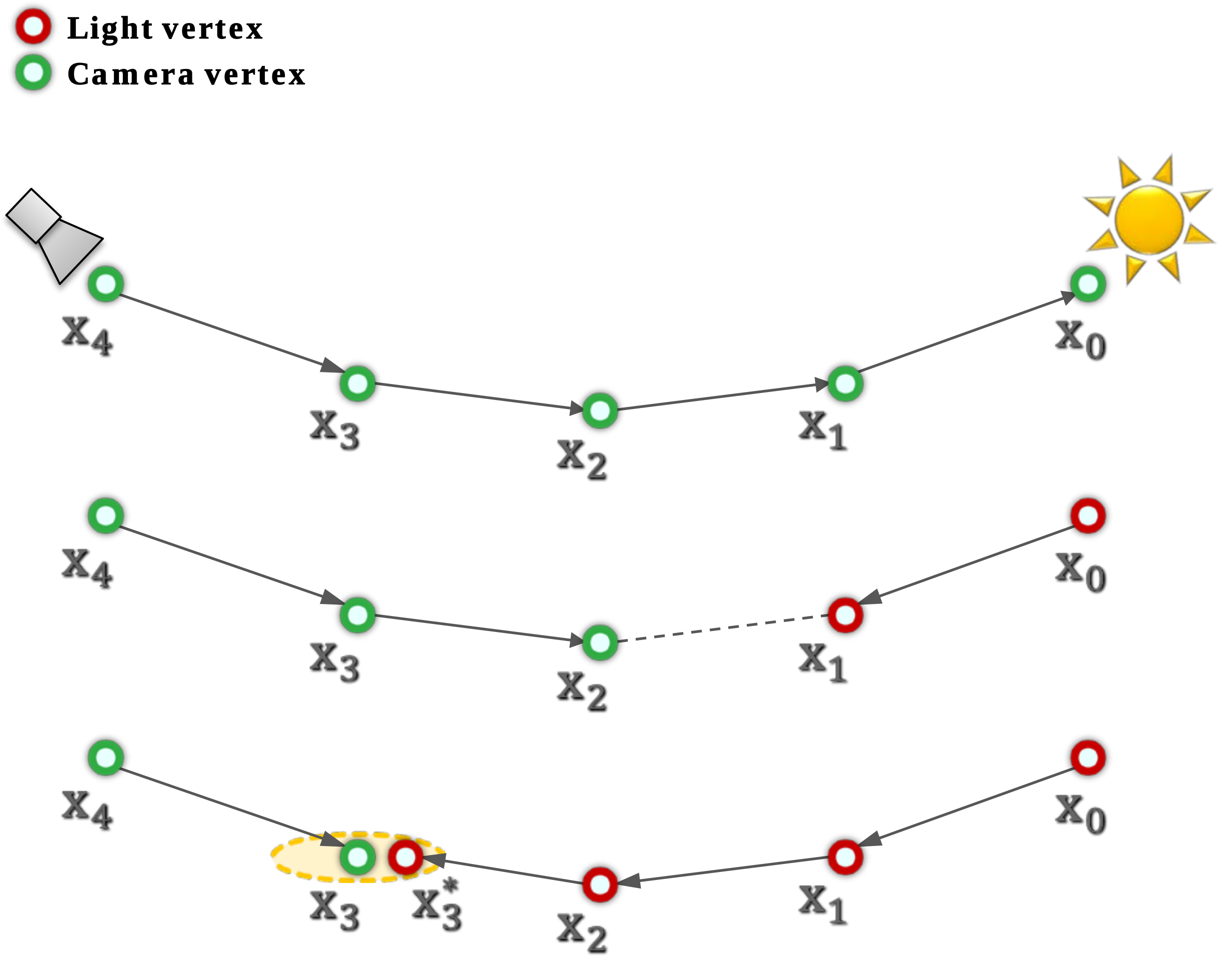
$$p(\bar{\mathbf{x}}) = p(\mathbf{x}_0)p(\mathbf{x}_0 \to \mathbf{x}_1)\, p(\mathbf{x}_3)p(\mathbf{x}_3 \to \mathbf{x}_2)/(\pi r^2)$$



Photon mapping

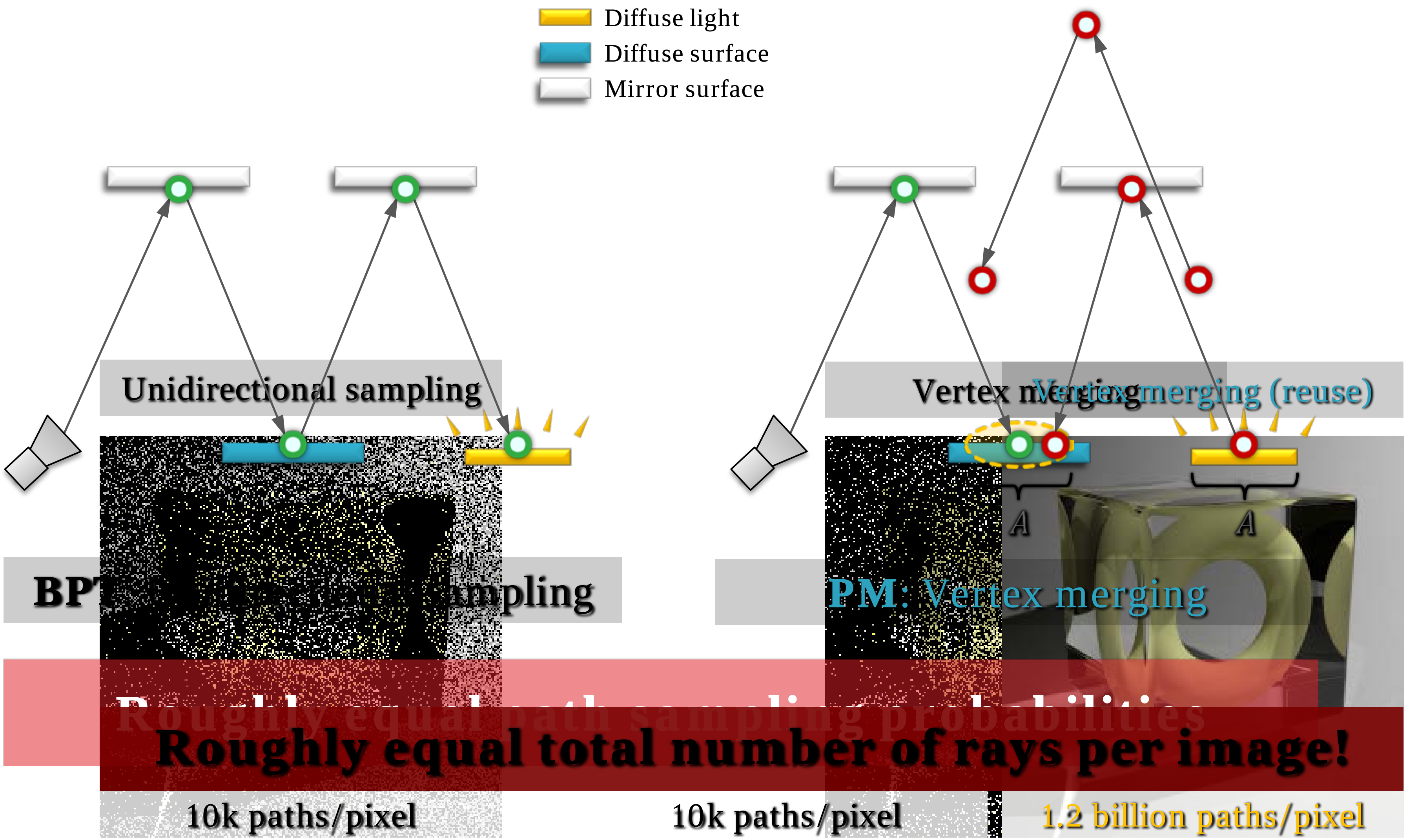
$$p(\bar{\mathbf{x}}) = p(\mathbf{x}_0)p(\mathbf{x}_0 \to \mathbf{x}_1)\ p(\mathbf{x}_1 \to \mathbf{x}_2^*)\, p(\mathbf{x}_3)p(\mathbf{x}_3 \to \mathbf{x}_2)$$

Sampling techniques



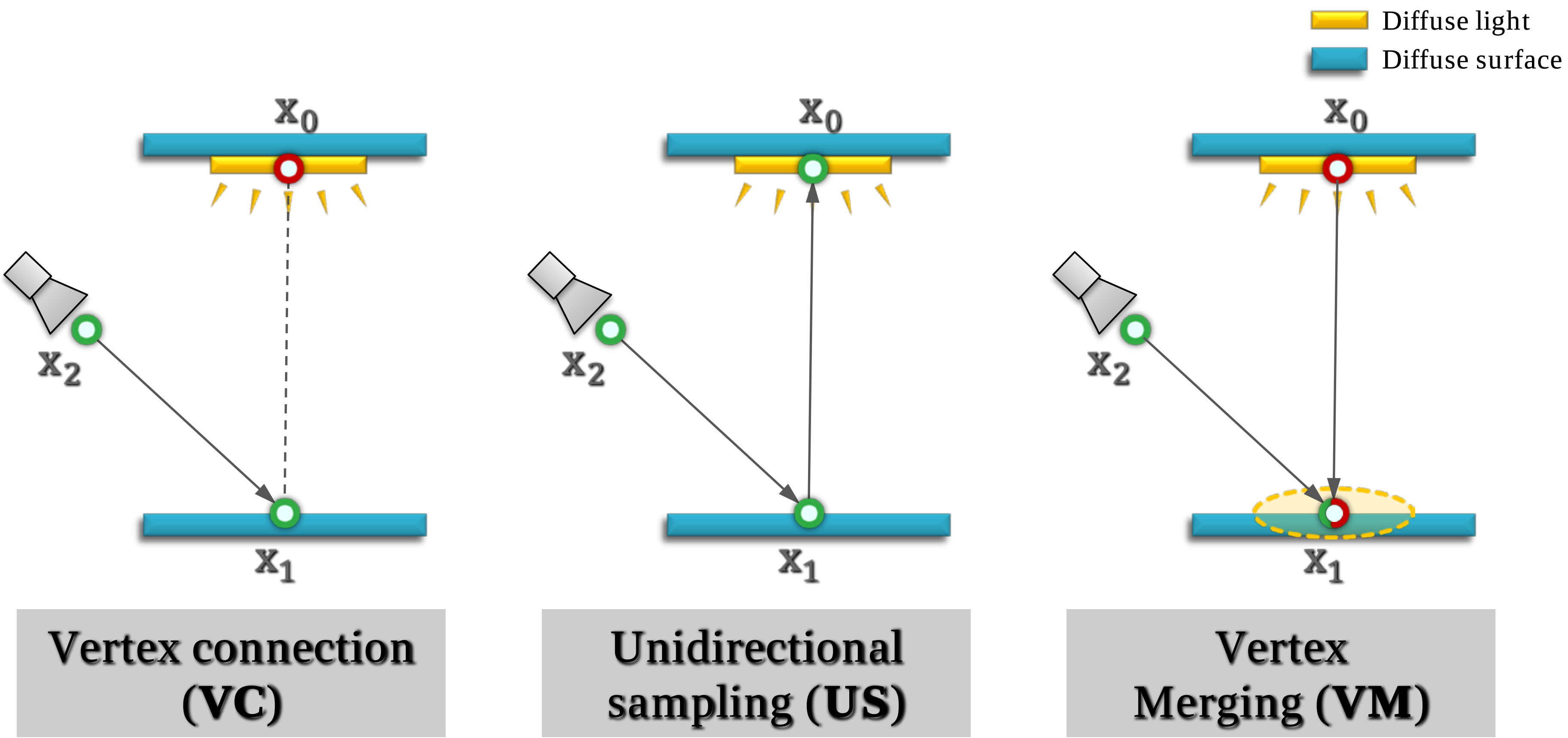
Unidirectional	2 ways
Vertex connection	4 ways
Vertex merging	**3 ways**
Total	**9 ways**

Technique comparison – SDS Paths



Photon mapping (Density estimation) (30 min)

Technique comparison – Diffuse Illum.



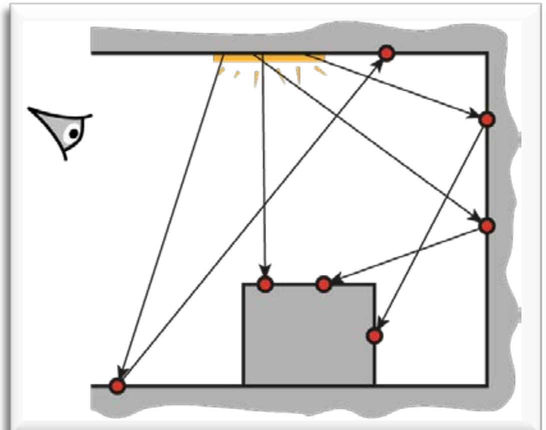
Roughly equal sampling densities

$$p_{US} \approx p_{VM} \approx \frac{p_{VC}}{100{,}000}$$

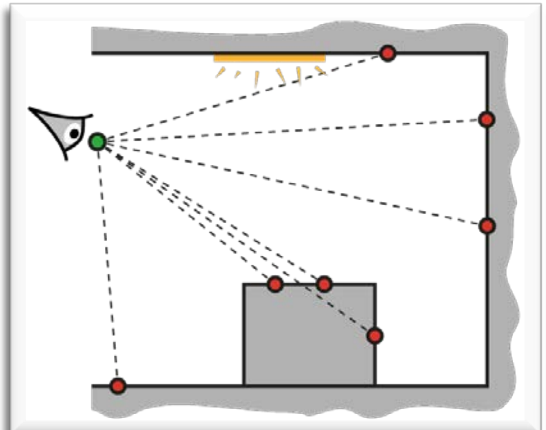

Bidirectional path tracing (30 min)

VCM – Algorithm overview

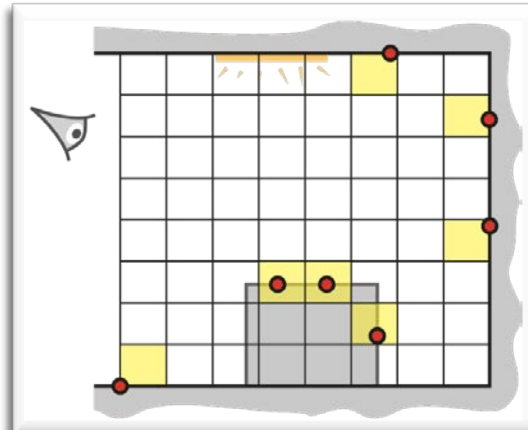
Stage 1: Light sub-path sampling



a) Trace sub-paths

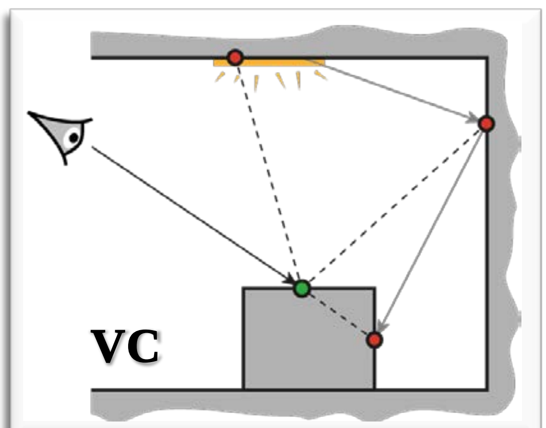


b) Connect to eye

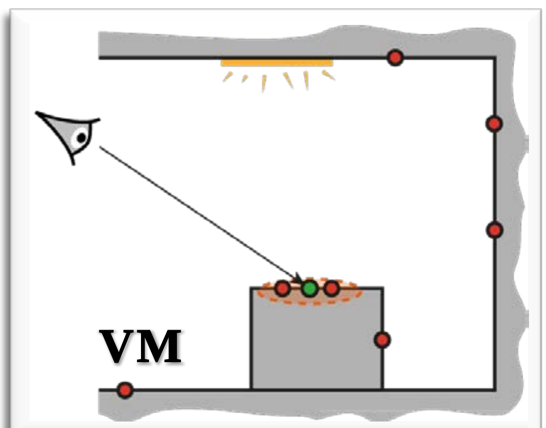


c) Build search struct.

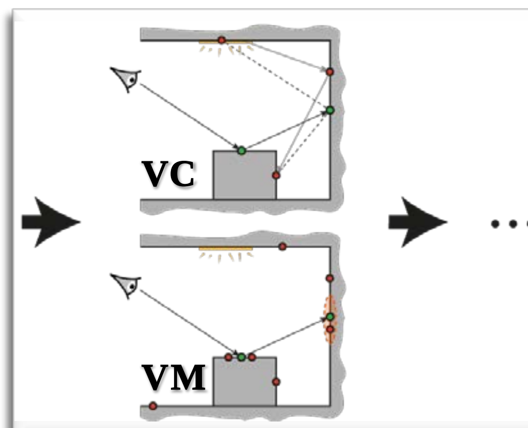
Stage 2: Eye sub-path sampling



a) Vertex connection



b) Vertex merging



c) Continue sub-path

Bidirectional path tracing (30 min)

Stochastic progressive photon mapping (30 min)

Vertex connection and merging (30 min)


VM
VC
Relative technique contributions

Bidirectional path tracing (30 min)

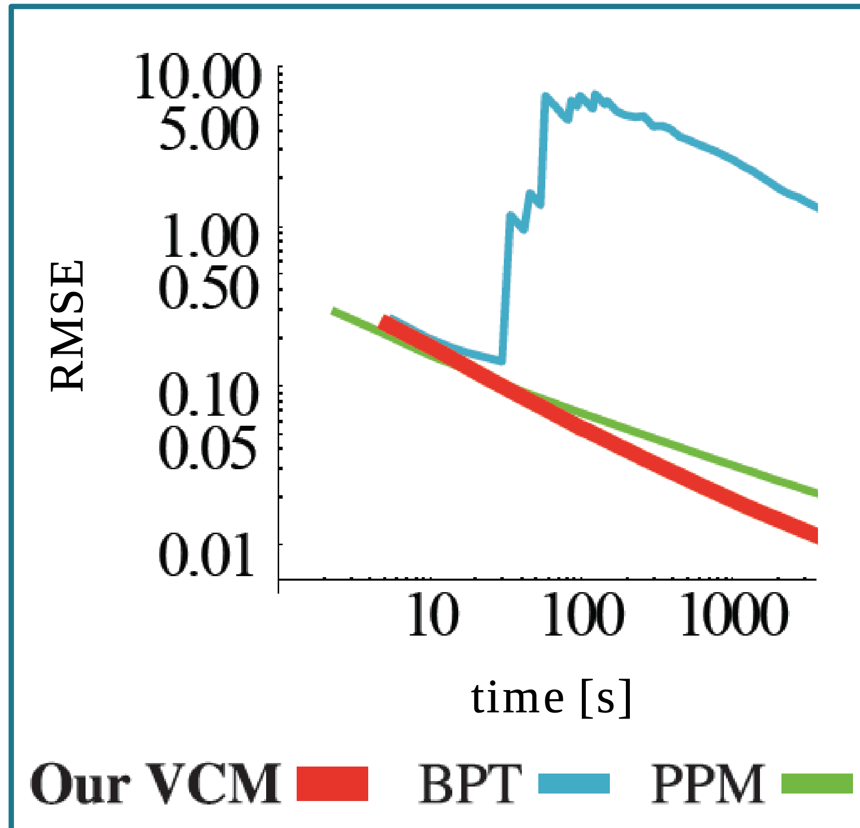
Stochastic progressive photon mapping (30 min)

Vertex connection and merging (30 min)


PM
BPT
AA-340
Relative technique contributions

Convergence rate

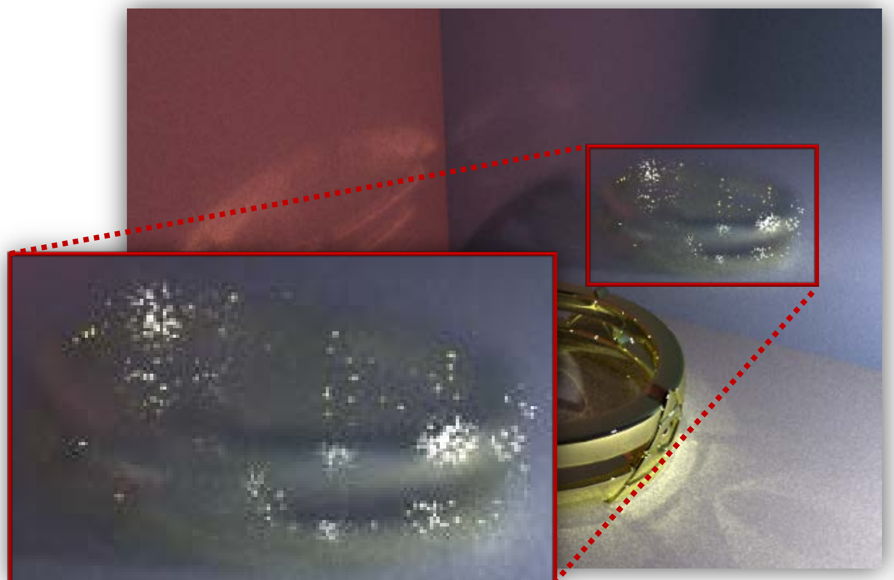
- **RMSE convergence**
  - **BPT**: $O(N^{-0.5})$
  - **PPM**: $O(N^{-0.33})$
  - **VCM**: $O(N^{-0.5})$



Summary

- Image synthesis requires **robust** estimators
- **Vertex Connection and Merging**
  - Bidirectional path tracing + Photon mapping (density est.)
  - Photon mapping (density est.) as a path sampling technique
- **Invaluable tools**
  - Multiple importance sampling
  - Path integral view of light transport

Remaining challenges



Resources

- **Implementation technical report**
  Image comparisons
  [iliyan.com]

- **SmallVCM** – *open-source VCM implementation*
  [SmallVCM.com]

VCM in production

PIXAR's RenderMan®

v·ray

corona

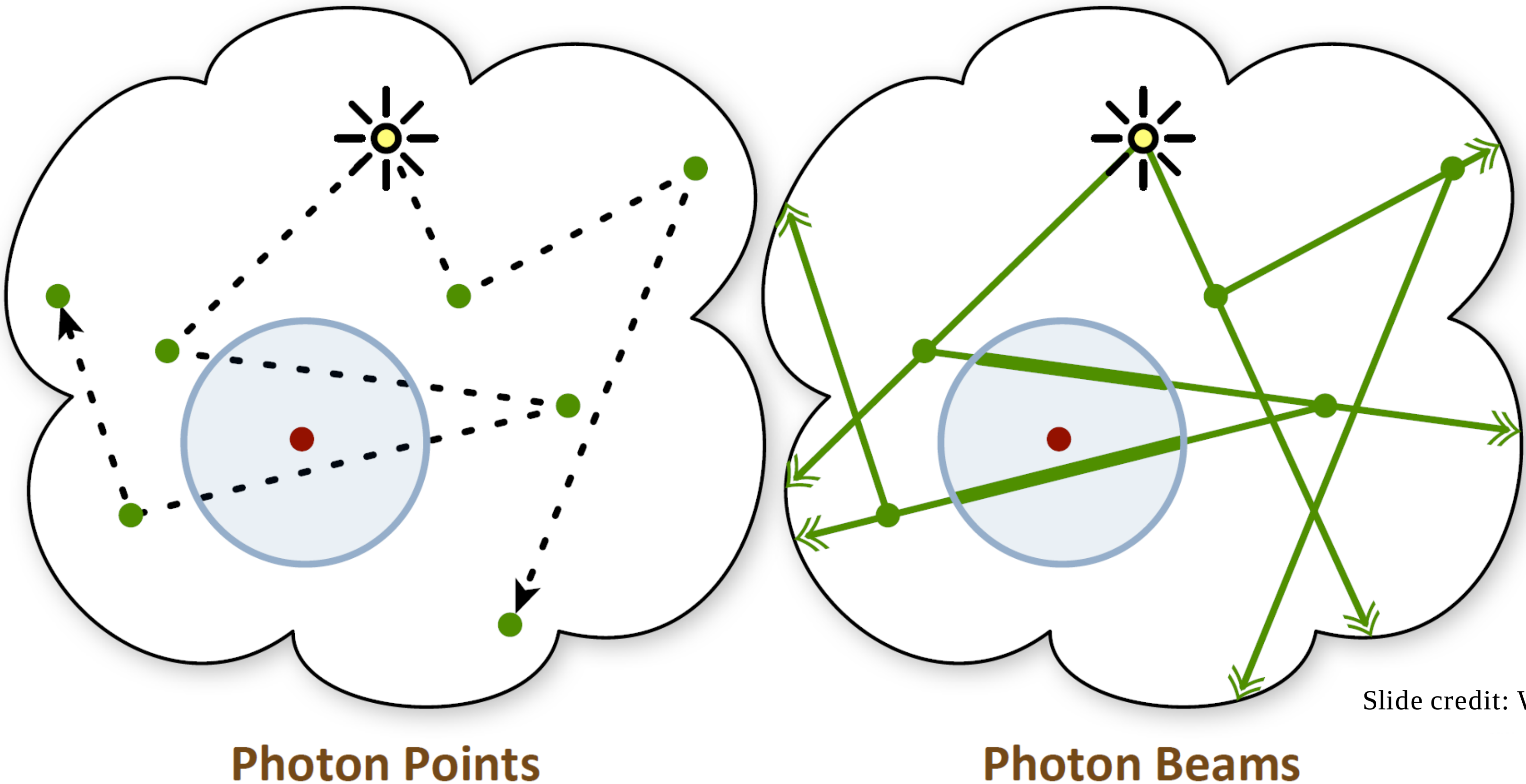
Developed by Ondra Karlík
http://www.corona-renderer.com

weta DIGITAL

EXTENSION TO VOLUMETRIC SCATTERING

Volumetric Light Transport

- "Photon beams" [Jarosz et al. 2011]



Slide credit: W. Jarosz

Volumetric Light Transport: Unified Points, Beams & Paths

SIGGRAPH 2014 (to appear)

References

- *Georgiev et al.*, "Light Transport Simulation with Vertex Connection and Merging", SIGGRAPH Asia 2012
- *Hachisuka et al.* "A Path Space Extension for Robust Light Transport Simulation" , SIGGRAPH Asia 2012
- *Křivánek et al.* "Unifying Points, Beams, and Paths in Volumetric Light Transport Simulation" , SIGGRAPH 2014 (to appear)

Acknowledgements

- **Ondra Karlík**
- **Czech Science Foundation**
  - grant no. P202-13-26189S

THANK YOU!

Questions?

Jaroslav Křivánek

Combining path integral and particle density estimators in light transport simulation

Computer Graphics Charles University

[*cgg.mff.cuni.cz/~jaroslav*]