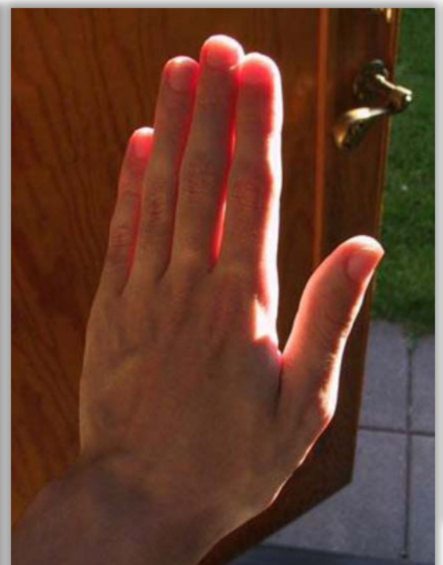
Subsurface scattering

Jaroslav Křivánek, KSVI, MFF, UK
Jaroslav.Krivanek@mff.cuni.cz

Subsurface scattering examples



Real

Simulated

BSSRDF

- Bidirectional surface scattering distribution function [Nicodemus 1977]
  - 8D function (2x2 DOFs for surface + 2x2 DOFs for dirs)
  - Differential outgoing radiance per differential incident flux (at two possibly different surface points)

$$dL_o(x_o, \vec{\omega}_o) = S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)\, d\Phi_i(x_i, \vec{\omega}_i)$$

- Encapsulates all light behavior under the surface

BSSRDF vs. BRDF

- BRDF is a special case of BSSRDF (same entry/exit pt)

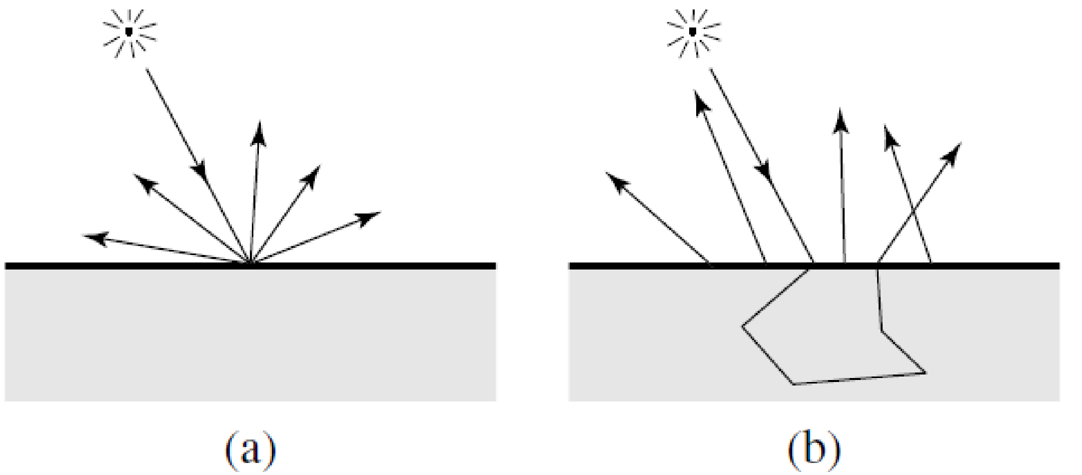


Figure 1: Scattering of light in (a) a BRDF, and (b) a BSSRDF.

BSSRDF vs. BRDF examples 1

BSSRDF vs. BRDF examples

- BRDF – hard, unnatural appearance

BSSRDF vs. BRDF examples

- Show video (SIGGRAPH 2001 Electronic Theater)

BSSRDF vs. BRDF

- Some BRDF model do take subsurface scattering into account (to model diffuse reflection)
  - [Kruger and Hanrahan 1993]
- BRDF assumes light enters and exists at the same point (not that there isn't any subsurface scattering!)

Generalized reflection equation

- Remember that $dL_o(x_o, \vec{\omega}_o) = S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)\, d\Phi_i(x_i, \vec{\omega}_i)$

- So total outgoing radiance at x_o in direction ω_o is

$$L_o(x_o, \vec{\omega}_o) = \int_A \int_{2\pi} S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)\, L_i(x_i, \vec{\omega}_i)\, (\vec{n} \cdot \vec{\omega}_i)\, d\omega_i dA(x_i)$$

  - (added integration over the surface)

Subsurface scattering simulation

- Path tracing – way too slow
- Photon mapping – practical [Dorsey et al. 1999]


Simulating SS with photon mapping

- Special instance of volume photon mapping [Jensen and Christensen 1998]
- Photons distributed from light sources, stored inside objects as they interact with the medium
- Ray tracing step enters the medium and gather photons

Problems with MC simulation of SS

- MC simulations (path tracing, photon mapping) can get very expensive for high-albedo media (skin, milk)
- High albedo means little energy lost at scattering events
  - Many scattering events need to be simulated (hundreds)

- Example: albedo of skim milk, a = 0.9987
  - After 100 scattering events, 87.5% energy retained
  - After 500 scattering events, 51% energy retained
  - After 1000 scattering events, 26% energy retained

- (compare to surfaces, where after 10 bounces most energy is usually lost)

Practical model for subsurface scattering

- Jensen, Marschner, Levoy, and Hanrahan, 2001
  - Won Academy award (Oscar) for this contribution

- Can find a diffuse BSSRDF $R_d(r)$, where r = $||\mathbf{x}_0 - \mathbf{x}_i||$
  - 1D instead of 8D !

Practical model for subsurface scattering

- Several **key approximations** that make it possible
  - Principle of similarity
    - Approximate highly scattering, directional medium by **isotropic** medium with modified ("reduced") coefficients
  - Diffusion approximation
    - Multiple scattering can be modeled as **diffusion** (simpler equation than full RTE)
  - Dipole approximation
    - Closed-form solution of diffusion can be obtained by placing two virtual point sources in and outside of the medium

Approx. #1: Principle of similarity

- Anisotropically scattering medium with **high albedo** approximated as **isotropic** medium with
  - reduced scattering coefficient: $\sigma_s' = \sigma_s(1-g)$
  - reduced extinction coefficient: $\sigma_t' = \sigma_s' + \sigma_a$
  - (absorption coefficient stays the same)

- Recall that g is the **mean cosine** of the phase function:

$$g = \int_{4\pi} (\vec{\omega} \cdot \vec{\omega}')p(\vec{\omega} \cdot \vec{\omega}')\, d\omega'$$

  - Equal to the anisotropy parameter for the Henyey-Greenstein phase function

Intuition behind the similarity principle

- Isotropic approximation
  - Even highly anisotropic medium becomes isotropic after many interactions because every scattering blurs light
- Reduced scattering coefficient $\sigma_s' = \sigma_s(1-g)$
  - Strongly forward scattering medium, g = 1
    - Actual medium: the light makes a strong forward progress
    - Approximation: small reduced coeff => large distance before light scatters
  - Strongly backward scattering medium, g = -1
    - Actual medium: light bounces forth and back, not making much progress
    - Approximation: large reduced coeff => small scattering distance

Approx. #2: Diffusion approximation

- We know that radiance mostly isotropic after multiple scattering; assume homogeneous, optically thick
- Approximate radiance at a point with just 4 SH terms:

$$L(x, \vec{\omega}) = \frac{1}{4\pi}\phi(x) + \frac{3}{4\pi}\vec{\omega} \cdot \vec{E}(x)$$

- Constant term: **scalar** irradiance, or **fluence**

$$\phi(x) = \int_{4\pi} L(x, \vec{\omega})\, d\omega$$

- Linear term: **vector irradiance**

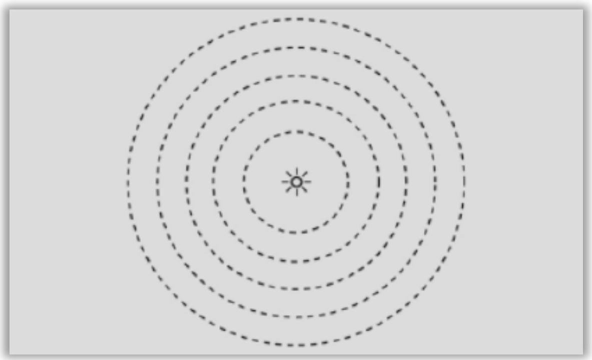
$$\vec{E}(x) = \int_{4\pi} L(x, \vec{\omega})\vec{\omega}\, d\omega$$

Diffusion approximation

- With the assumptions from previous slide, the full RTE can be approximated by the **diffusion equation**
  - Simpler than RTE (we're only solving for the scalar fluence, rather than directional radiance)
  - Skipped here, see [Jensen et al. 2001] for details

Solving diffusion equation

- Can be solved numerically
- Simple analytical solution for point source in infinite homogeneous medium:



$$\phi(x) = \frac{\Phi}{4\pi D} \frac{e^{-\sigma_{tr} r(x)}}{r(x)}$$

- Diffusion coefficient: $D = \frac{1}{3\sigma_t'}$
- Effective transport coefficient: $\sigma_{tr} = \sqrt{3\sigma_a \sigma_t'}$

Solving diffusion equation

- Our medium not infinite, need to enforce **boundary condition**
  - Radiance at boundary going down equal to radiance incident at boundary weighed by Fresnel coeff (accounting for reflection)

- Fulfilled, if $\phi(0,0,2AD) = 0$ (zero fluence at height 2AD)
  - where

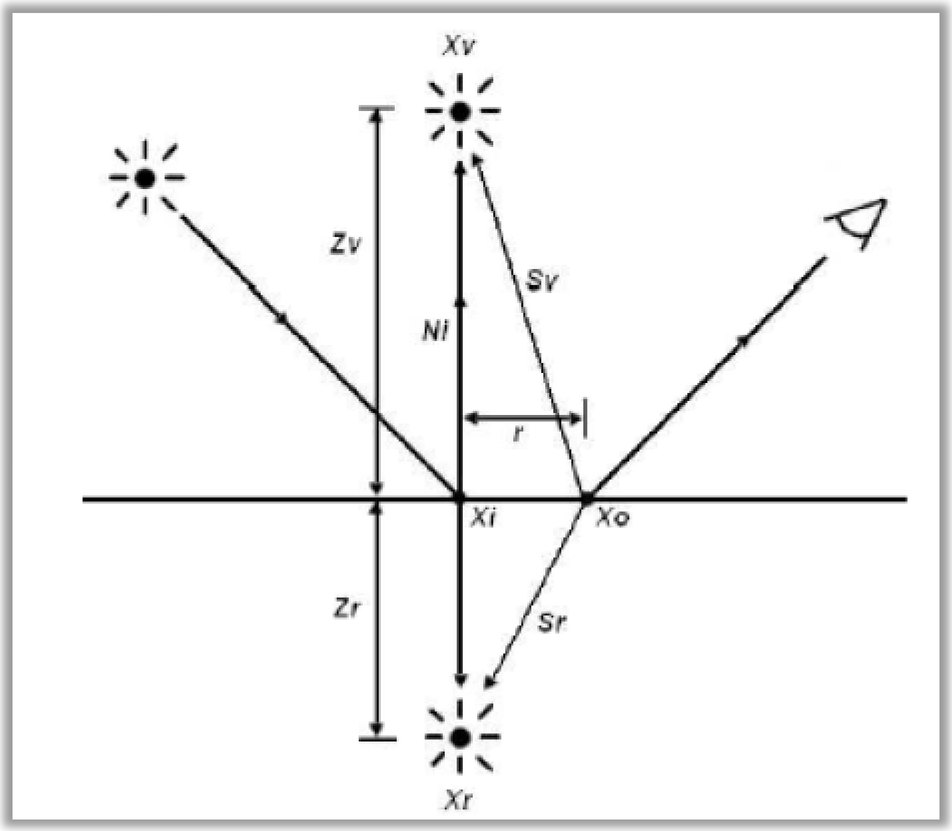
$$A = \frac{1 + F_{dr}}{1 - F_{dr}}$$

  - **Diffuse Fresnel reflectance**

$$F_{dr} = \int_{2\pi} F_r(\eta, \vec{n} \cdot \vec{\omega}')(\vec{n} \cdot \vec{\omega}')\, d\omega' \text{ approx as } -\frac{1.440}{\eta^2} + \frac{0.710}{\eta} + 0.668 + 0.0636\eta$$

Dipole approximation

- Fulfill $\phi(0,0,2AD) = 0$ by placing **two point sources** (positive and negative) inside and above medium



$$z_r = 1/\sigma_t',$$

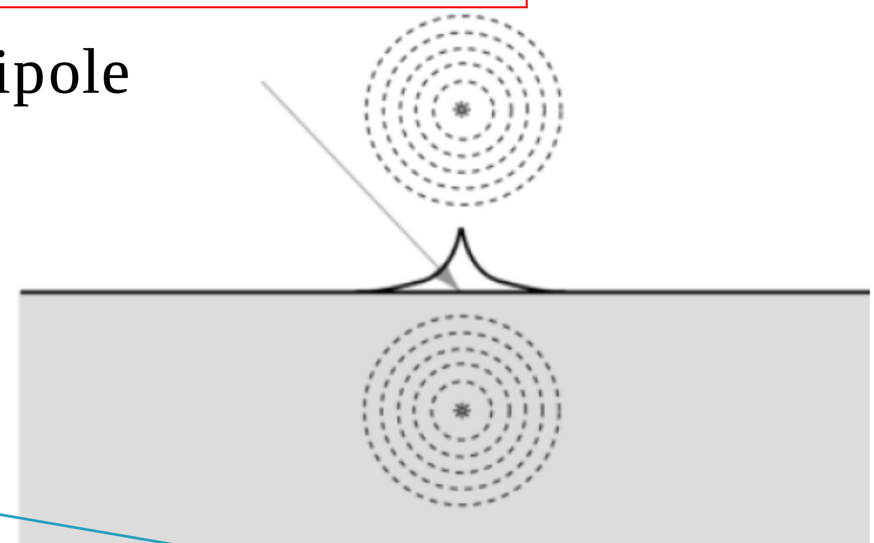
one mean free path below surface

Dipole approximation

- Fluence due to the dipole (d_r … dist to real, d_v .. to virtual)

$$\phi(x) = \frac{\Phi}{4\pi D}\left(\frac{e^{-\sigma_{tr} d_r}}{d_r} - \frac{e^{-\sigma_{tr} d_v}}{d_v}\right)$$

- Diffuse reflectance due to dipole



  - We want radiant exitance (radiosity) at surface…
    - (gradient of fluence)
  - … per unit incident flux

$$R_d(r) = -\frac{\vec{\mathbf{N}} \bullet (\nabla\phi_1(r) - \nabla\phi_2(r))}{\Phi_i}$$

Diffuse reflectance due to dipole

- Gradient of fluence per unit incident flux

gradient in the normal direction = derivative w.r.t. z-axis

$$R_d(r) = -D\frac{(\vec{n}\cdot\vec{\nabla}\phi(x_s))}{d\Phi_i}$$

$$= \frac{\alpha'}{4\pi}\left[(\sigma_{tr}d_r + 1)\frac{e^{-\sigma_{tr}d_r}}{\sigma_t' d_r^3} + z_v(\sigma_{tr}d_v + 1)\frac{e^{-\sigma_{tr}d_v}}{\sigma_t' d_v^3}\right]$$

Final diffusion BSSRDF

Normalization term
(like for surfaces)

Diffuse multiple-scattering
reflectance

$$S_d(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) = \frac{1}{\pi} F_t(\eta, \vec{\omega}_i) R_d(||x_i - x_o||) F_t(\eta, \vec{\omega}_o)$$

Fresnel term for
incident light

Fresnel term for
outgoing light

Diffusion profile

- Plot of R_d

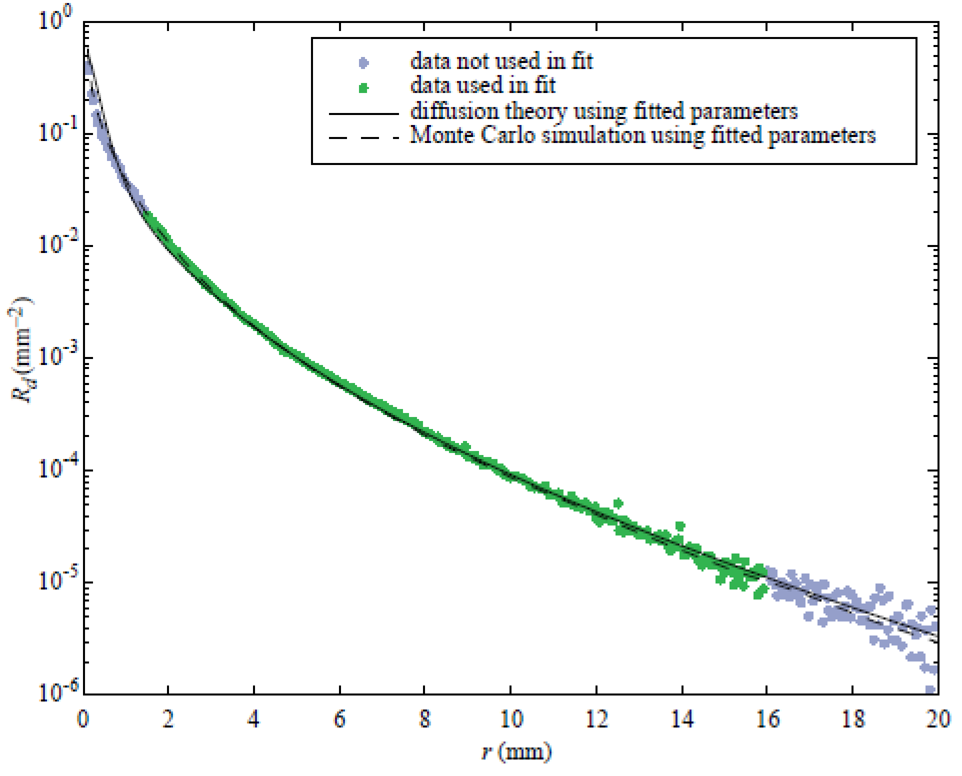


Figure 6: Measurements for marble (green wavelength band) plotted with fit to diffusion theory and confirming Monte Carlo simulation.

Single scattering term

- Cannot be described by diffusion
- Much shorter influence than multiple scattering
- Computed by classical MC techniques (marching along ray, connecting to light source)

Rendering BSSRDFs

Rendering with BSSRDFs

1. Monte Carlo sampling [Jensen et al. 2001]
2. Hierarchical method [Jensen and Buhler 2002]
3. Real-time approximations exist but are skipped here

Monte Carlo sampling

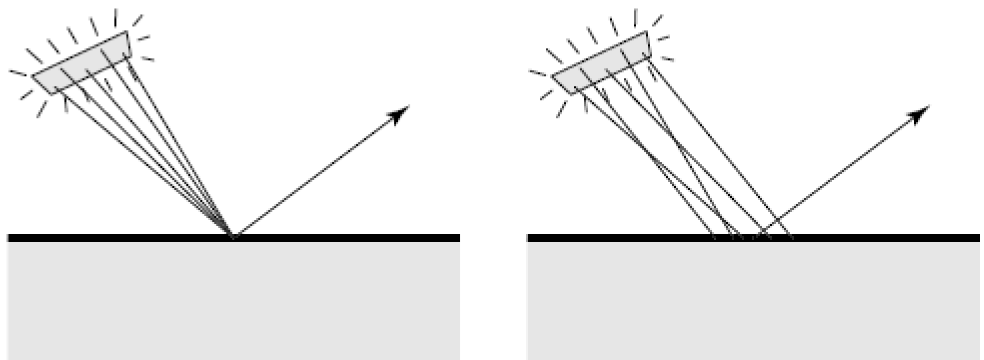


Figure 7: (a) Sampling a BRDF (traditional sampling), (b) sampling a BSSRDF (the sample points are distributed both over the surface as well as the light).

Hierarchical method

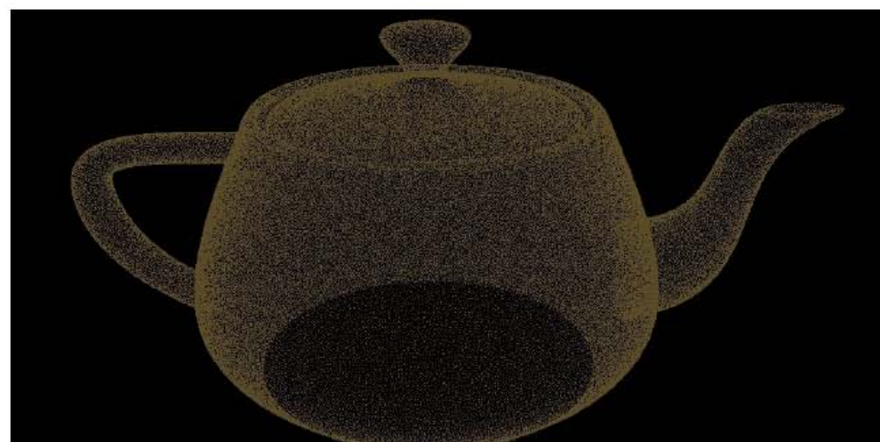
- Key idea: decouple computation of surface irradiance from integration of BSSRDF
- Algorithm
  - Distribute many points on translucent surface
  - Compute irradiance at each point
  - Build hierarchy over points (partial avg. irradiance)
  - For each visible point, integrate BSSRDF over surface using the hierarchy (far away point use higher levels)

Hierarchical method - Results

BSSRDF: sampled evaluation - 18 minutes

Illumination from a HDR environment

BSSRDF: hierarchical evaluation - 7 seconds



The sample locations on the teapot

Multiple Dipole Model

Donner and Jensen, SIGGRAPH 2005

Multiple Dipole Model

- Dipole approximation assumed semi-infinite homogeneous medium
- Many materials, namely skin, has multiple layers of different optical properties and thickenss
- Solution: infinitely many point sources

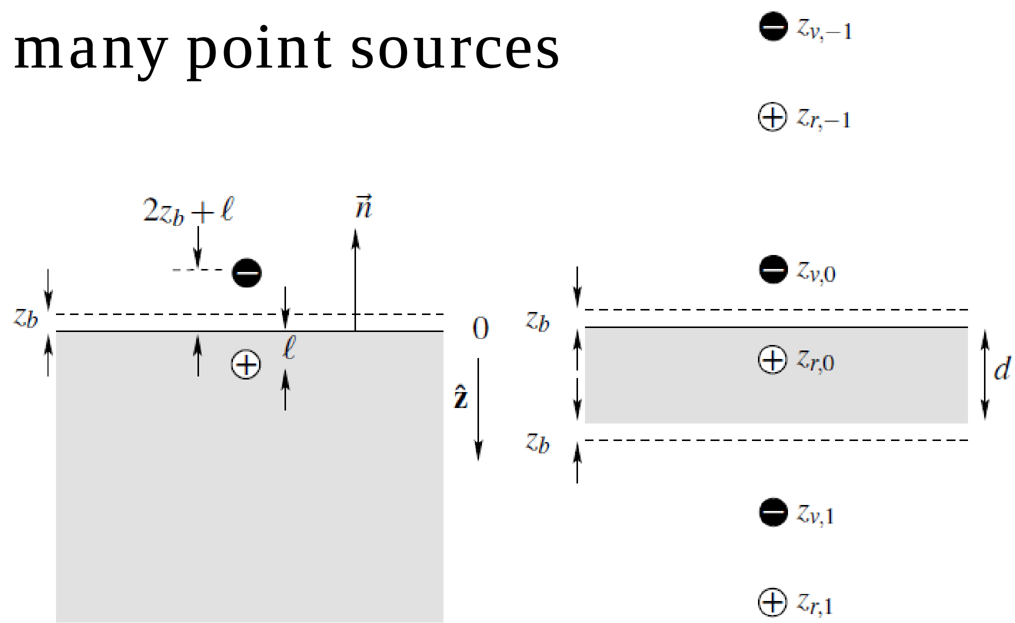
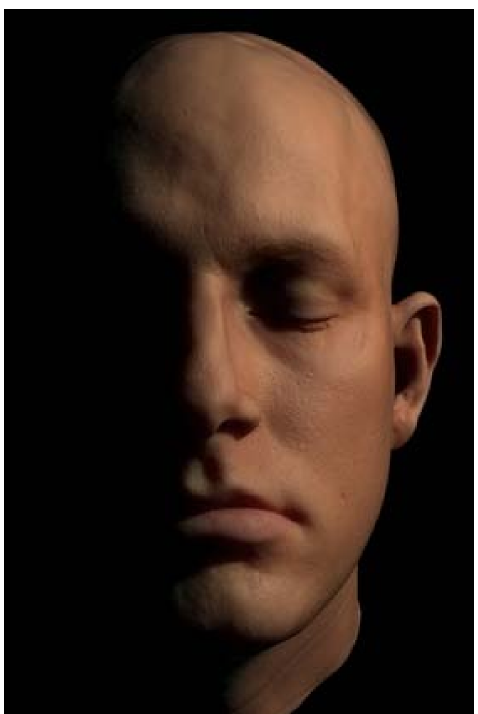
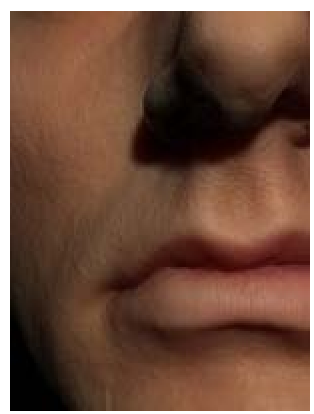


Figure 1: *Dipole configuration for semi-infinite geometry (left), and the multipole configuration for thin slabs (right).*

Multiple Dipole Model - Results

Epidermis Reflectance | Epidermis Transmittance | Upper Dermis Reflectance | Upper Dermis Transmittance | Bloody Dermis Reflectance | Surface Roughness | All Layers





References

- PBRT, section 16.5