Product Importance Sampling for Light Transport Path Guiding

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Hendrik Lensch\textsuperscript{1} \hspace{1cm} Jaroslav Křivánek\textsuperscript{2}

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Motivation
Reference
(4 weeks)
BDPT
(1hr)
Our
(1hr)

Vorba2014
(1hr)
Our (1hr) vs Vorba2014 (1hr)

VCM (3 weeks)

Reference
Light Transport: Rendering Equation

\[ L_o = L_e + \int_{\Omega} f_r \cdot L_i \cdot \cos\theta \cdot d\vec{w}_i \]

emission + direct + indirect = combined
Light Transport: Rendering Equation

\[ L_0 = L_e + \int_\Omega f_r \cdot L_i \cdot \cos \theta \cdot d\omega_i \]

\( L_R \)
Bidirectional Reflectance Distribution Function (BRDF)

\[ L_R = \int_{\Omega} f_r \cdot L_i \cdot \cos \theta \cdot d\omega_i \]
Incomming Illumination

\[ L_R = \int_{\Omega} f_r \cdot L_i \cdot \cos\theta \ d\omega_i \]
Reflectance Integral

\[ L_R = \int_{\Omega} f_r \cdot L_i \cdot \cos \theta \cdot d\omega_i \]
BRDF-based Sampling

\[ p_{fr}(\omega_i|\omega_o, x) \propto f_r(x, \vec{\omega}_o, \vec{\omega}_i) \]
Guided Illumination Sampling

\[ p_L(\omega_i|\omega_o, x) \propto L_i(x, \vec{w}_i) \cdot \cos\theta \]
Optimal (Product) Sampling

$$p_{opt}(\omega_i | \omega_o, x) \propto f_r(x, \vec{\omega}_o, \vec{\omega}_i)L_i(x, \vec{\omega}_i) \cdot \cos \theta$$
Related Work

[CAM08]: Practical product importance sampling for direct illumination

[TCE05]: Importance resampling for global illumination

[CJAMJ05]: Wavelet importance sampling: efficiently evaluating products of complex functions

[JCJ09]: Importance sampling spherical harmonics

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Product Importance Sampling
BRDF GMM Representation

\[ p_{f_r}(\omega_i | \omega_o, x) \approx G_{f_r}(y, \Theta) \]
Illumination GMM Representation

\[ p_L(\omega_i | \omega_o, x) \approx G_L(y, \Theta) \]
Gaussian Mixture Model (GMM)

\[ G(\mathbf{y}, \Theta) = \sum_{i=1}^{K} \pi_i N(\mathbf{y}, \mu_i, \Sigma_i) \]

\[ \Theta = \{\pi_0 \ldots, \mu_0 \ldots, \Sigma_0 \ldots\} \]
Gaussian Mixture Model (GMM)

\[
G(y, \Theta) = \sum_{i=0}^{K} \pi_i N(y, \mu_i, \Sigma_i)
\]

\[
\Theta = \{\pi_0, \mu_0, \Sigma_0\}
\]
Gaussian Mixture Model (GMM)

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\[ \Theta = \{ \pi_0 \ldots, \mu_0 \ldots, \Sigma_0 \ldots \} \]
Product of two Gaussians is a Gaussian

\[
\pi_i N(y, \mu_i, \Sigma_i) \cdot \pi_j N(y, \mu_j, \Sigma_j) = \pi_{ij} N(y, \mu_{ij}, \Sigma_{ij})
\]
Product of two Gaussians is a Gaussian

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\[ \pi_i N(y, \mu_i, \Sigma_i) \cdot \pi_j N(y, \mu_j, \Sigma_j) = \pi_{ij} N(y, \mu_{ij}, \Sigma_{ij}) \]

• Full GMM product contains \( K^2 \) components
Product GMM Representation

\[ p_{fr} \otimes p_L \approx G_{fr} \otimes G_L = G_{\otimes}(y, \Theta) \]
Product GMM Representation

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Pipeline

Pre-Processing
- Illum. Fit
- BRDF Fit

Rendering
- Path Tracer
  - GuidedProduct BRDF
  - Weight Window RR
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Pipeline

Pre-Processing
- Illum. Fit
- BRDF Fit

Rendering
- Path Tracer
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Illumination Fit: [Vorba2014]

GMM Illumination caches

[S. Herholz: Product Importance Sampling for Light Transport Path Guiding]
BRDF Fitting and Caching
Fitting BRDF GMM

Weighted EM

• Weighted MAP EM [Vorba2014]

• Sample BRDF (N=512)

• \( w_i = \frac{f_r(x, \omega_i, \omega_o)}{p(\omega_o)} \)

• Init components using K BRDF samples (QMC sampler)

CERES

• Non-linear optimization

• Init with weighted EM

• Objective function:

\[
\sum_{i}^{N} \left[ 1 - \frac{\tilde{f}_r(\omega_i)}{G(y|\Theta)} \right]^2
\]
Fitting BRDF GMM

Weighted EM

• Weighted MAP EM [Vorba2014]

• Sample BRDF (N=512)

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\[ \sum_{i=1}^{N} \left[ 1 - \frac{\hat{f}_r(\omega_i)}{G(y|\Theta)} \right]^2 \]
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wEM vs CERES

$\text{rough conductor } \alpha = 0.3$

$K = 8$
wEM vs CERES

rough conductor $\alpha = 0.3$

$K = 8$
wEM vs CERES

\[ \begin{align*}
\text{rough conductor } & \alpha = 0.3 \\
& K = 8
\end{align*} \]
wEM vs CERES

rough conductor $\alpha = 0.15$

$K = 8$
Caching

- Isotropic
  - 512 different elevation angles

- Anisotropic
  - 4096 spherical Fibonacci points [KISS15]
Isotropic

Caching
Caching

- Isotropic
Caching

• Isotropic
• Isotropic
• Isotropic
GMM Component Reduction
Component Reduction

\[ c_{i,j} = d_{KL}(\{(\pi_i, \mu_i, \Sigma_i), (\pi_j, \mu_j, \Sigma_j)\}, (\pi_{ij}, \mu_{ij}, \Sigma_{ij})) \]
Component Reduction

\[ c_{i,j} = d_{KL}(\{(\pi_i, \mu_i, \Sigma_i), (\pi_j, \mu_j, \Sigma_j)\}, (\pi_{ij}, \mu_{ij}, \Sigma_{ij})) \]
Component Reduction

\[ c_{i,j} = d_{KL}(\{(\pi_i, \mu_i, \Sigma_i), (\pi_j, \mu_j, \Sigma_j)\}, (\pi_{ij}, \mu_{ij}, \Sigma_{ij})) \]
Component Reduction

\[ c_{i,j} = d_{KL}(\{(\pi_i, \mu_i, \Sigma_i), (\pi_j, \mu_j, \Sigma_j)\}, (\pi_{ij}, \mu_{ij}, \Sigma_{ij})) \]

- Adapted from [Runnals2007], used also in [Jacob2011]

- Kullback-Leibler discrimination: \( d_{KL} \)
Component Reduction

Cumulative cost

Component count: 8
Cost: 0.000 (0.000%)
Component Reduction

Cumulative cost

Component count: 7
Cost: 0.083 (3.364%)
Component Reduction

Cumulative cost

Component count: 6
Cost: 0.174 (7.062%)
Component Reduction

Cumulative cost

Component count: 4
Cost: 0.498 (20.210%)
Component Reduction

Cumulative cost

Component count: 3
Cost: 0.931 (37.773%)
Component Reduction

Cumulative cost

Component count: 2
Cost: 1.443 (58.549%)
Component Reduction

Cumulative cost

Component count: 1
Cost: 2.464 (100.000%)
Component Reduction

Cumulative cost

Component count: 6
Cost: 0.174 (7.062%)
Component Reduction

Reduction

- BRDF:
  - Full $K = 8$

- Illumination
  - Full $K = 8$

- Product GMM:
  - Avg. $K_{ij} = 64$
Component Reduction

**Reduction**

- **BRDF:**
  - Full $K = 8$
  - Red. avg. $K = 2$

- **Illumination**
  - Full $K = 8$
  - Red. 50% to 4 comp.

- **Product GMM:**
  - Avg. $K_{ij} = 64$
  - Red. avg. $K_{ij} = 12$

**Cumulative cost**
<table>
<thead>
<tr>
<th>Motivation</th>
<th>Product Importance Sampling</th>
<th>BRDF Fitting</th>
<th>Component Reduction</th>
<th>Results</th>
<th>Future Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>S. Herholz: Product Importance Sampling for Light Transport Path Guiding</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Reference
equal time 1hr
Path Tracer

4736 / 2.1830

Reference

SPP / MSE

equal time 1hr
Path Tracer     Vorba2014     Reference

4736 / 2.1830   882 / 0.0331   SPP / MSE

equal time 1hr
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**Results:**

- **Path Tracer:** 4736 / 2.1830
- **Vorba2014:** 882 / 0.0331
- **Our:** 1128 / 0.0211
- **Reference:**

**NOTE:** Equal time 1hr
equal time 1hr

Path Tracer  Vorba2014  Our  Reference

4335 / 0.0081  1528 / 0.0025  1322 / 0.0007  SPP / MSE

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Reference

Vorba2014
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<table>
<thead>
<tr>
<th>Method</th>
<th>Samples</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Tracer</td>
<td>4335</td>
<td>0.0081</td>
</tr>
<tr>
<td>Vorba2014</td>
<td>1528</td>
<td>0.0025</td>
</tr>
<tr>
<td>Our</td>
<td>1322</td>
<td>0.0007</td>
</tr>
<tr>
<td>Reference</td>
<td></td>
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</tr>
</tbody>
</table>

equal time 1hr

SPP / MSE
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Reference

3120 / 0.9715
1616 / 0.006
712 / 0.007
SPP / MSE

equal time 1hr
Average Path Lengths

Vorba: 9.8  Ours: 4.9

normalized per-path segment contribution

LivingRoom
Average Path Lengths
Vorba: 9.8       Ours: 4.9

normalized per-path segment contribution

LivingRoom

path depth
Average Path Lenghts

Vorba: 9.8  
Ours: 4.9

normalized per-path segment contribution

LivingRoom

path depth

S. Herholz: Product Importance Sampling for Light Transport Path Guiding
Average Path Lengths

**Vorba:** 6.5  
**Ours:** 9.0

normalized per-path segment contribution

**Kitchen**

- **Contribution**
- **Vorba**
- **Ours**

path depth
Discussion / Future work
Discussion / Future Work

• Path Length and Russian Roulette
  - Adjoint-driven RR and Splitting [Vorba2016]
Discussion / Future Work

- SVBRDFs
  - Enlarge BRDF caches
  - Direct function transform BRDF-\(\rightarrow\)GMM

<table>
<thead>
<tr>
<th>Scene</th>
<th># BRDFs</th>
<th># Caches</th>
<th>Avg. # comp.</th>
<th>Mem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LivingRoom</td>
<td>41</td>
<td>15k</td>
<td>2.5</td>
<td>7.7 MB</td>
</tr>
<tr>
<td>Kitchen</td>
<td>72</td>
<td>2.5k</td>
<td>1.8</td>
<td>10 MB</td>
</tr>
<tr>
<td>Jewelry</td>
<td>6</td>
<td>1.5k</td>
<td>1.44</td>
<td>0.7 MB</td>
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</table>
Discussion / Future Work

- Extension to other MC-algorithms
  - BDPT
  - MCMC
  - Gradient domain
GD-PT (full)

Reference

GD-BDPT (full)
GD-PT (estimate)

Reference

GD-BDPT (estimate)
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Our

Reference
Discussion/Future Work

- Optimizing Illumination caches
  - Poorly fitted illumination caches cause inconsistent convergence rates
Product Importance Sampling for Light Transport Path Guiding

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- Martin Šík, Ivo Kondapaneni, Ludvík Koutný, Anton Kaplanyan, Johannes Hanika
- Anonymous reviewers
- You!

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Backup
wEM vs CERES
Gradient Domain

<table>
<thead>
<tr>
<th>GD-PT Est.</th>
<th>GD-PT Rec.</th>
<th>GD-BDPT Est.</th>
<th>GD-BDPT Rec.</th>
<th>Our</th>
<th>Reference</th>
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<tr>
<td>[Image]</td>
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</tbody>
</table>
Reference

Uncorrected  Corrected  Standard
Table 1: *Timings for the cache fitting stages (in minutes).*

<table>
<thead>
<tr>
<th>Scene</th>
<th>Illumination</th>
<th>BRDF wEM</th>
<th>Ceres</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIVINGROOM</td>
<td>14.0</td>
<td>0.31</td>
<td>25.5</td>
</tr>
<tr>
<td>KITCHEN</td>
<td>20.1</td>
<td>0.44</td>
<td>42.3</td>
</tr>
<tr>
<td>JEWELRY</td>
<td>6.1</td>
<td>0.04</td>
<td>6.3</td>
</tr>
</tbody>
</table>
### Statistics:

#### Table 2:

Left and middle: BRDF and illumination caching statistics for the scenes in Fig. 8. Right: Overhead of the product sampling relative to illumination-only sampling, without (‘naïve’) and with the reduction of the BRDF and illumination mixtures.

<table>
<thead>
<tr>
<th>Scene</th>
<th>BRDF caching</th>
<th></th>
<th>Mem.</th>
<th>Illumination caching</th>
<th></th>
<th>Mem.</th>
<th>Sampling overhead [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td># BRDFs</td>
<td># Caches</td>
<td>Avg. # comp.</td>
<td>7.7 MB</td>
<td># Caches</td>
<td># Reduced</td>
<td>192.9 MB</td>
</tr>
<tr>
<td>LIVINGROOM</td>
<td>41</td>
<td>15k</td>
<td>2.5</td>
<td>7.7 MB</td>
<td>82k</td>
<td>57 %</td>
<td>192.9 MB</td>
</tr>
<tr>
<td>KITCHEN</td>
<td>72</td>
<td>2.5k</td>
<td>1.8</td>
<td>10 MB</td>
<td>107k</td>
<td>62 %</td>
<td>236.9 MB</td>
</tr>
<tr>
<td>JEWELRY</td>
<td>6</td>
<td>1.5k</td>
<td>1.44</td>
<td>0.7 MB</td>
<td>16k</td>
<td>33 %</td>
<td>19.5 MB</td>
</tr>
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