



Volumetric Zero-Variance-Based Path Guiding

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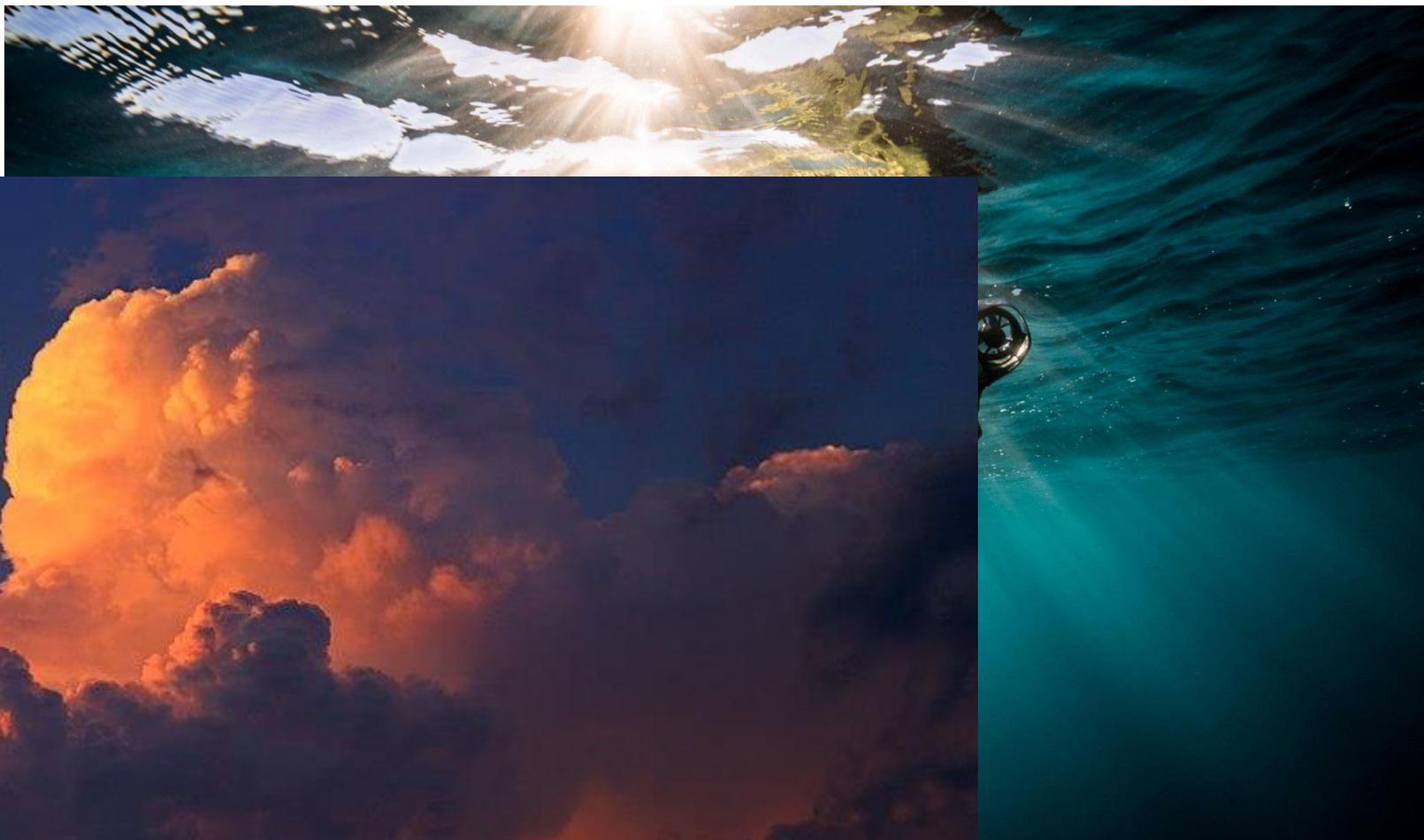
Yangyang Zhao²

Hendrik P. A. Lensch¹

Oskar Elek³

Jaroslav Křivánek³

MOTIVATION



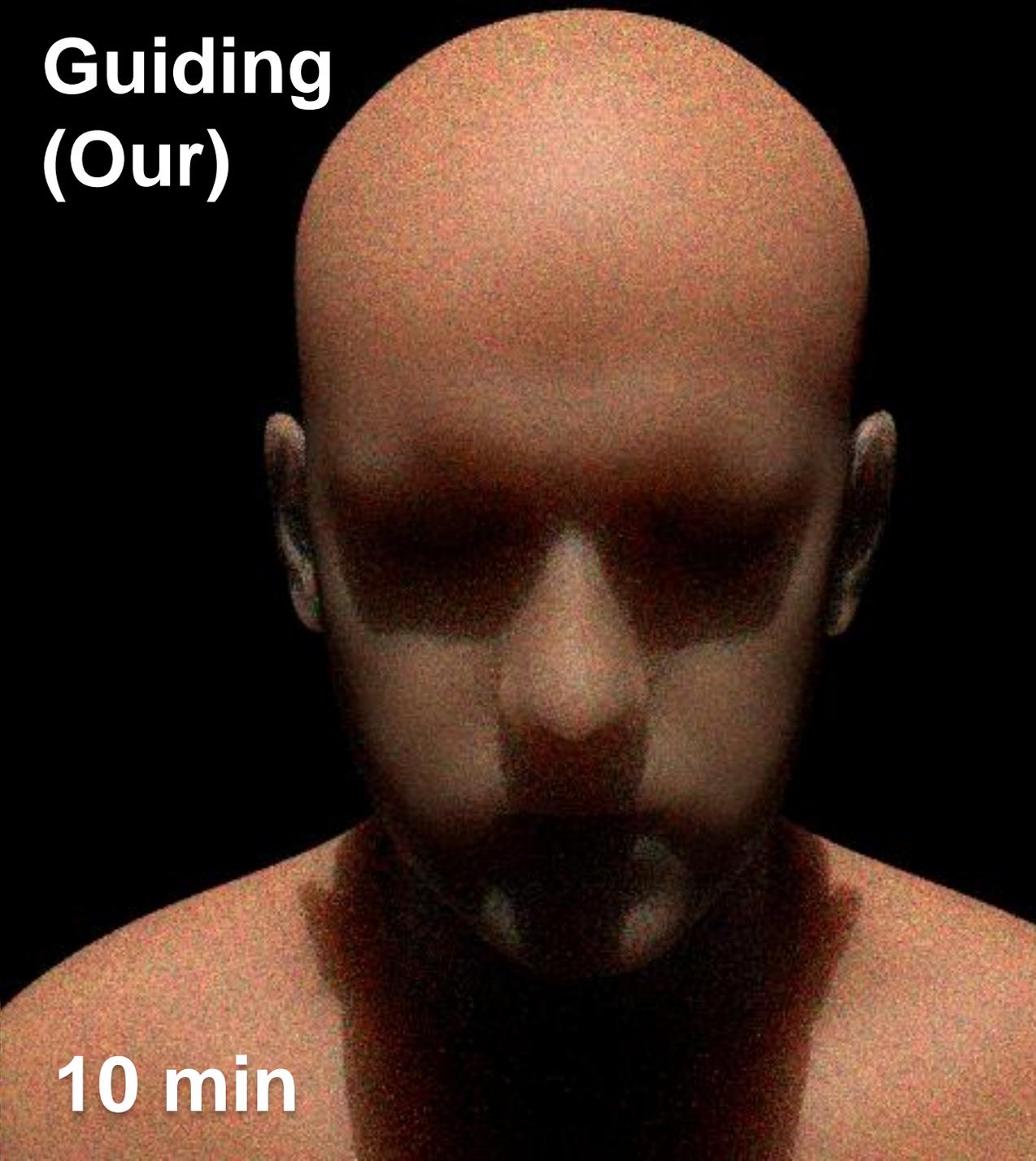
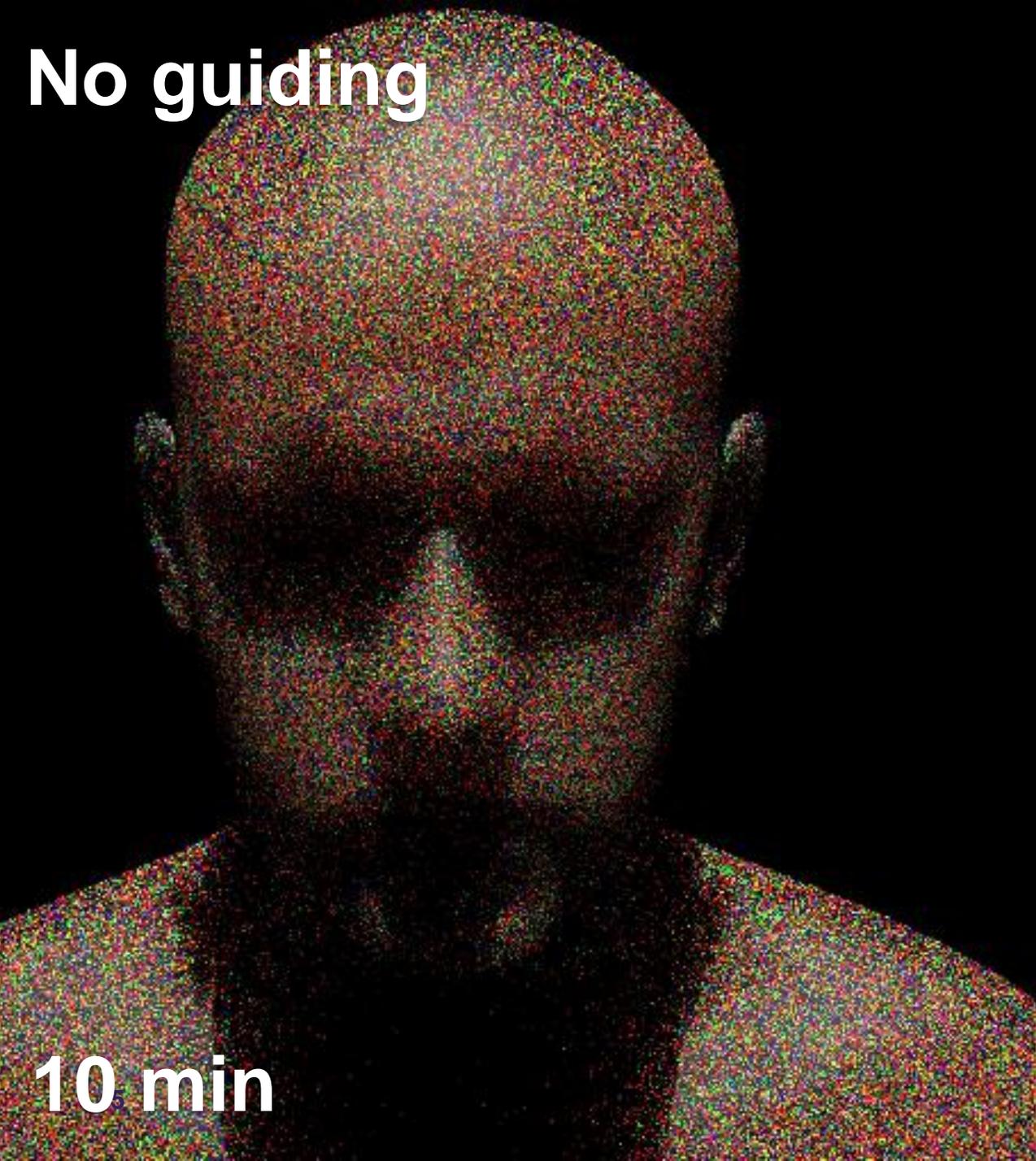
MOTIVATION



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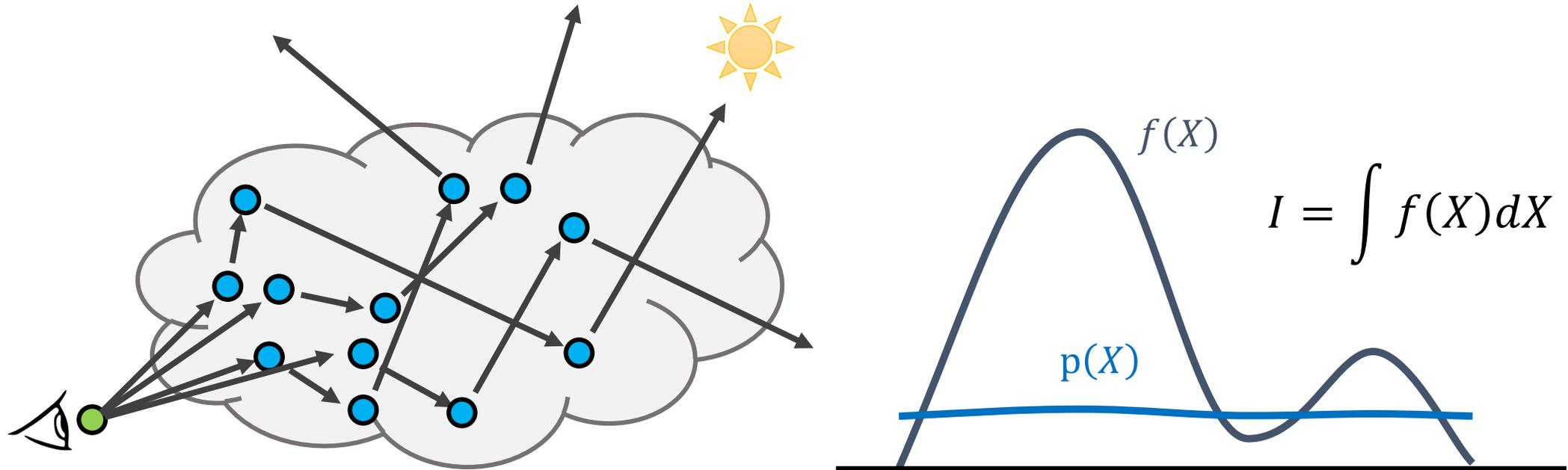


- A correct physically-based simulation of volumetric effects is crucial for rendering realistic scenes
- In the recent years, brute-force path tracing these effects started to become applicable in production environments ([Fong2017], [Novak2018])
- **Increased complexity of the light transport makes it still challenging**





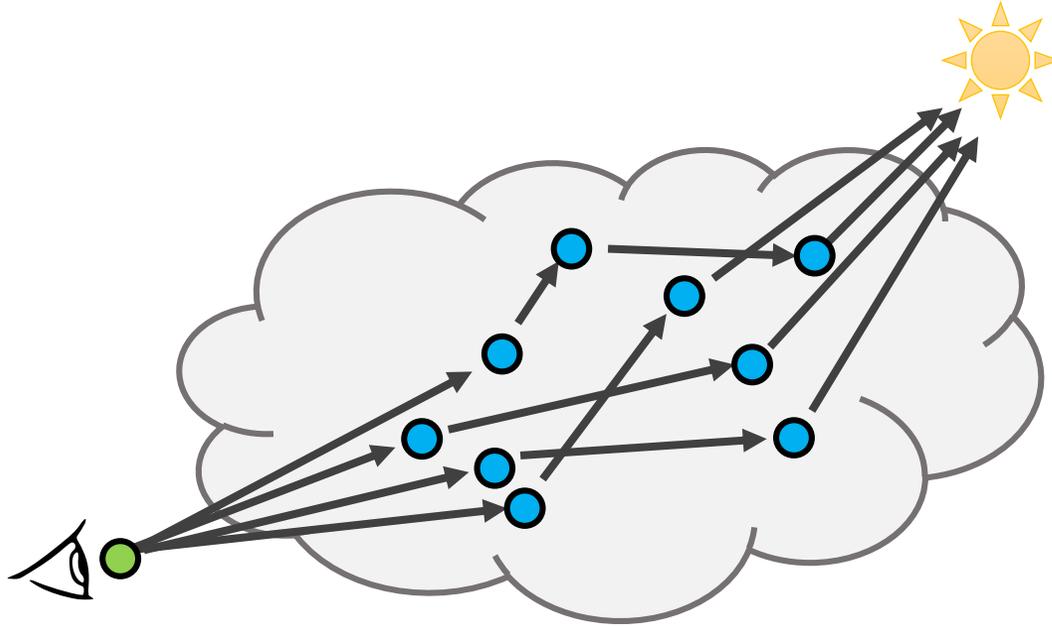
VOLUMETRIC MONTE-CARLO PATH TRACING



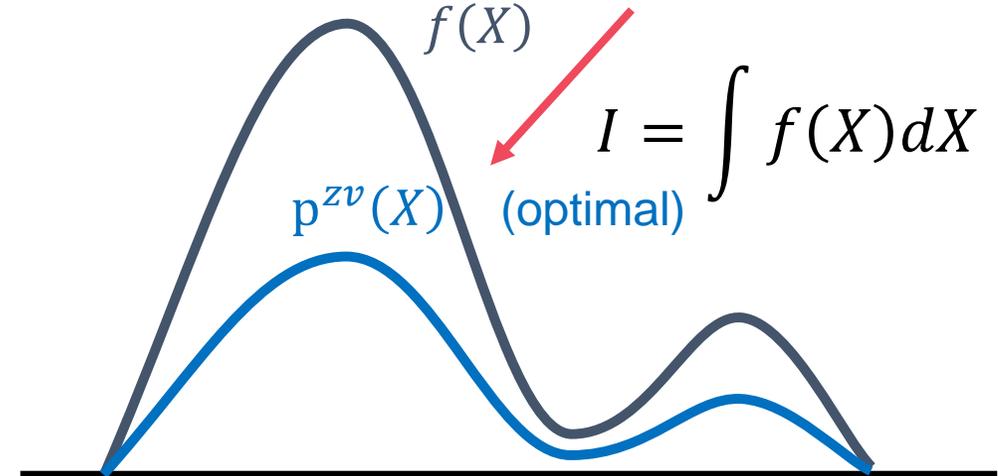
- The variance is defined by how well we can generate random paths proportional to the volumetric light transport:

$$\sigma^2 = V \left[\frac{f(X)}{p(X)} \right]$$

VOLUMETRIC MONTE-CARLO PATH TRACING: ZERO-VARIANCE



We need to know the shape of $f(X)$!

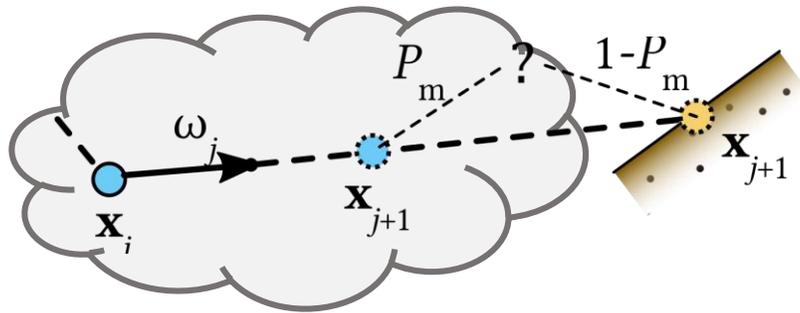


- If the PDF for all paths is proportional to the light transport function we would get a perfect **zero-variance** estimator:

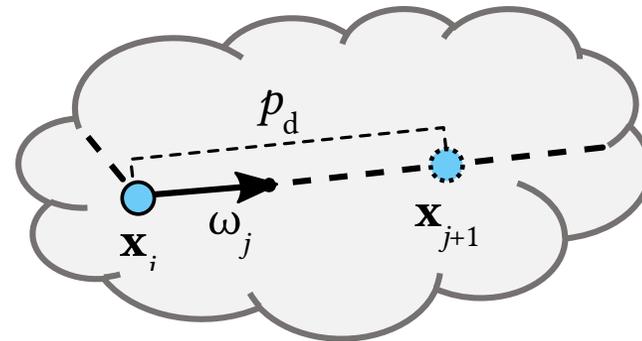
$$\sigma^2 = V \left[\frac{f(X)}{p^{zv}(X)} \right] = 0$$



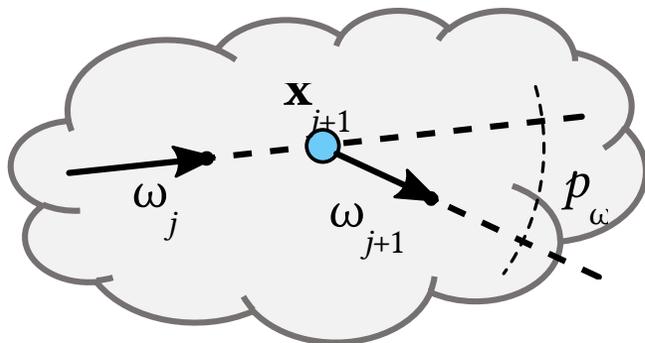
THE 4 SAMPLING DECISIONS:



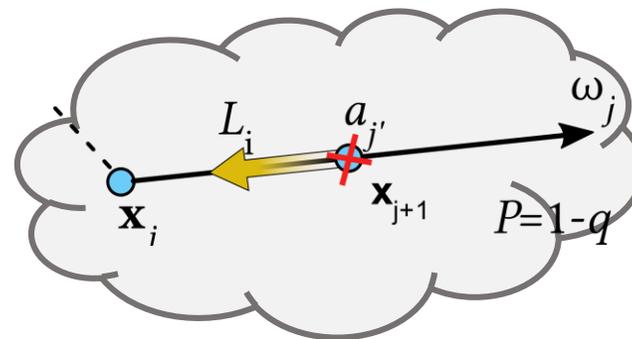
1. Scatter: $P_m(\mathbf{x}_j, \omega_j)$



2. Distance: $p_d(d_{j+1}|\mathbf{x}_j, \omega_j)$



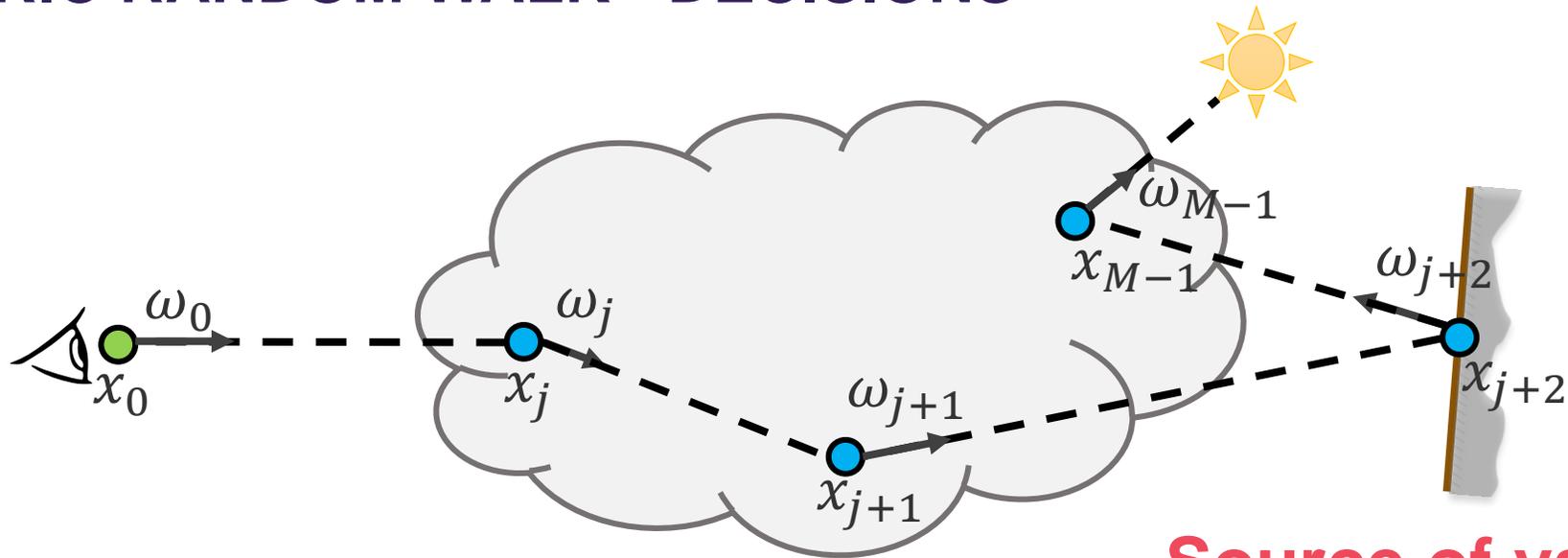
3. Direction: $p_\omega(\omega_{j+1}|\mathbf{x}_{j+1}, \omega_j)$



4. Termination: $P_{RR}(\mathbf{x}_j, \omega_{j-1})$



VOLUMETRIC RANDOM WALK - DECISIONS



Source of variance

- Path PDF :

$$p(\mathbf{X}) = \prod_{j=1}^{M-1} \underbrace{P_m(\dots) \cdot p_d(\dots) \cdot p_\omega(\dots) \cdot (1 - P_{RR}(\dots))}_{\text{Path segment PDF}}$$



VOLUME RENDERING EQUATION

- Incident radiance (volume):

$$L(x, \omega) = \underbrace{T(\dots) \cdot L_o(\dots)}_{\text{Surface contribution}} + \underbrace{\int T(\dots) \cdot \sigma_s(\dots) \cdot L_i(\dots) dd}_{\text{Volume contribution}}$$

- In-scattered radiance:

$$L_i(\dots) = \int f(\dots) \cdot L(\dots) d\omega'$$



VOLUME RENDERING EQUATION

- Incident radiance (volume): **Transmittance**

$$L(x, \omega) = T(\dots) \cdot L_o(\dots) + \int T(\dots) \cdot \sigma_s(\dots) \cdot L_i(\dots) dd$$

**Known Local
Quantities**

- In-scattered radiance:

$$L_i(\dots) = \int f(\dots) \cdot L(\dots) d\omega'$$

Phase function



VOLUME RENDERING EQUATION

- Incident radiance (volume):

$$L(x, \omega) = T(\dots) \cdot L_o(\dots) + \int T(\dots) \cdot \sigma_s(\dots) \cdot L_i(\dots) dd$$

Diagram annotations: A red arrow points from the text "surface radiance" to the $L_o(\dots)$ term, which is enclosed in a red box. Another red arrow points from the text "In-scattered radiance" to the $L_i(\dots)$ term, which is also enclosed in a red box.

Unknown Light Transport Quantities

- In-scattered radiance:

$$L_i(\dots) = \int f(\dots) \cdot L(\dots) d\omega'$$

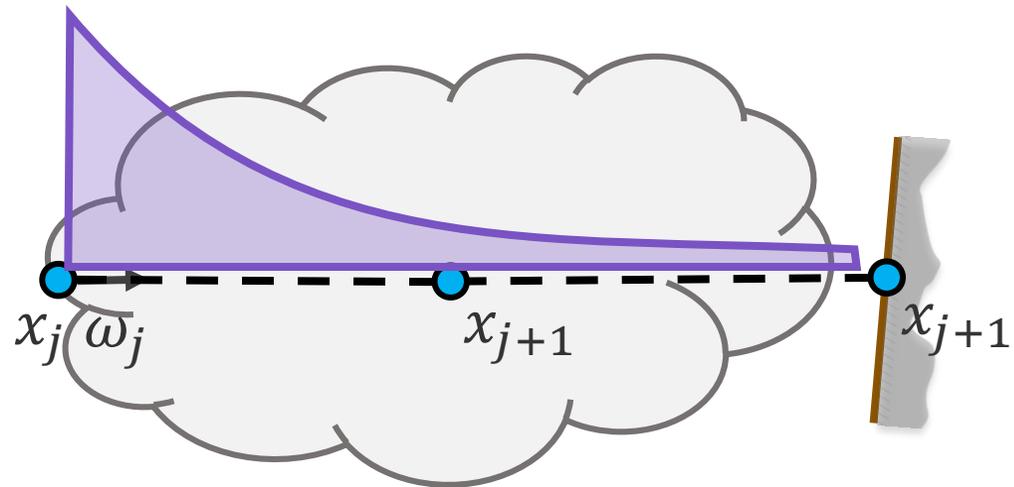
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Incident radiance



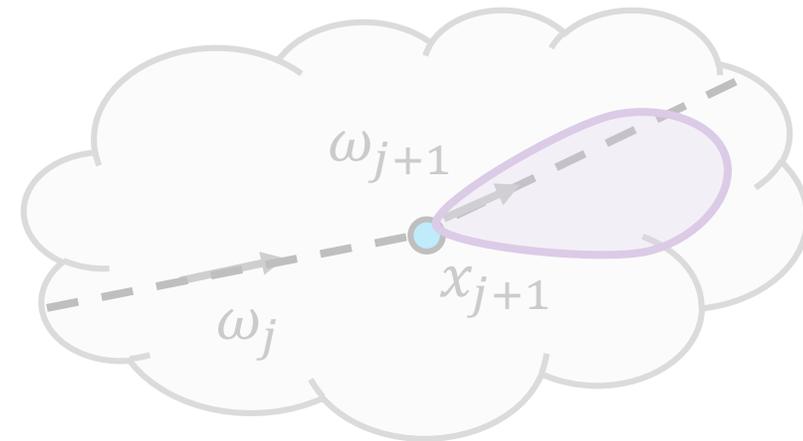
STANDARD SAMPLING

1+2 Scatter and Distance:



- Based on the transmittance

3 Direction:

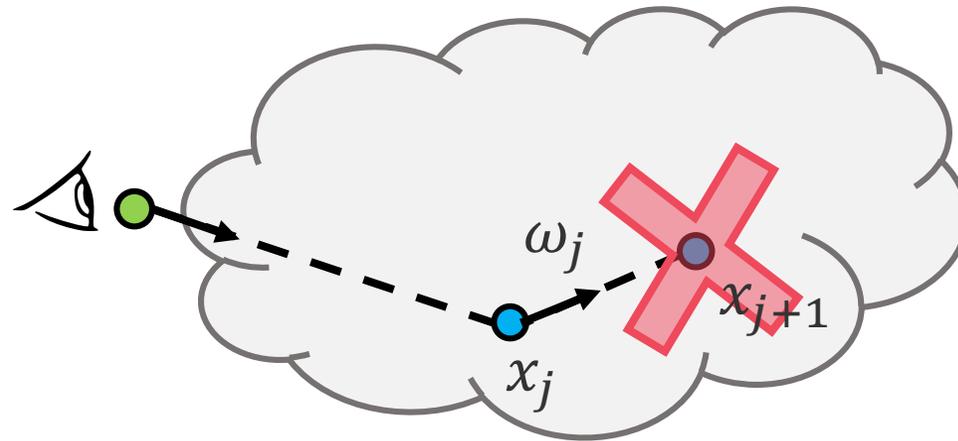


- Based on the phase function

STANDARD SAMPLING



4 Termination:



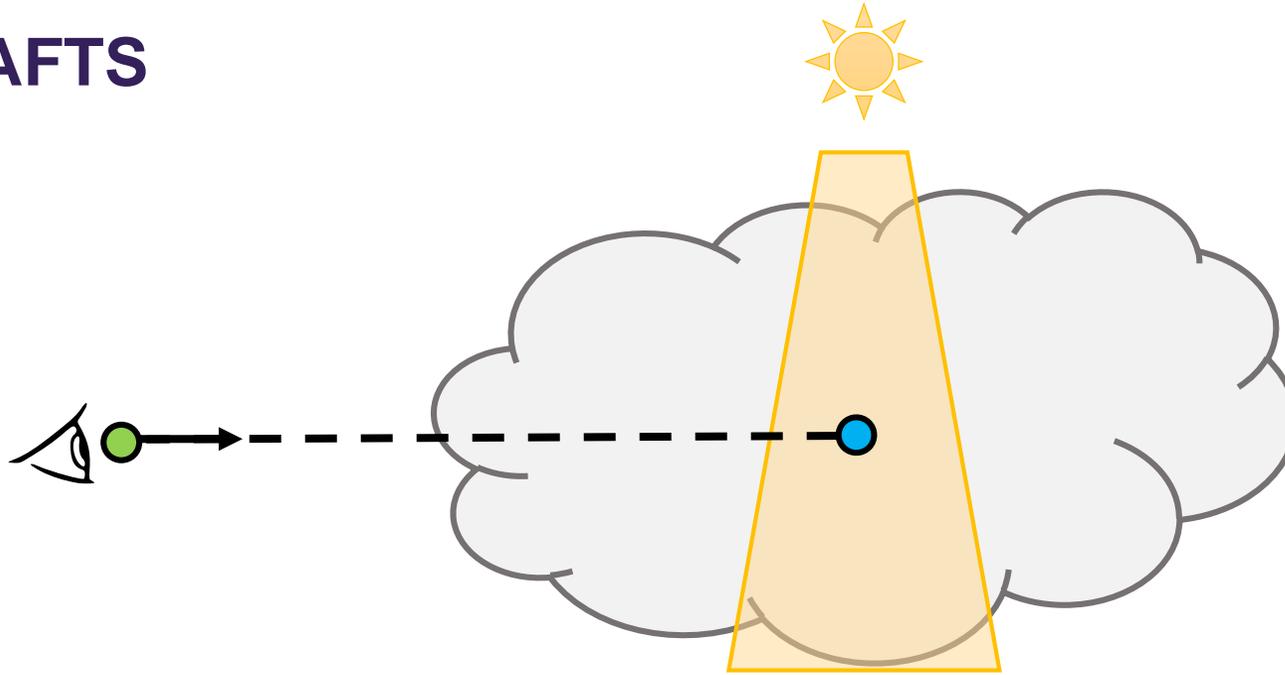
- Based on local albedo or throughput



CHALLENGES FOR VOLUME SAMPLING

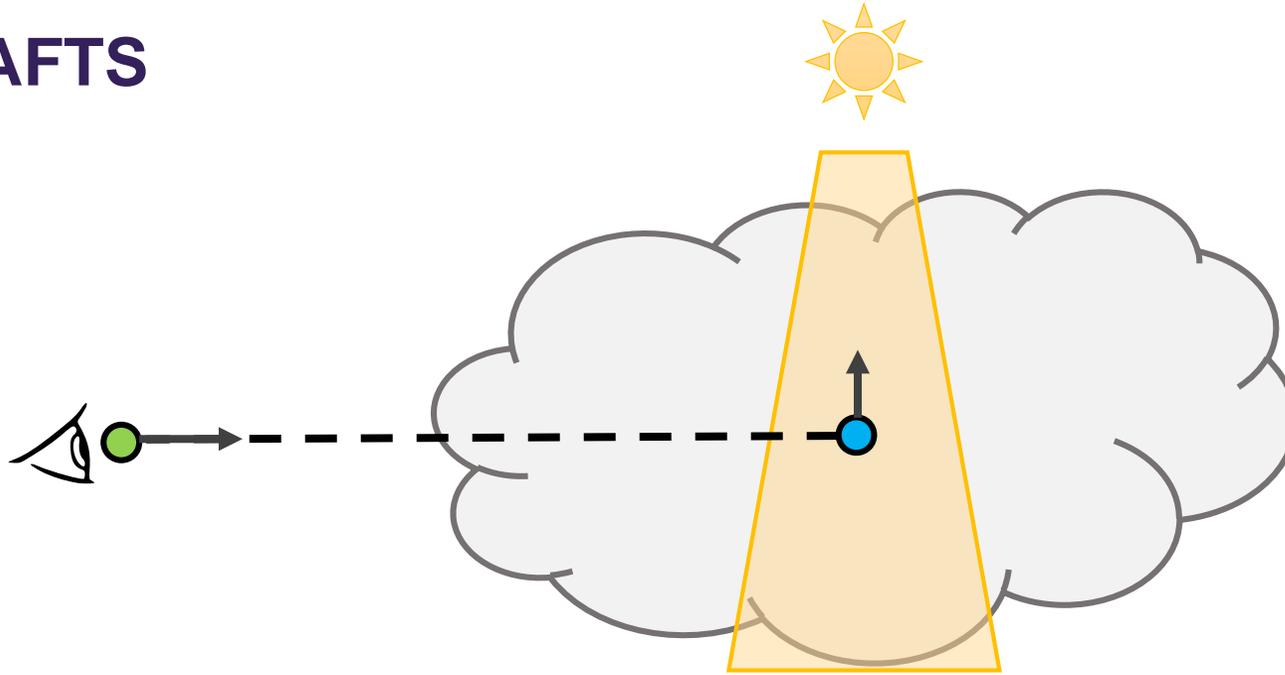
Why do we need volumetric path guiding?

LIGHT SHAFTS



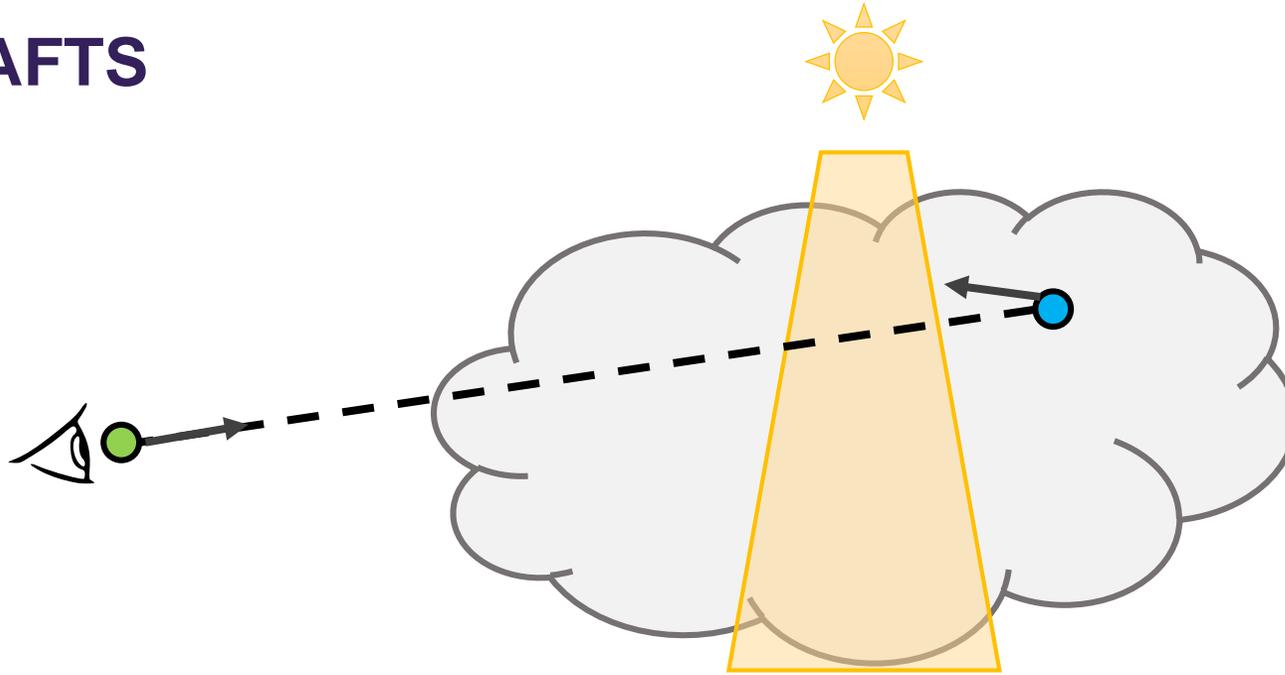
- Light shafts:
 - We need to scatter inside the light shaft.

LIGHT SHAFTS



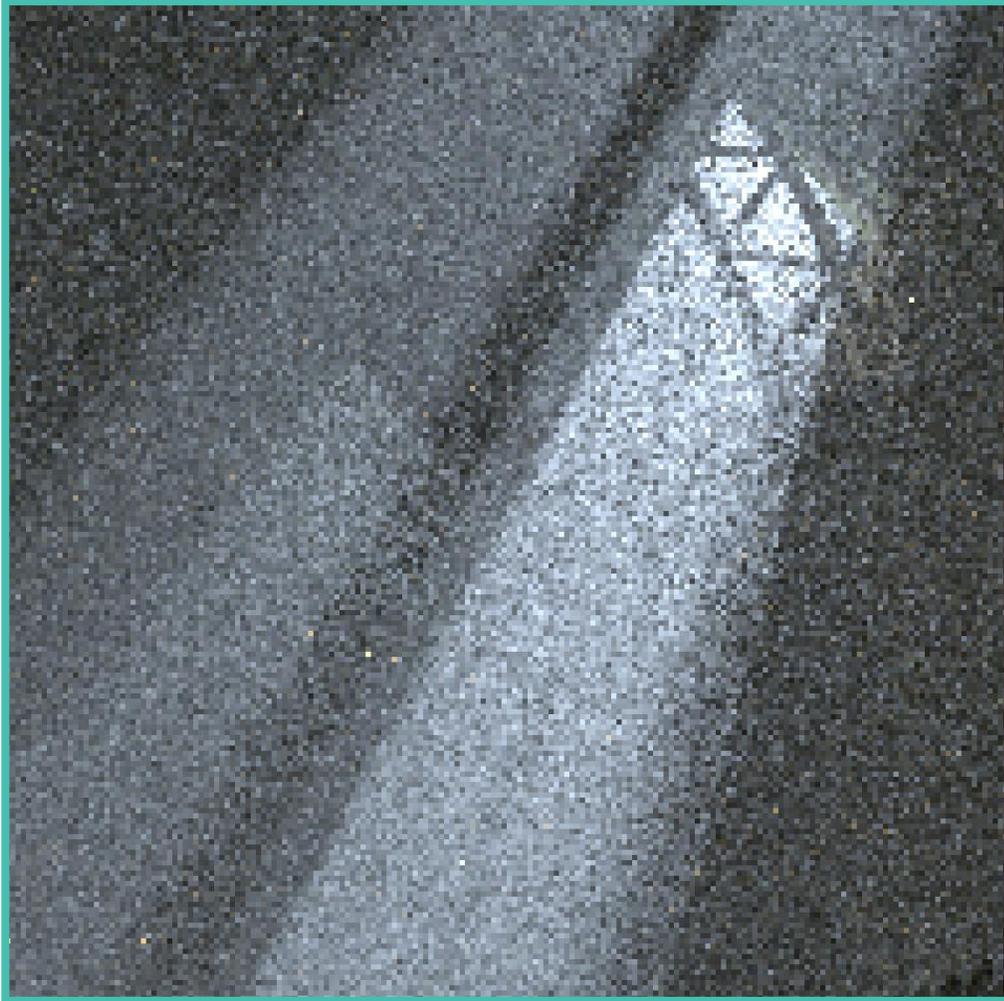
- Light shafts:
 - We need to scatter inside the light shaft.
 - We need to follow the direction of the light shaft.

LIGHT SHAFTS

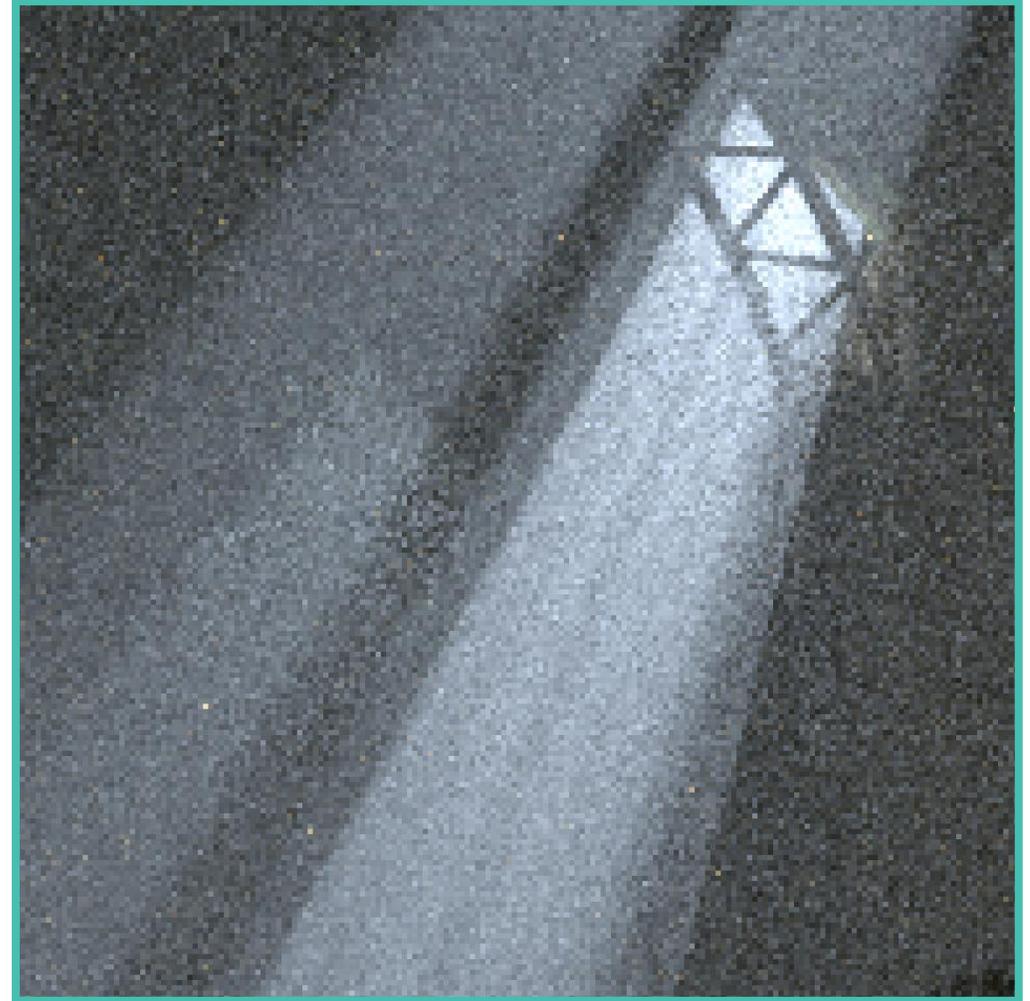


- Light shafts:
 - We need to scatter inside the light shaft.
 - We need to follow the direction of the light shaft.
 - We need to scatter towards the light shaft.

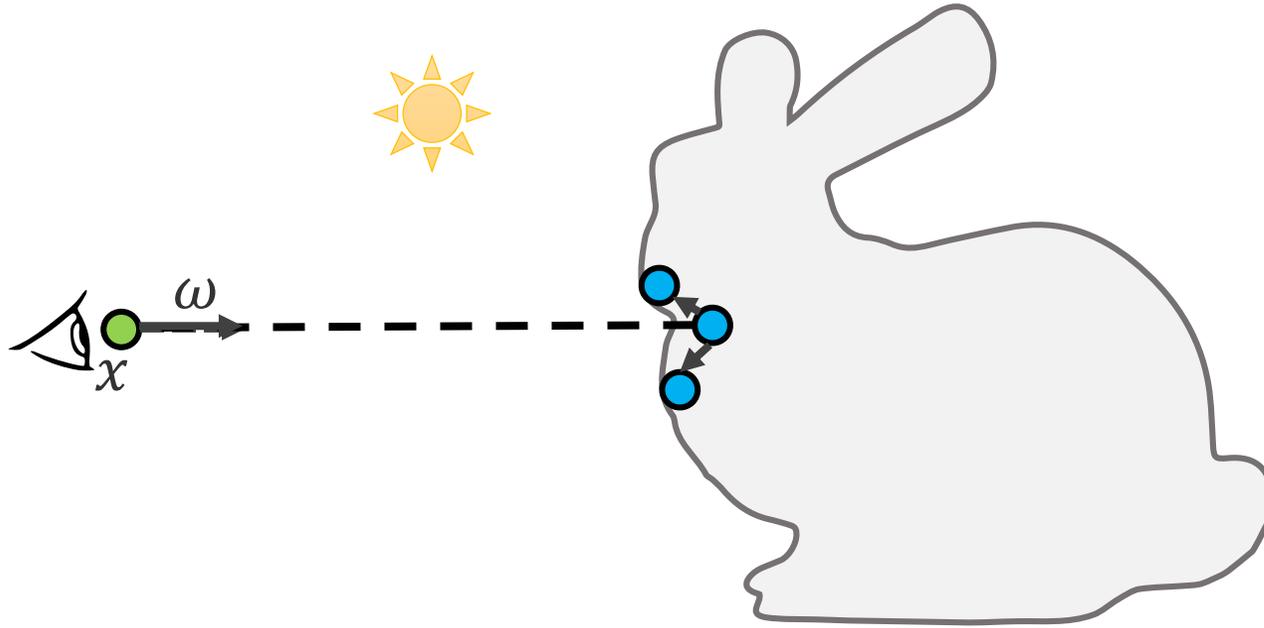
No guiding (1024 spp)



Our guiding (1024 spp)



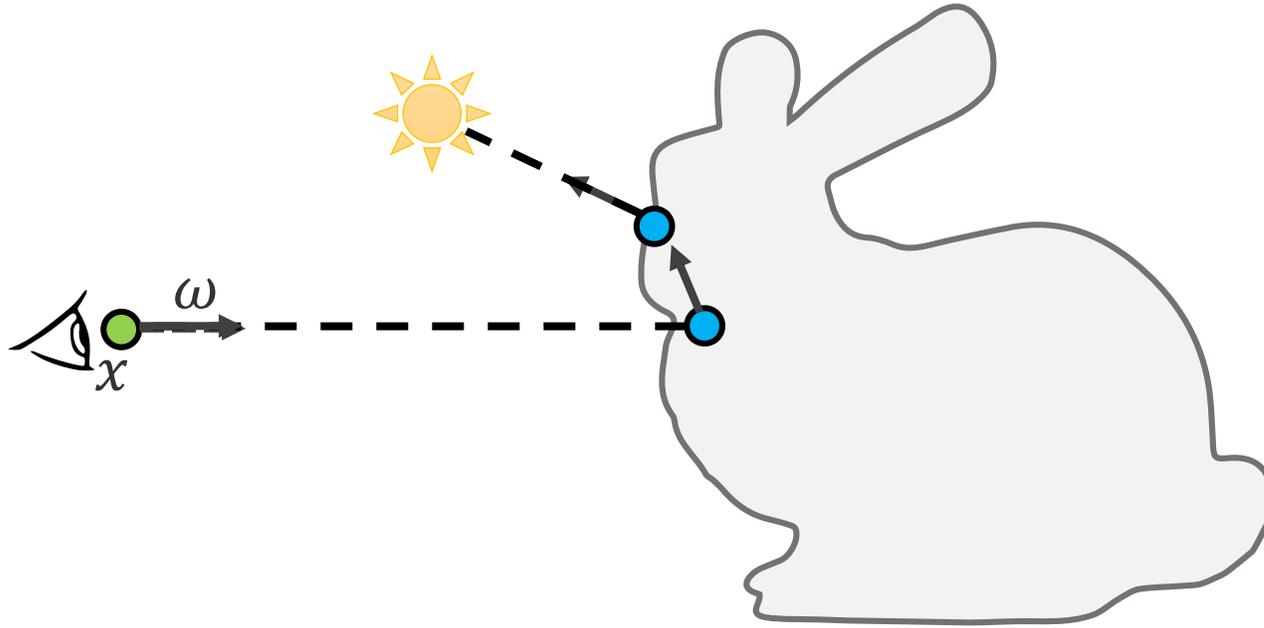
SUB-SURFACE-SCATTERING



- Sub-Surface-Scattering:
 - We 'often' need stay close to the surface

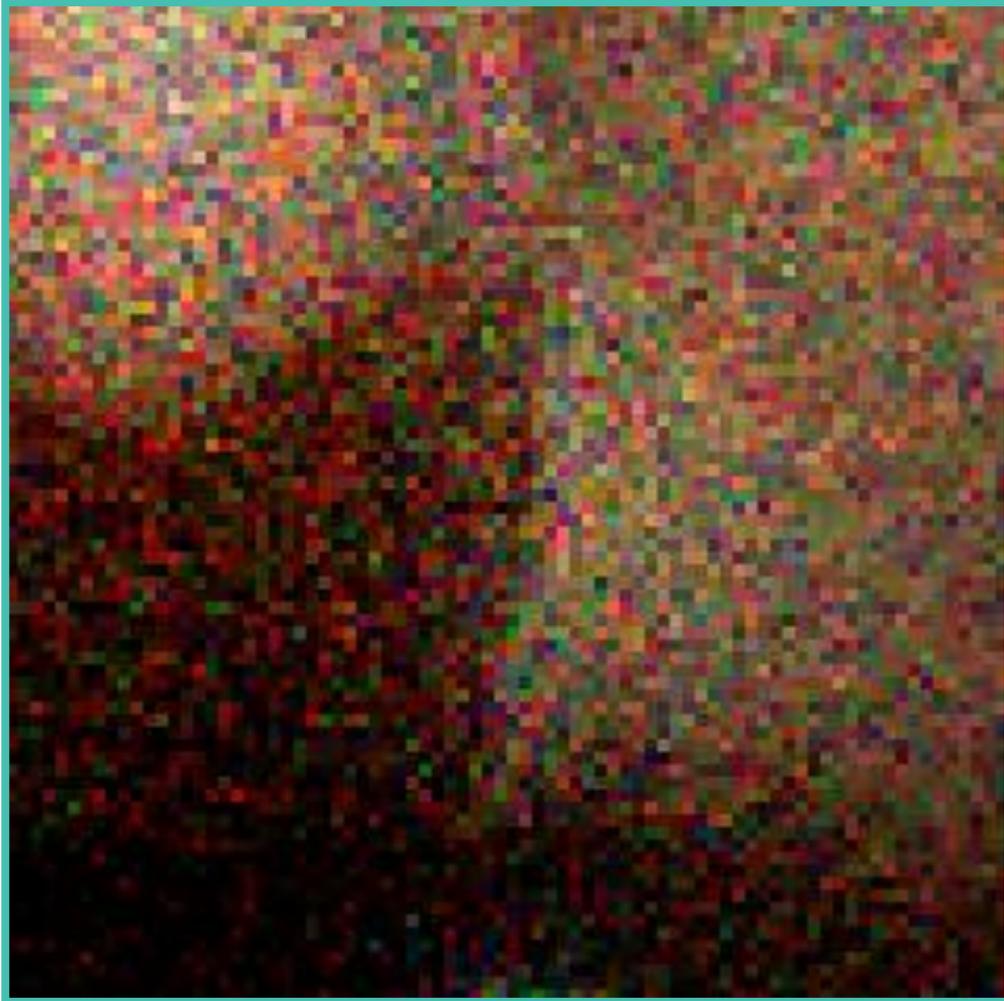


SUB-SURFACE-SCATTERING

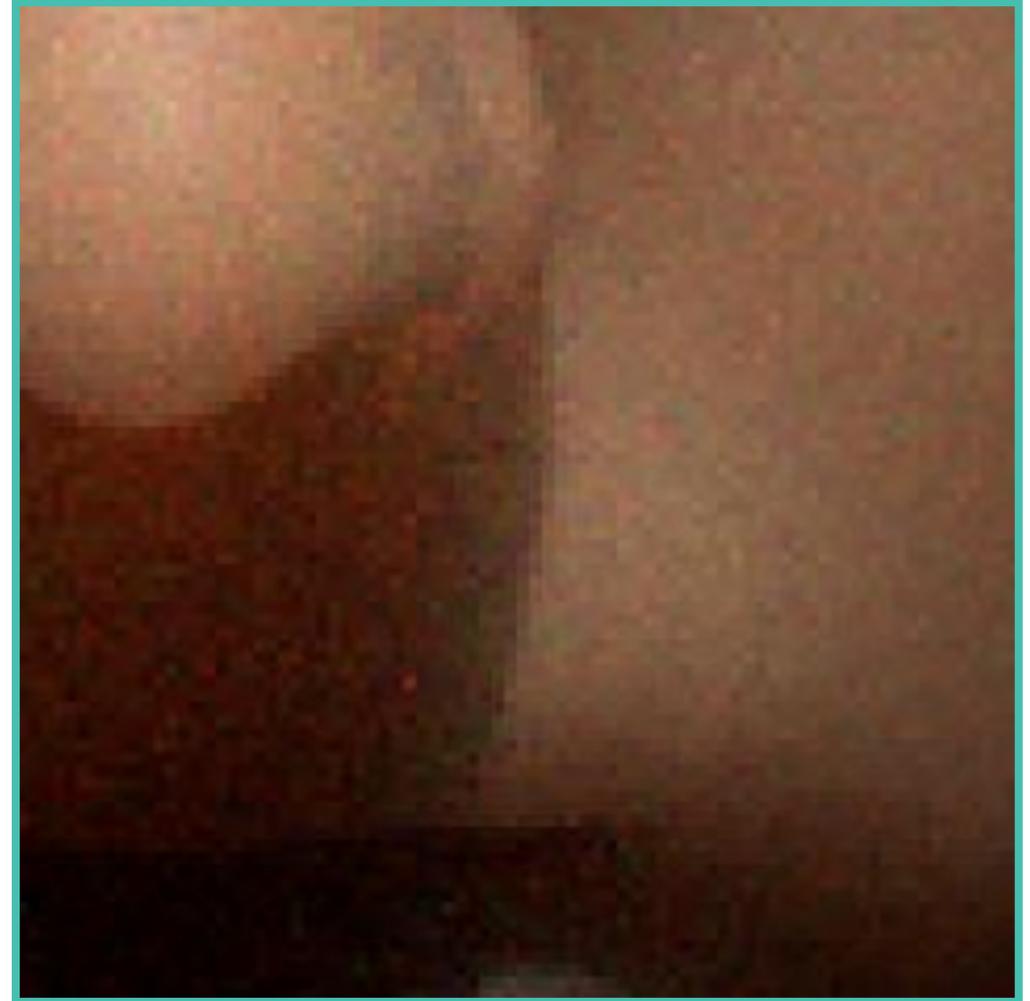


- Sub-Surface-Scattering:
 - We 'often' need to stay close to the surface
 - We need to leave the object with the right direction

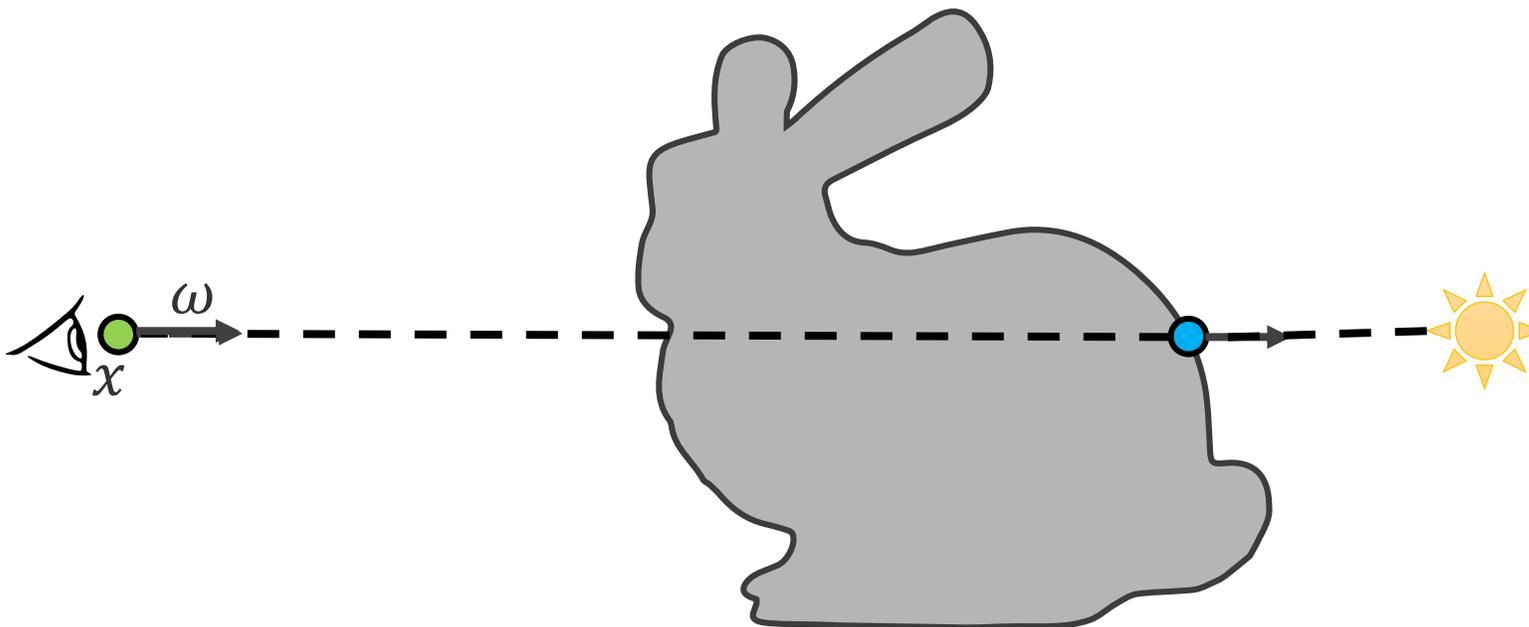
No guiding (256 spp)



Our guiding (256 spp)



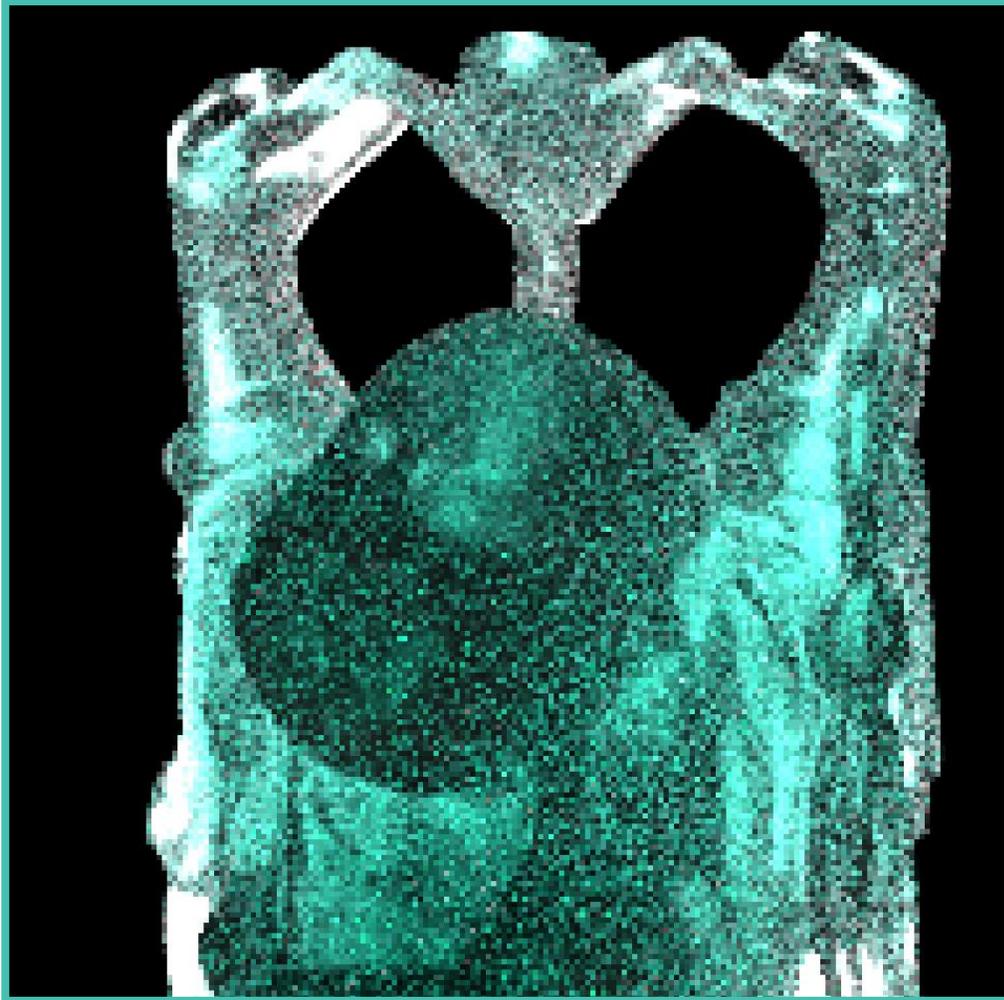
DENSE MEDIA



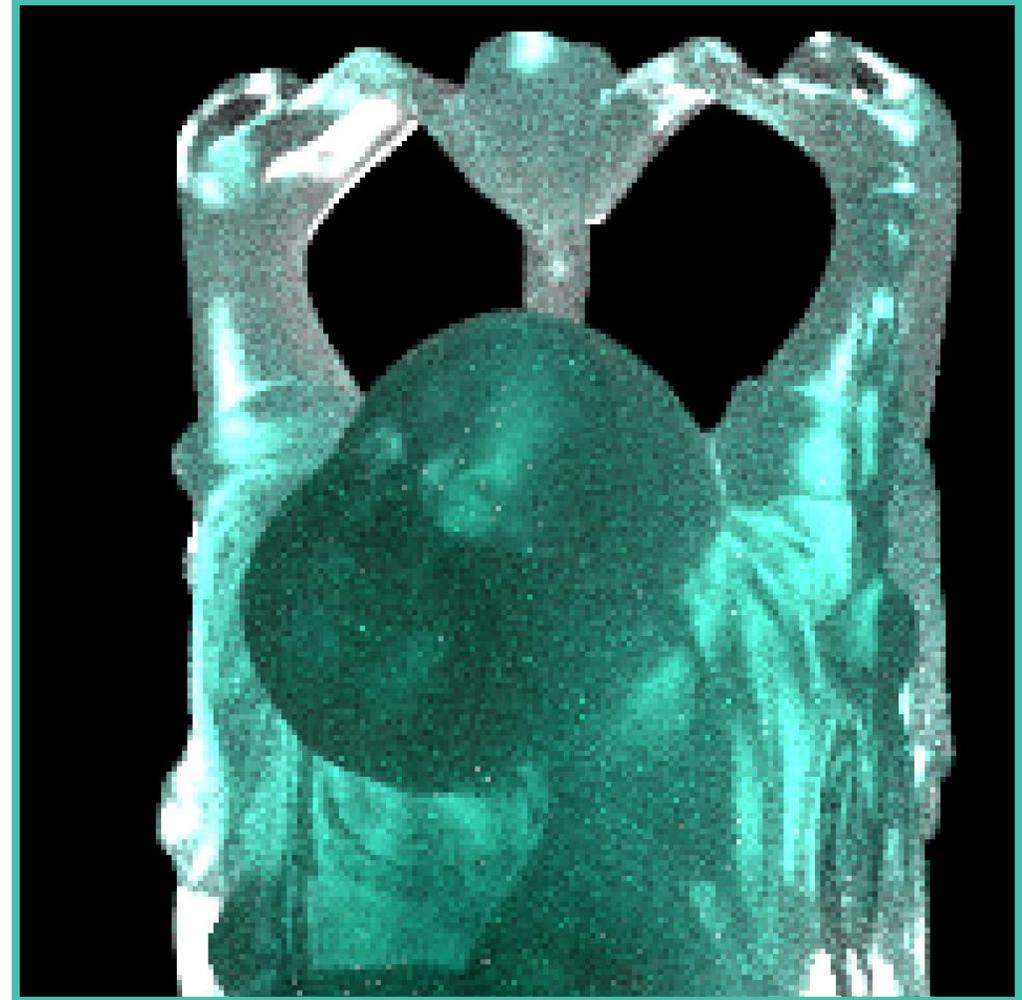
- Dense media (back illuminated):
 - We may need to **'avoid'** generating a scattering event even if the transmittance is low (e.g. strong light source behind the volume).



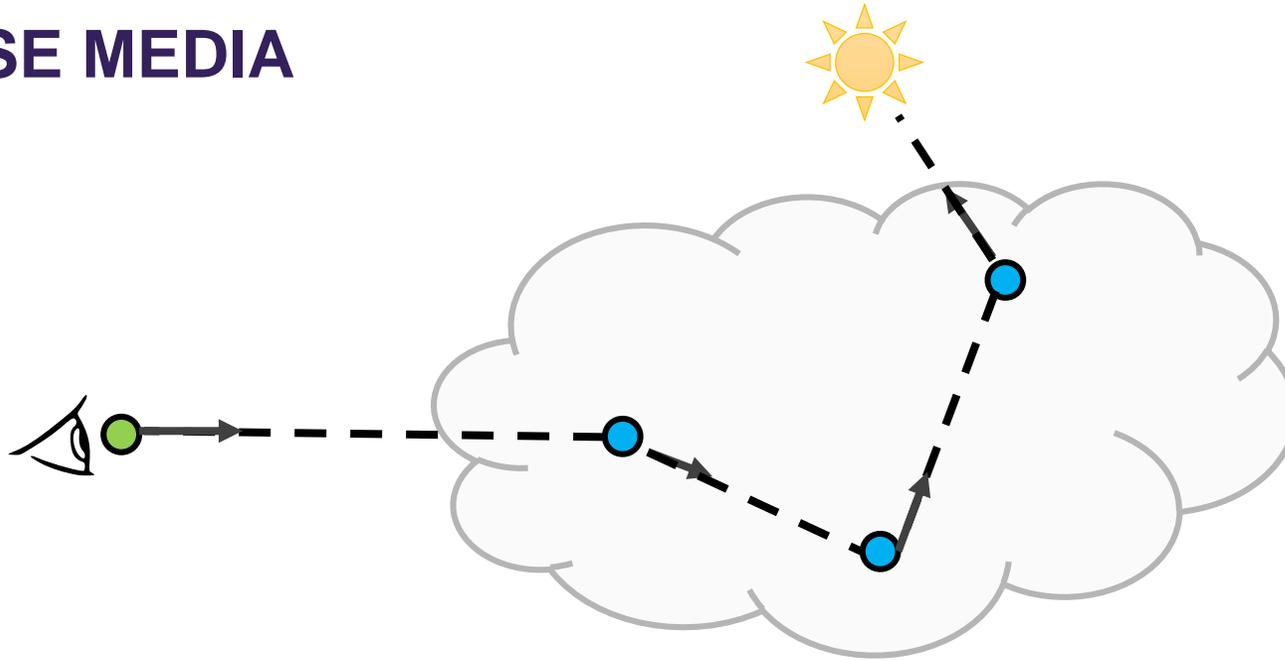
No guiding (256 spp)



Our guiding (256 spp)

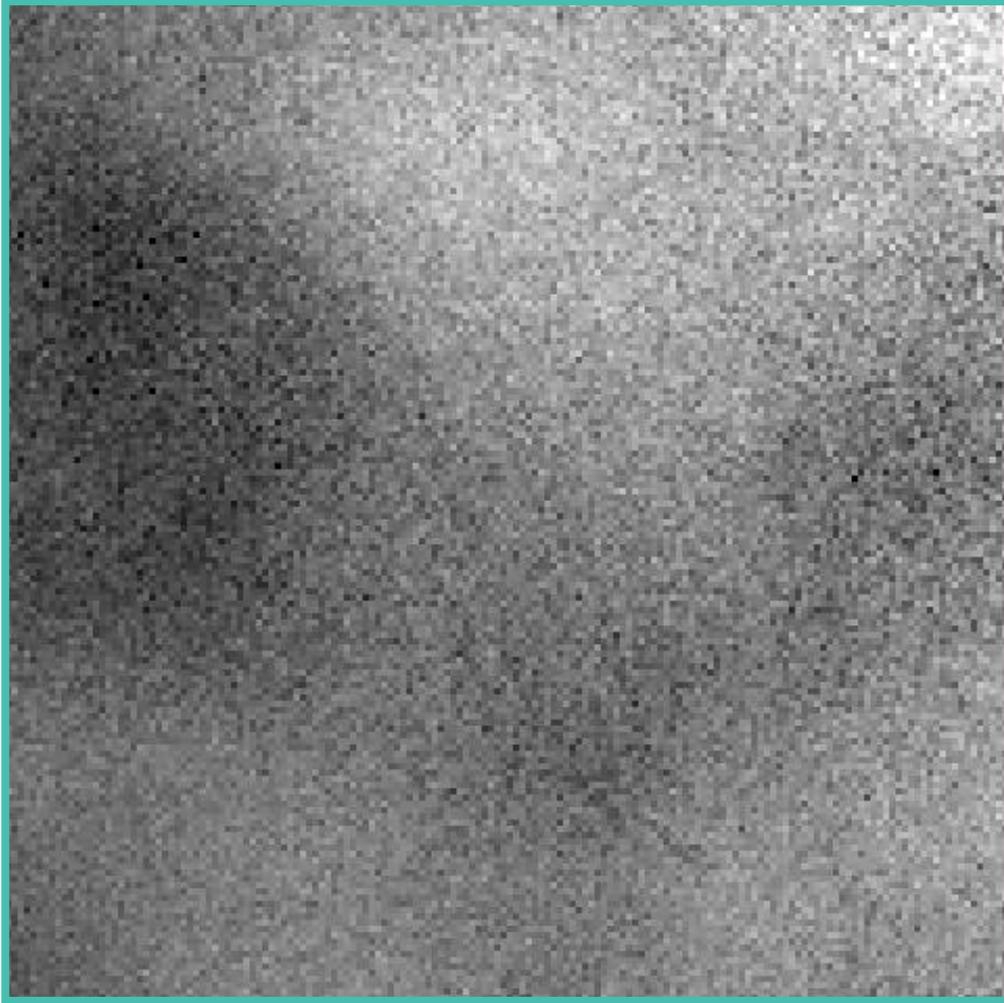


NON-DENSE MEDIA

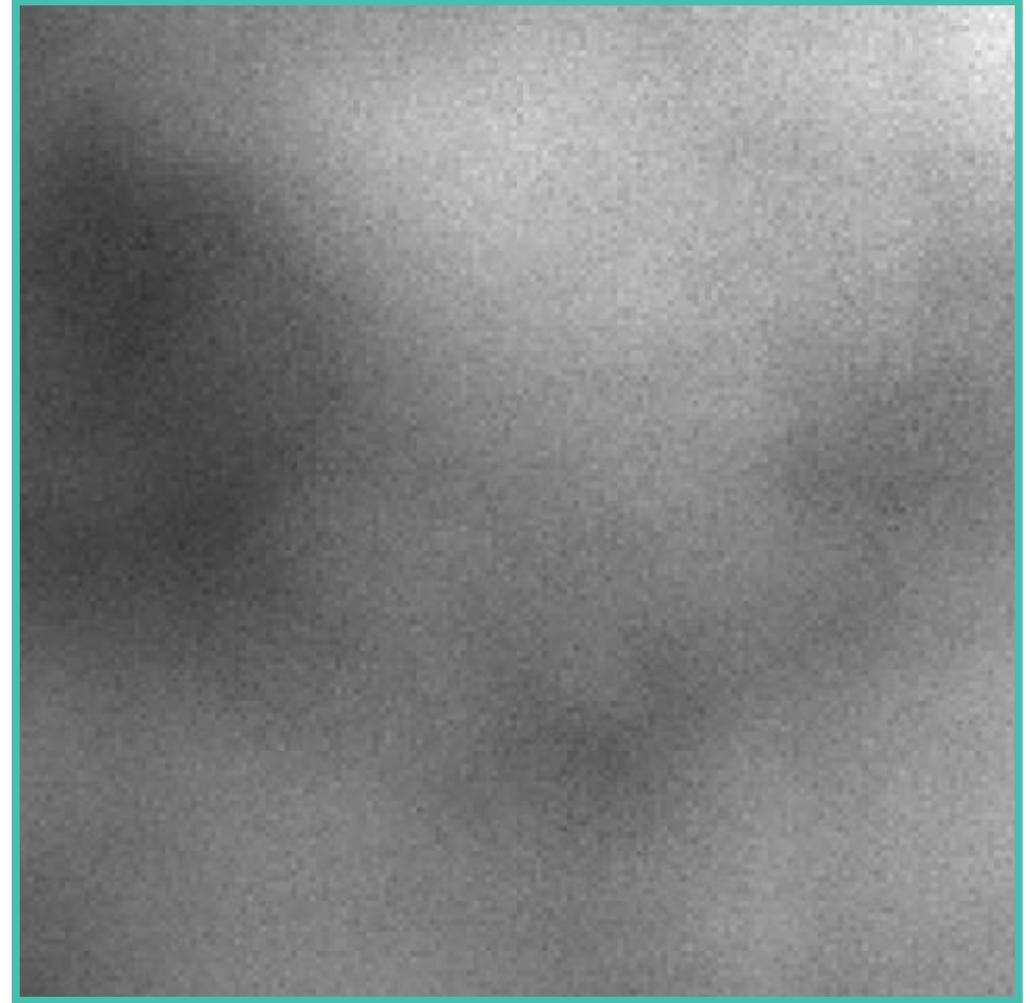


- Non-dense media (no back illumination):
 - We may need to **'force'** a scattering event even if the transmittance is high.

No guiding (256 spp)



Our guiding (256 spp)



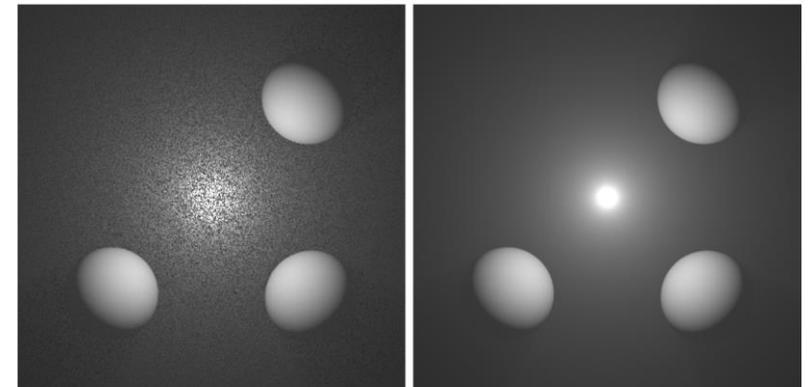


SPECIALIZED SOLUTIONS: SHORTCOMINGS

- Many individual solutions:
 - Equiangular Sampling: [Kulla2012]
 - Joint-Importance Sampling: [Georgiev2012]
 - Zero-Variance Dwivedi Sampling: [Krivanek2014]
 - Directional (illumination-based) guiding: [Meng2016]
 - Directional (illumination-based) guiding: [Pegoraro2008]
 - Directional (illumination-based) guiding: [Bashford2012]
- Only considering sub-sets or special cases:
 - Surface-bounded volumes
 - Homogenous or isotropic volumes
 - Single scattering
- **None of the current methods importance samples the full volumetric light transport!**



(a) Path traced light transport in clouds



(a) Distance Sampling

(b) Equi-angular Sampling

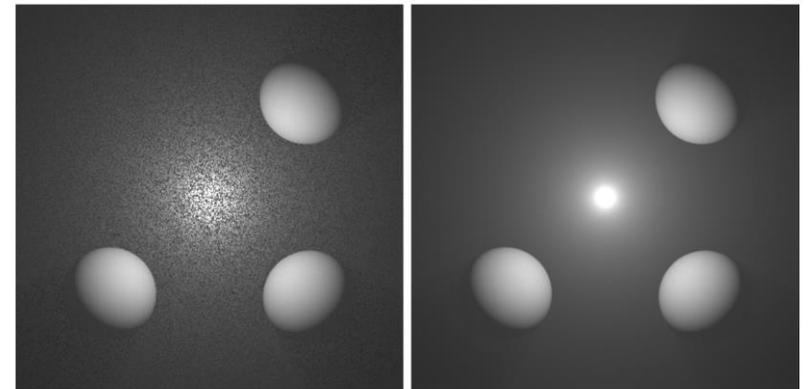


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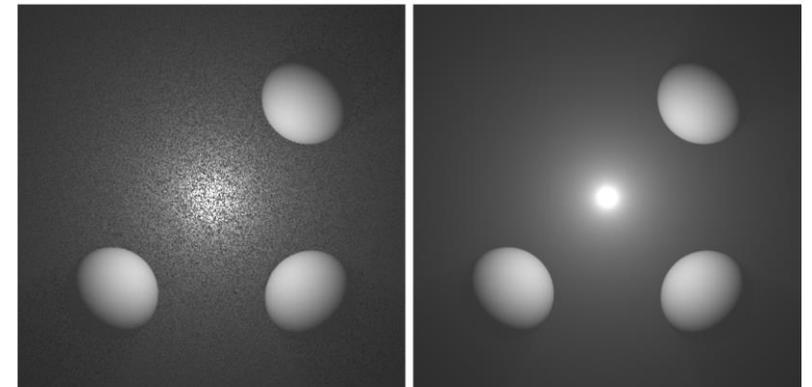


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(b) Equi-angular Sampling



ZERO-VARIANCE-BASED VOLUMETRIC PATH GUIDING

TECH TALK: TUE: 30TH JULY
TIME: 9:00 AM
ROOM: 152



ZV-BASED VOLUMETRIC PATH GUIDING: GOALS

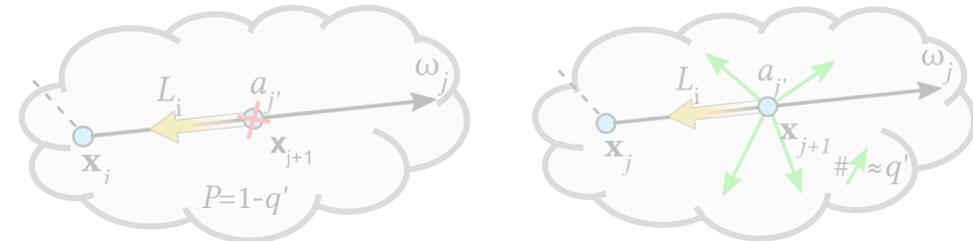
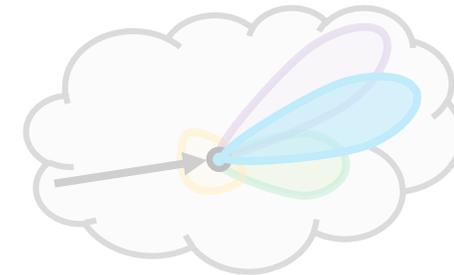
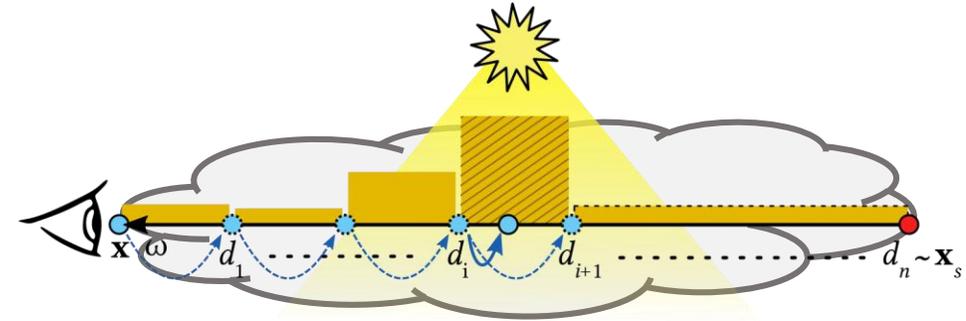
- Leverage recent success of local surface guiding methods: [Vorba2014], [Herholz2016], [Mueller2017]
 - Extend the concept to volumes
- Consider the **complete** volumetric light transport:
 - No prior assumptions or special cases
 - Guide based on the **optimal** zero-variance decisions [Hoogenboom 2008]
- Replace unknown quantities by estimates:

$$L(\mathbf{x}, \omega) = \tilde{L}(\mathbf{x}, \omega) \quad L_i(\mathbf{x}, \omega) = \tilde{L}_i(\mathbf{x}, \omega)$$



ZV-BASED VOLUMETRIC PATH GUIDING: CONTRIBUTIONS

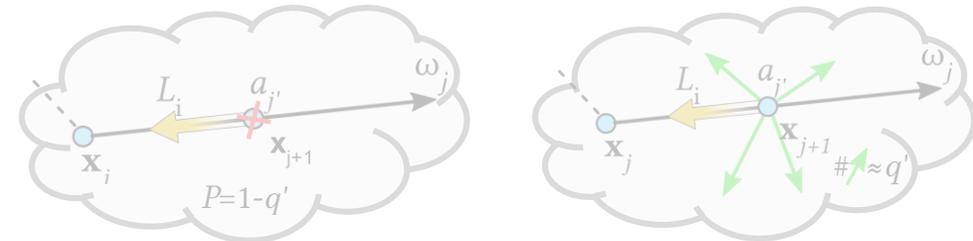
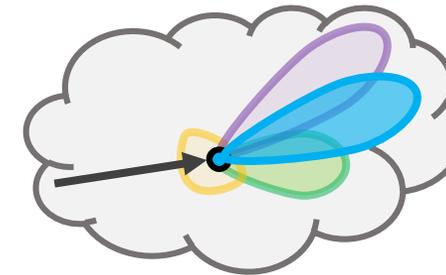
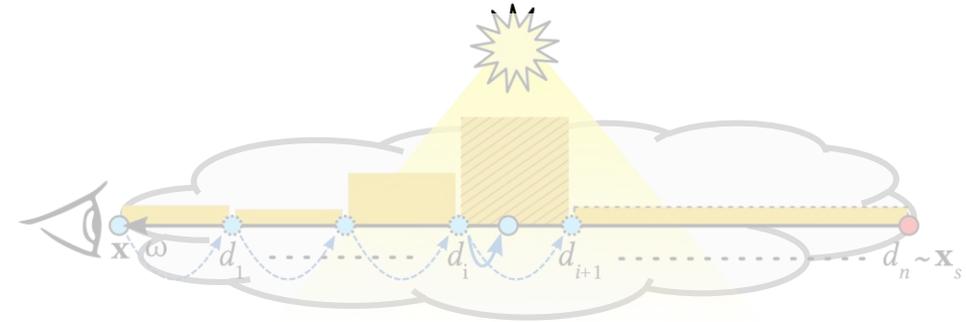
- Guiding **all** local sampling decisions:
 - 1+2 Guided product distance sampling:
 - 3 Guided product directional sampling:
 - 4 Guided Russian roulette and Splitting:





ZV-BASED VOLUMETRIC PATH GUIDING: CONTRIBUTIONS

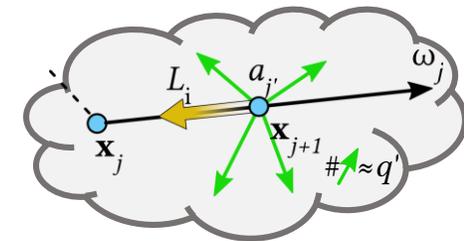
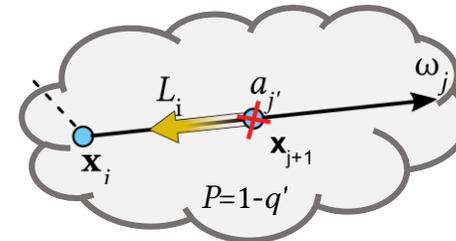
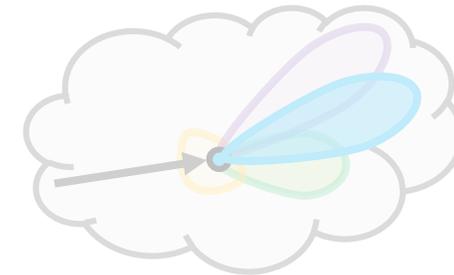
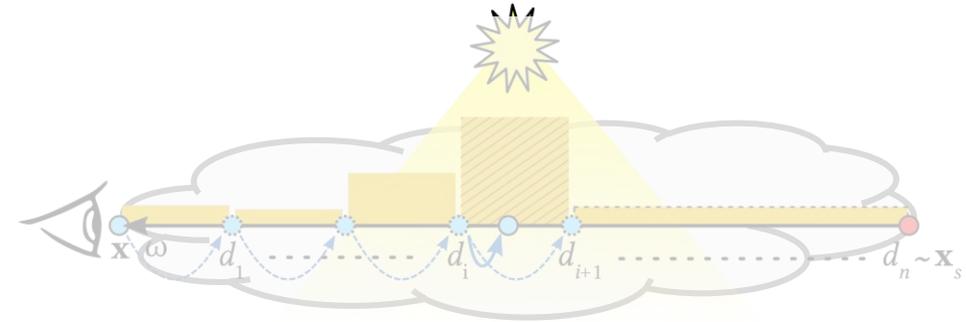
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ZV-BASED VOLUMETRIC PATH GUIDING: CONTRIBUTIONS

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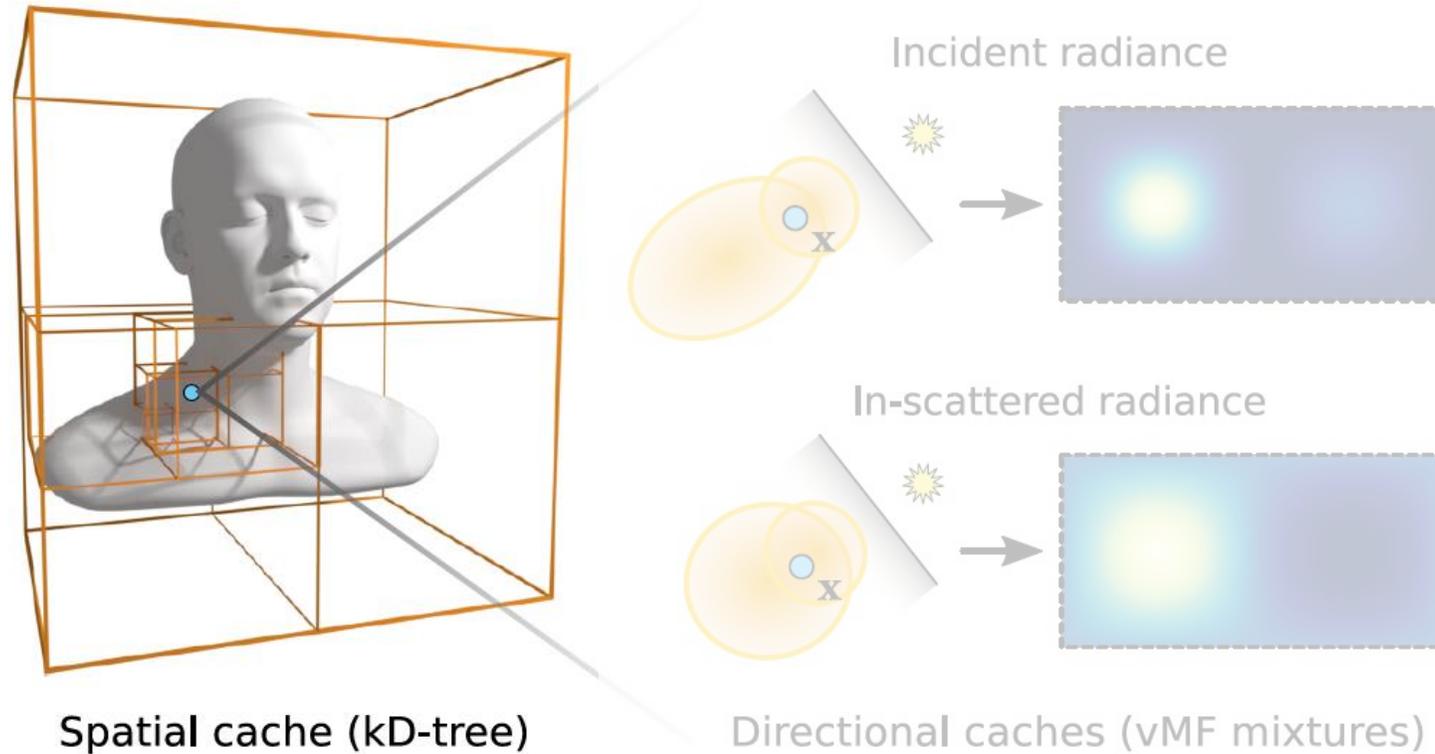




VOLUME RADIANCE ESTIMATES



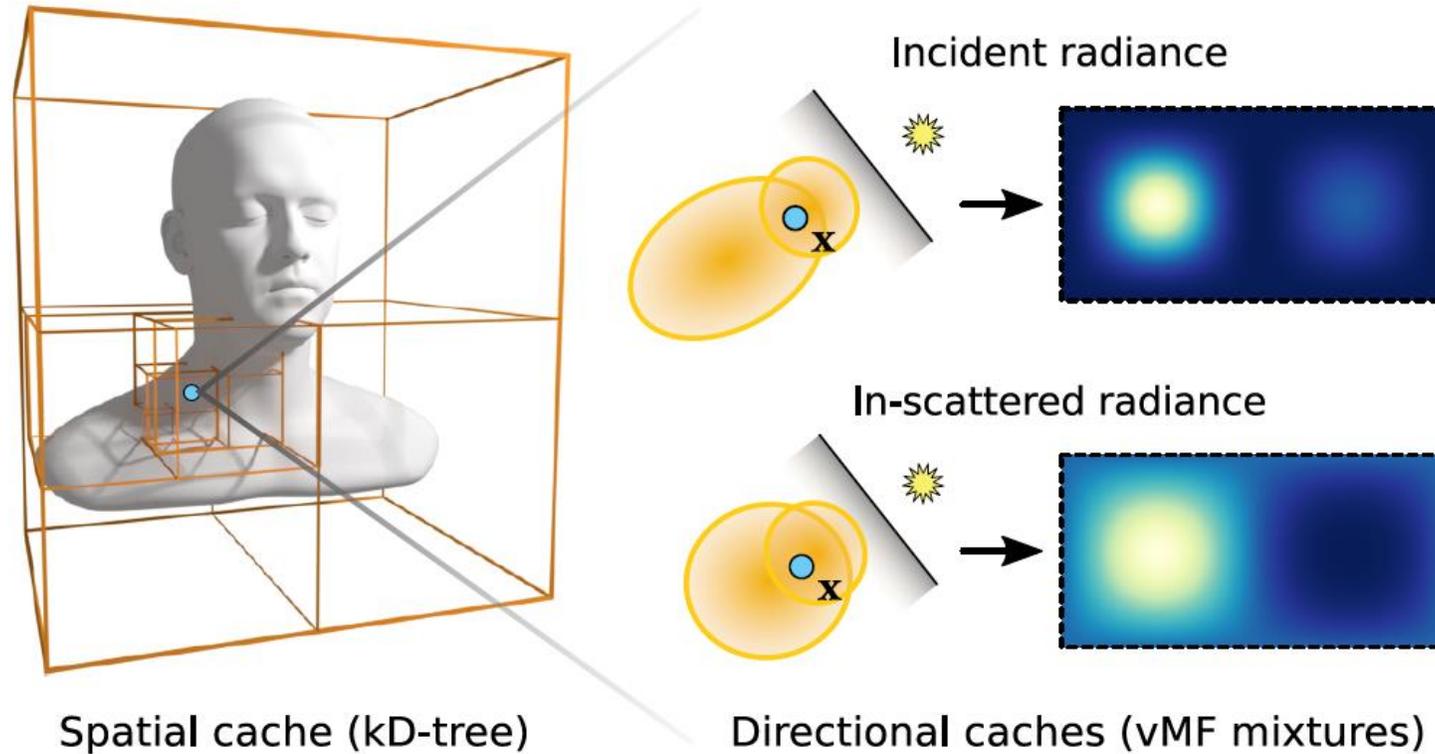
VOLUME RADIANCE ESTIMATES



- Spatial caches via BSP-tree: max. 2K photons per node:
 - Similar 3D structure as PPG [Mueller2017]

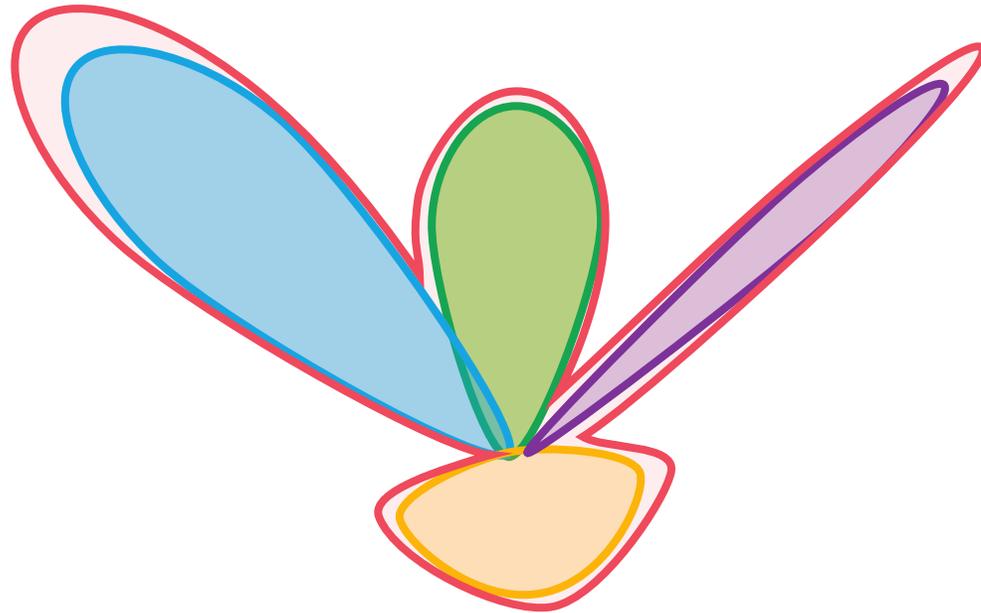


VOLUME RADIANCE ESTIMATES



- Pre-processing step to fit estimates from photons (50M):
 - EM-fitting of von Mises-Fisher mixtures (similar to [Vorba2014]’s GMMs)

VON MISES-FISHER MIXTURE MODELS

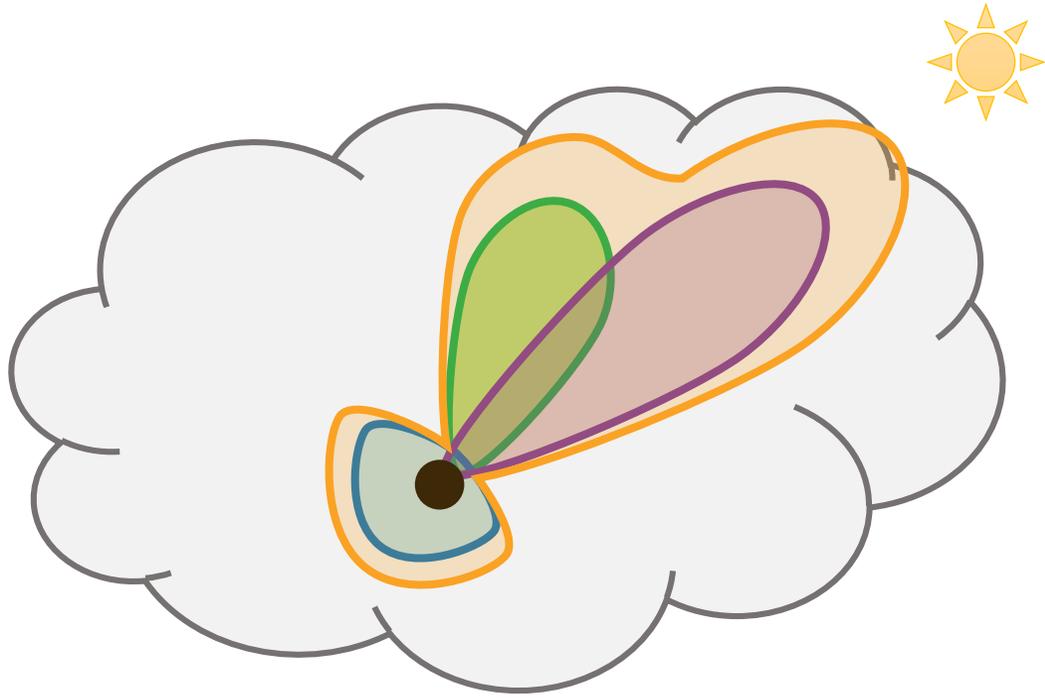


- Spherical Distribution:

$$V(\omega|\Theta) = \sum^K \pi_i v(\omega|\mu_i, \kappa_i)$$

- Features (closed-form):
 - Sampling
 - Convolution
 - Product

RADIANCE ESTIMATES

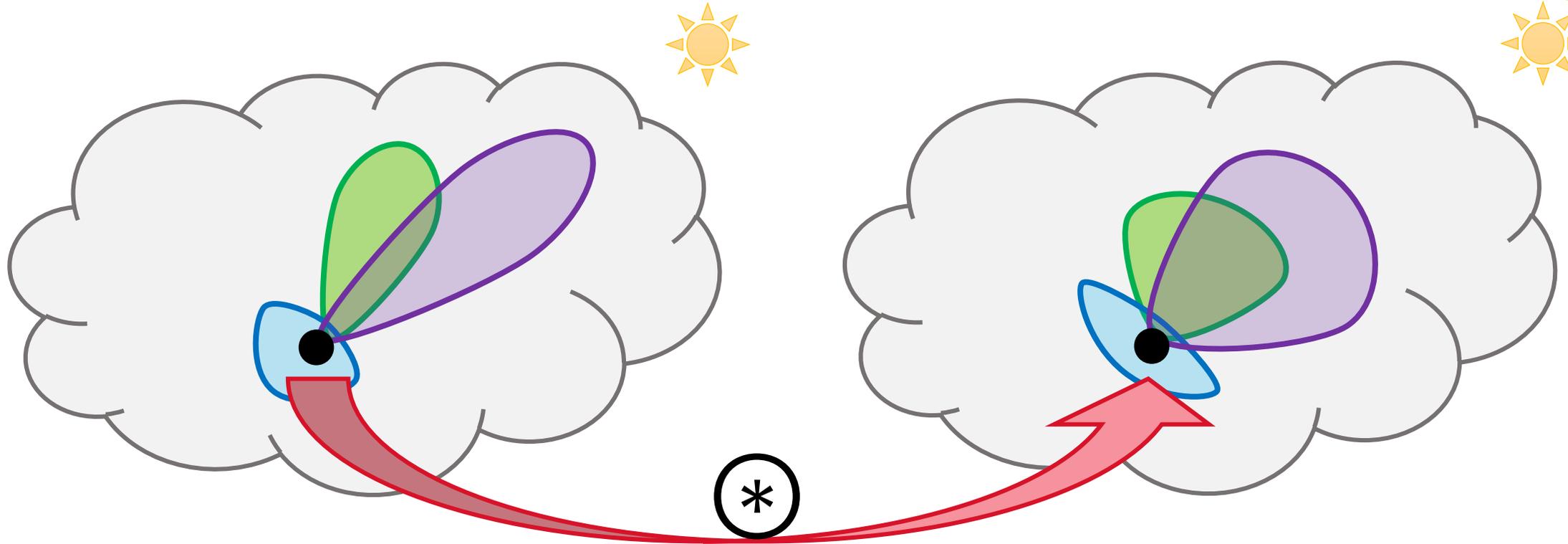


• Incident Radiance Distribution



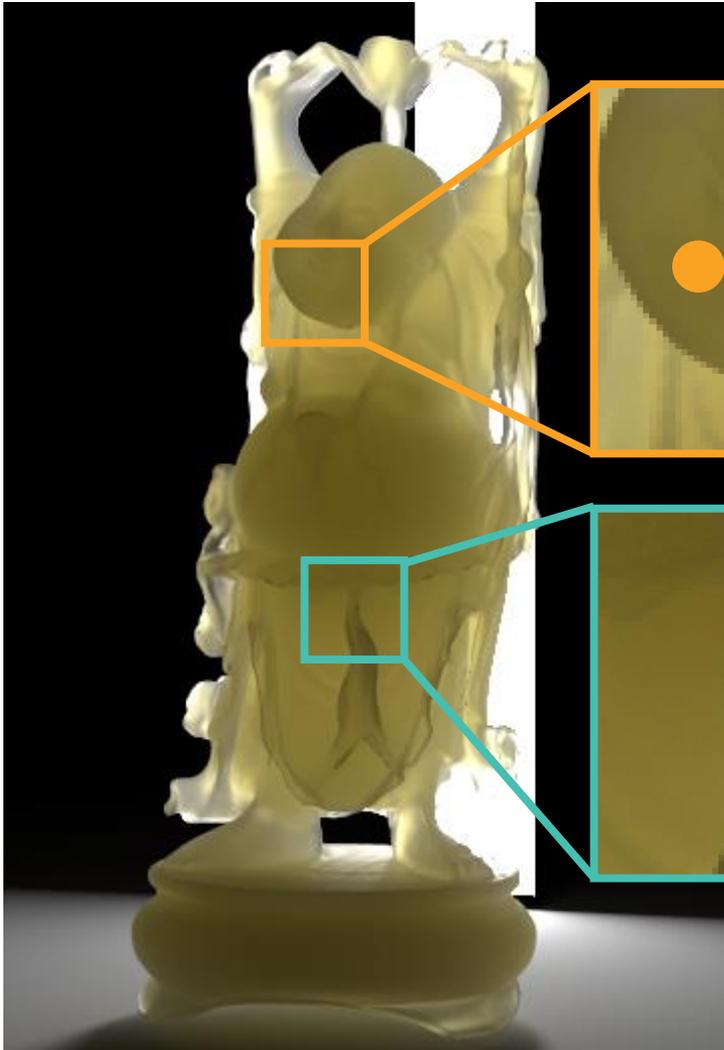
• In-Scattered Radiance Distribution

INCIDENT RAD. TO IN-SCATTERED RAD. TRANSFORMATION



- Convolution between incident radiance L and the phase function f

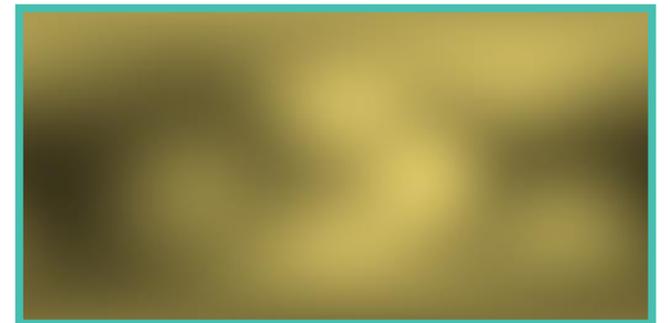
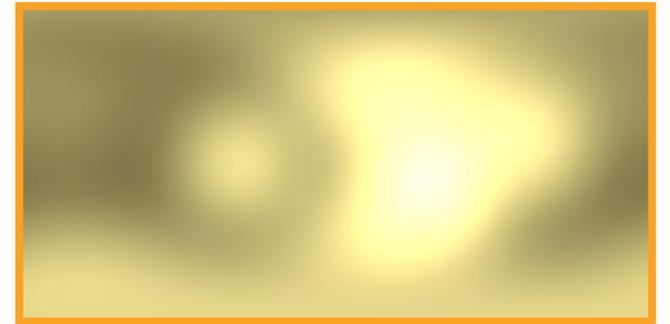
INCIDENT RADIANCE ESTIMATES



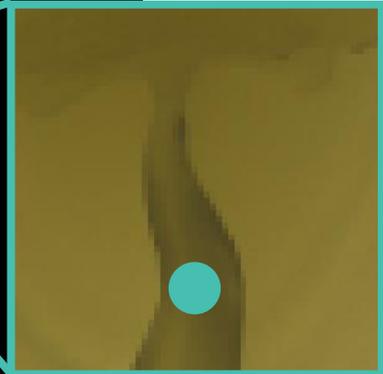
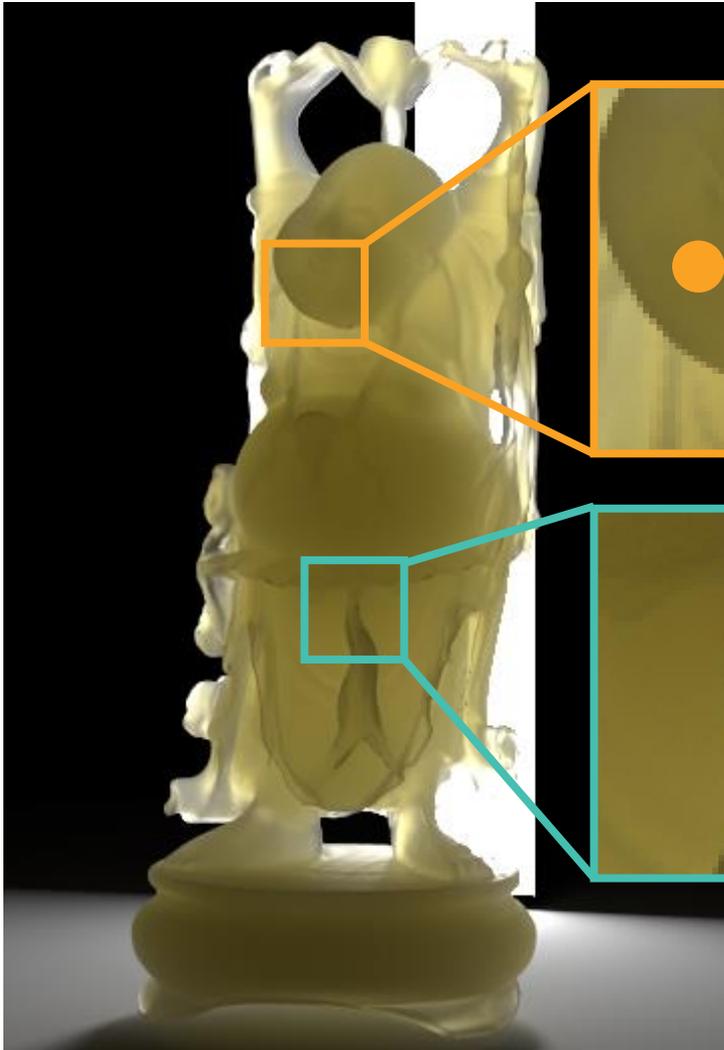
Ground truth (2K spp)



Our estimates (VMM)



IN-SCATTERED RADIANCE ESTIMATES



Ground truth (2K spp)



Our estimates (VMM)

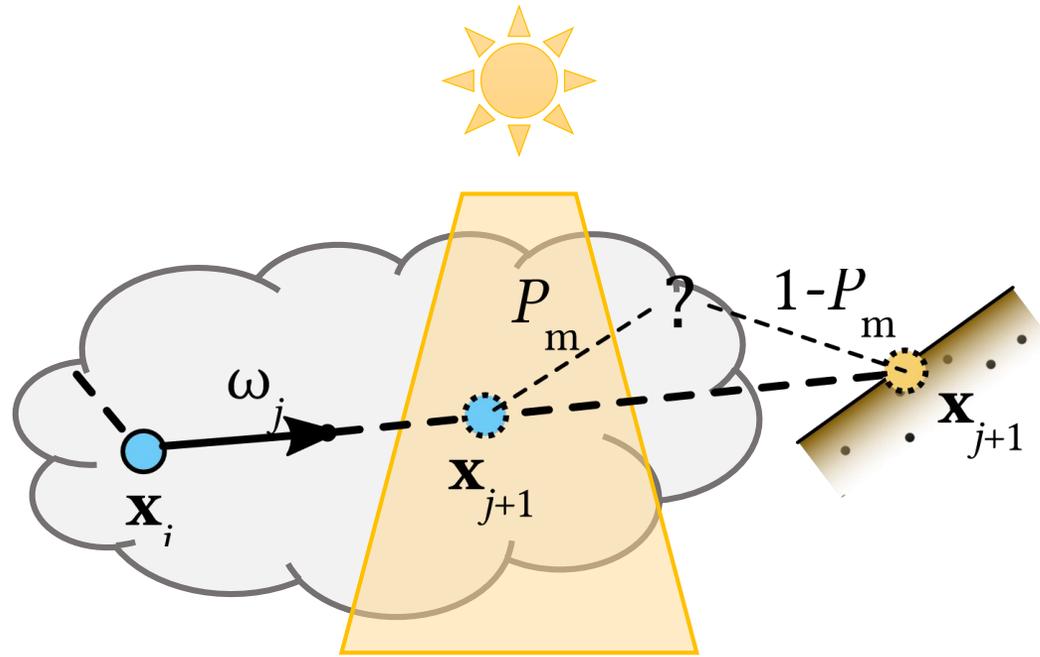




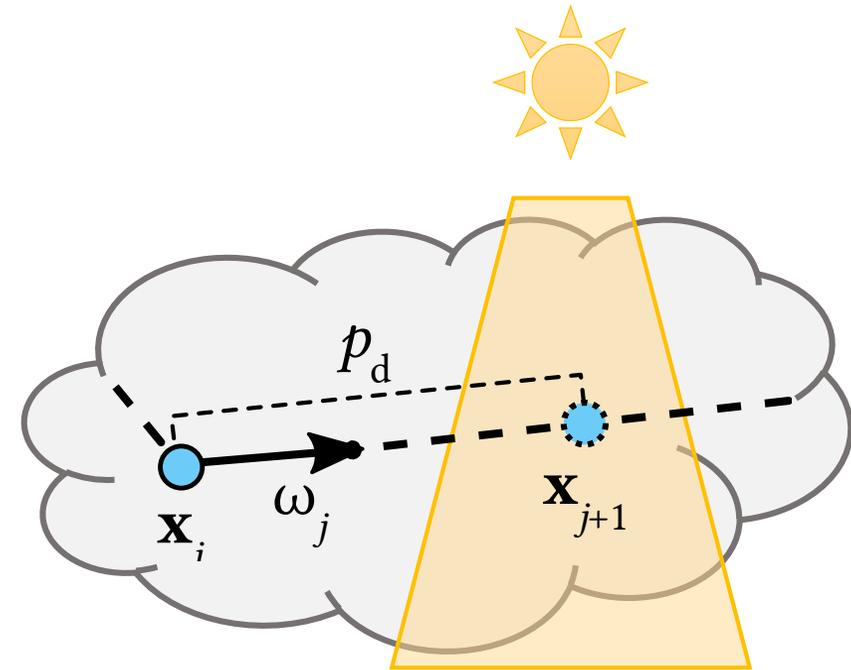
GUIDED SAMPLING DECISIONS



DISTANCE SAMPLING



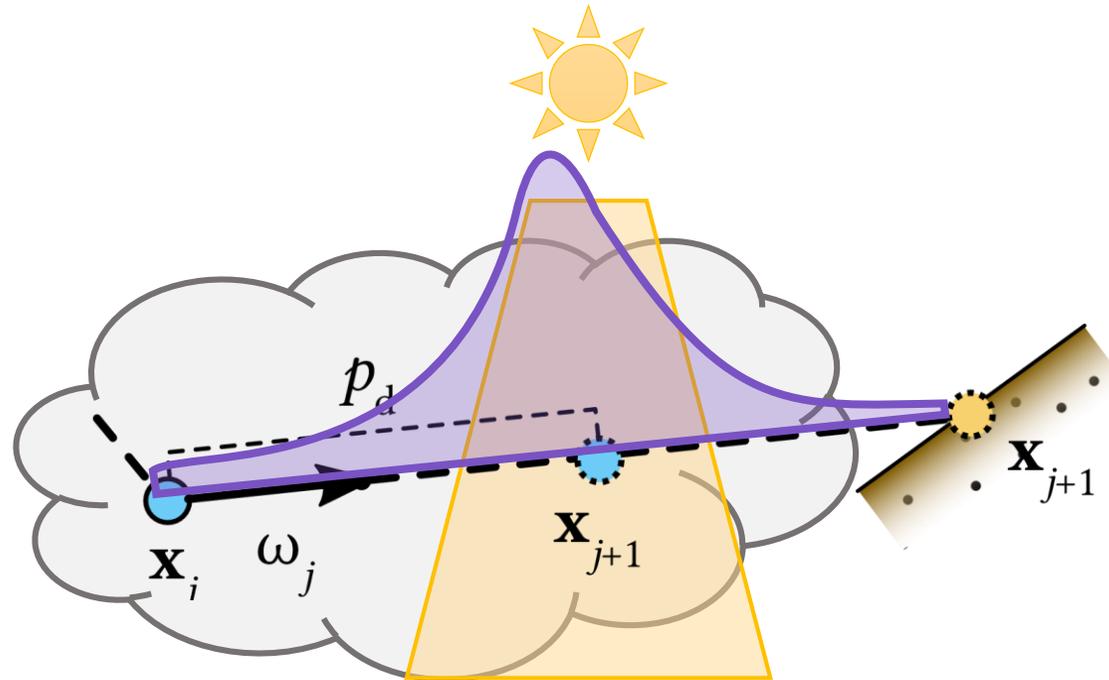
1. Volume or surface decision



2. Scatter distance



GUIDED PRODUCT DISTANCE SAMPLING



1+2. Event distance

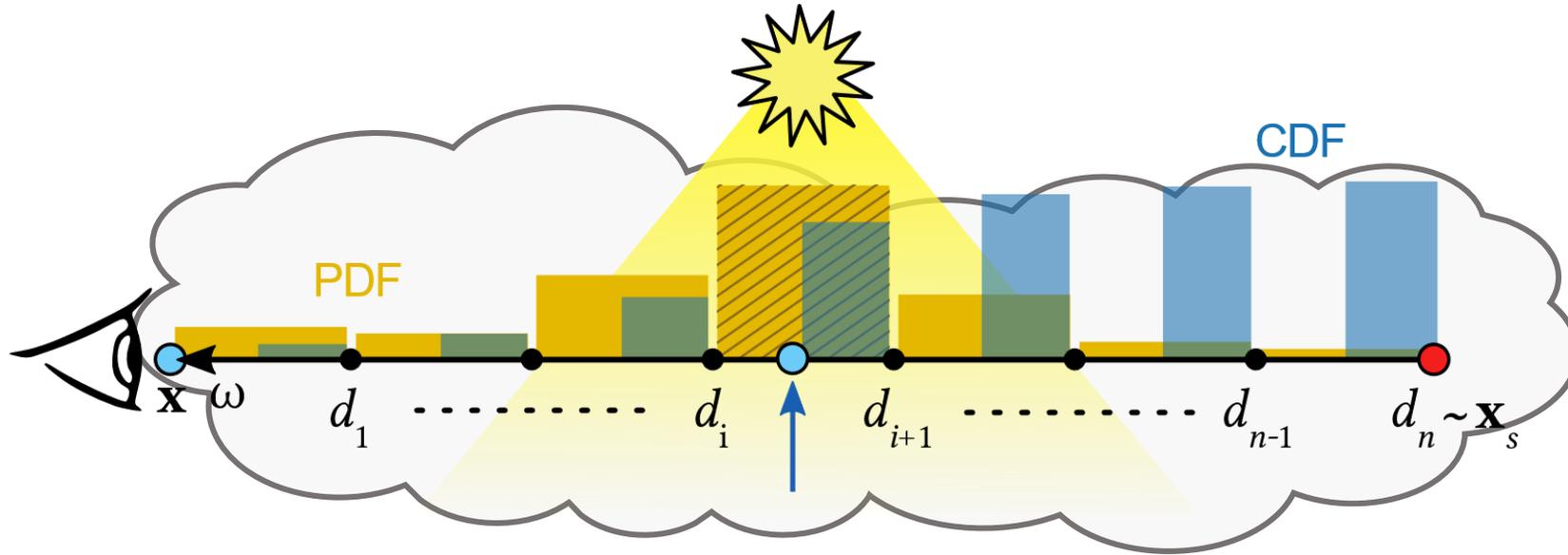
- Our guided PDF:

$$\tilde{p}_d^{zv}(\dots) = \frac{T(\dots) \cdot \sigma_s(\dots) \cdot \tilde{L}_i(\dots)}{\tilde{L}(\dots)}$$

Our estimates

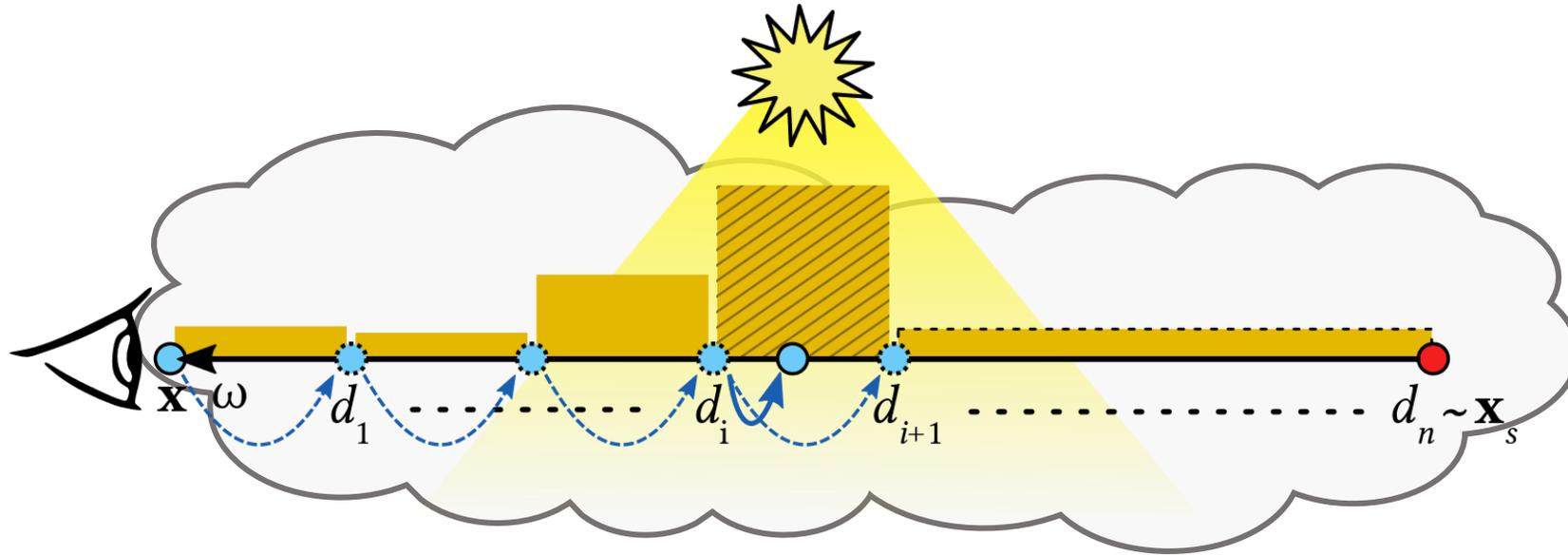


NAÏVE TABULATED APPROACH



- Naïve tabulated approach:
 - Step through the **complete** volume and build a tabulated PDF
- Inefficient (large scenes dense media):
 - we always need to evaluate all bins first

OUR INCREMENTAL GUIDED PRODUCT DISTANCE SAMPLING



- Incremental approach:
 - At each step make a local decision, if we scatter inside the bin
- **We only need to step until the scattering event happens**

Full CDF (30min)



Spp: 548

49 Avg. steps: 18

Our incremental (30min)



Spp: 1140

Avg. steps: 4

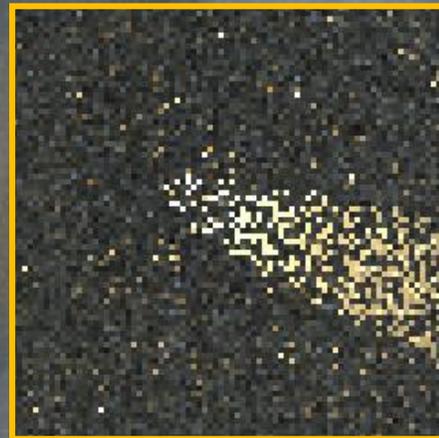
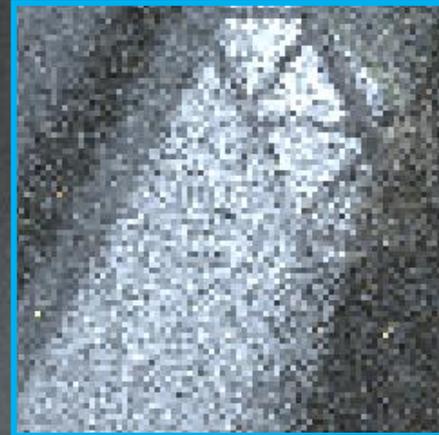


45 min



45 min

No guiding



Spp: 960
relMSE: 1.342

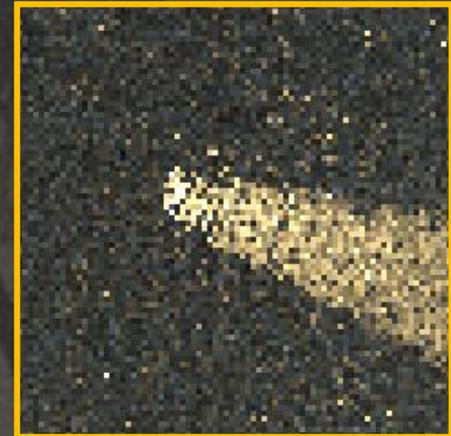


45 min



No guiding

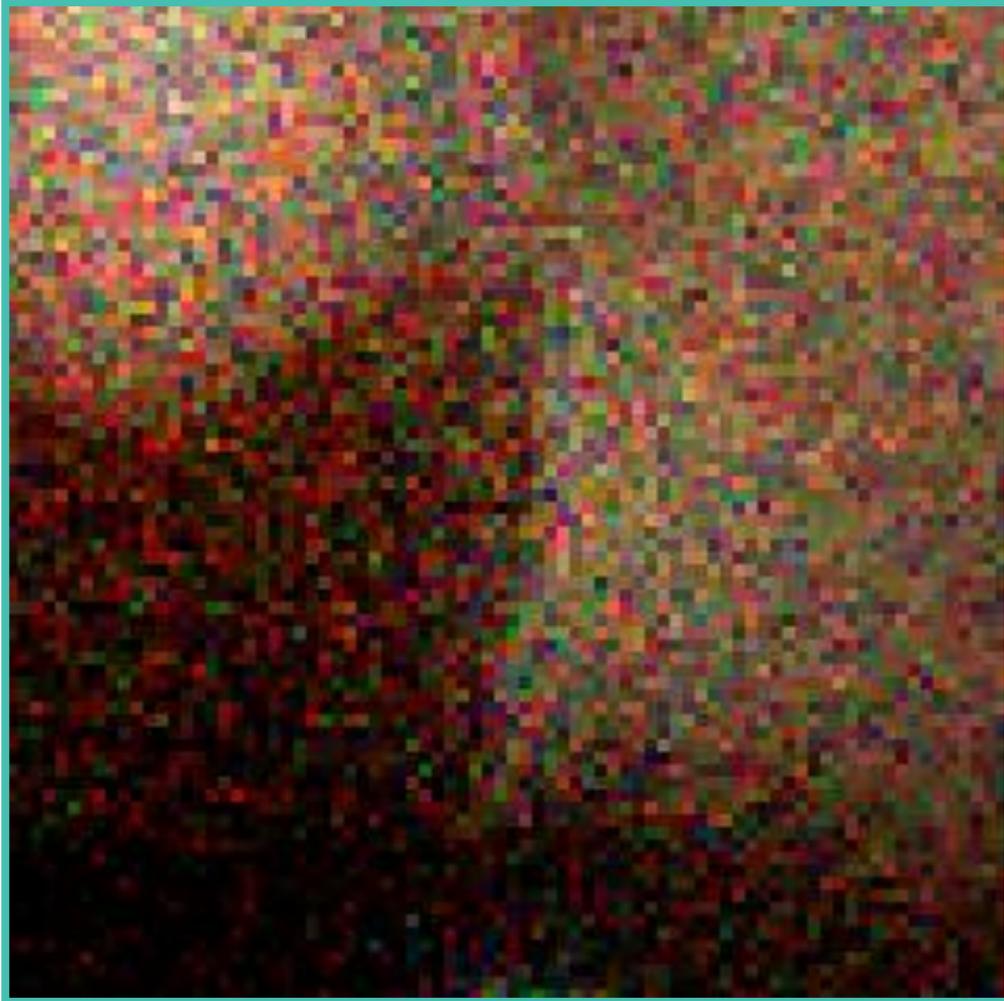
Distance guiding



Spp: 960
relMSE: 1.342

Spp: 424
relMSE: 0.901

No guiding (256 spp)

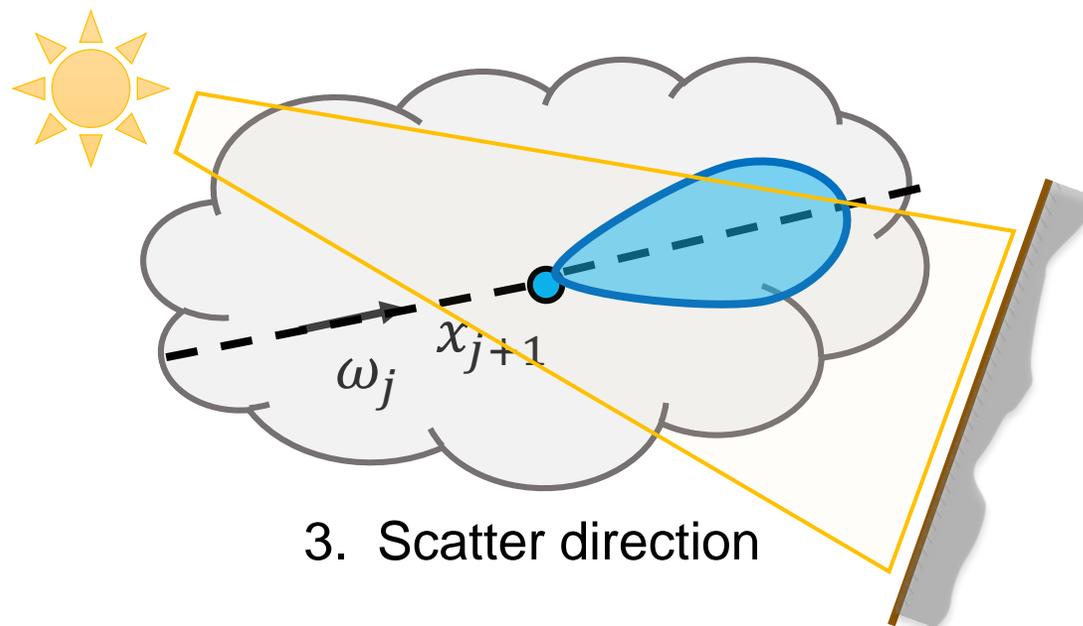


Distance guiding (256 spp)



- Here, distance sampling is not the main cause of variance!!

DIRECTIONAL SAMPLING

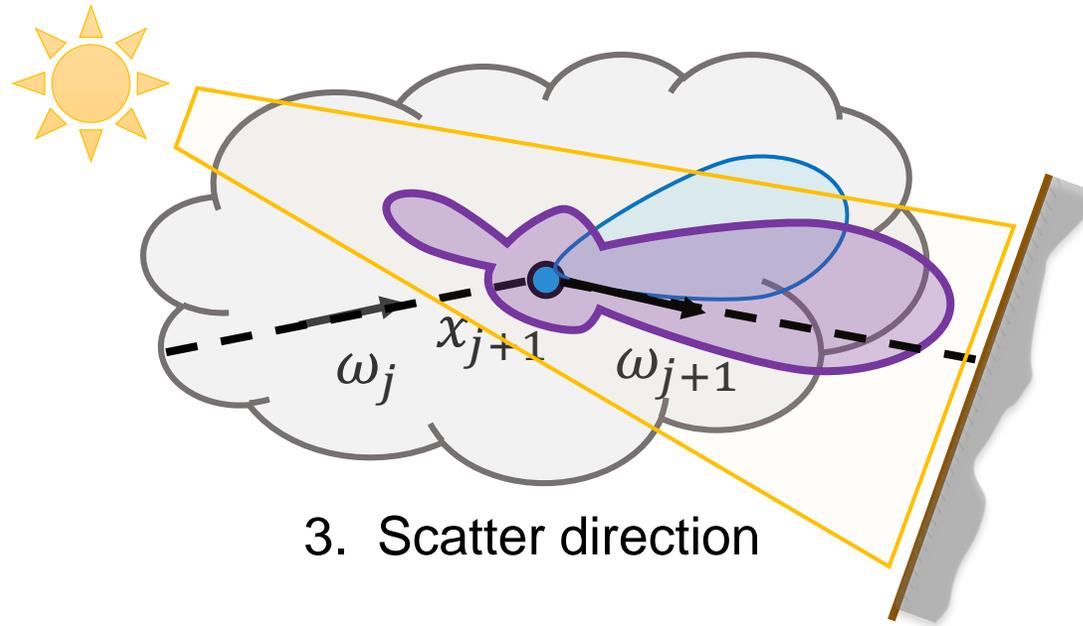


- Standard PDF:

$$p_{\omega}^{std}(\dots) \propto \tilde{f}(\dots)$$



GUIDED PRODUCT DIRECTIONAL SAMPLING

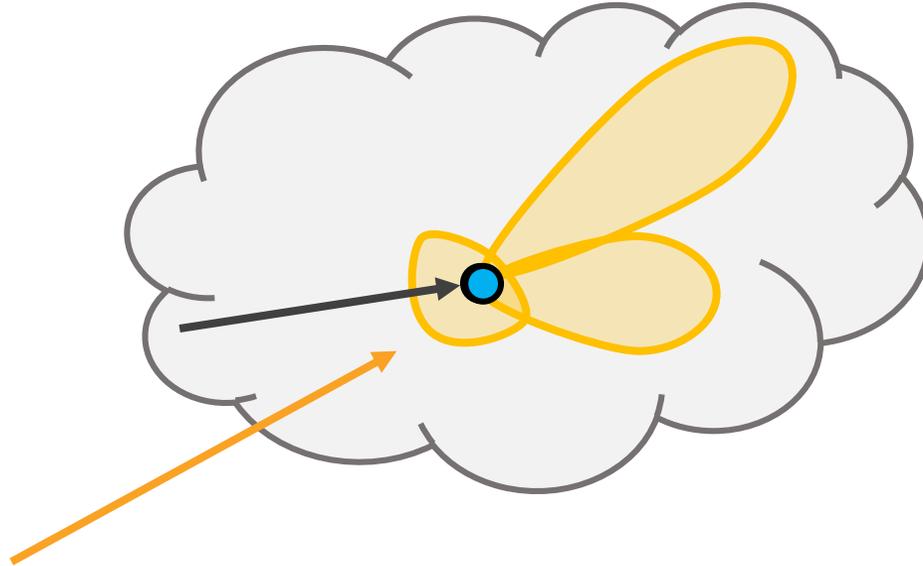


- Our guided PDF:

Our estimates

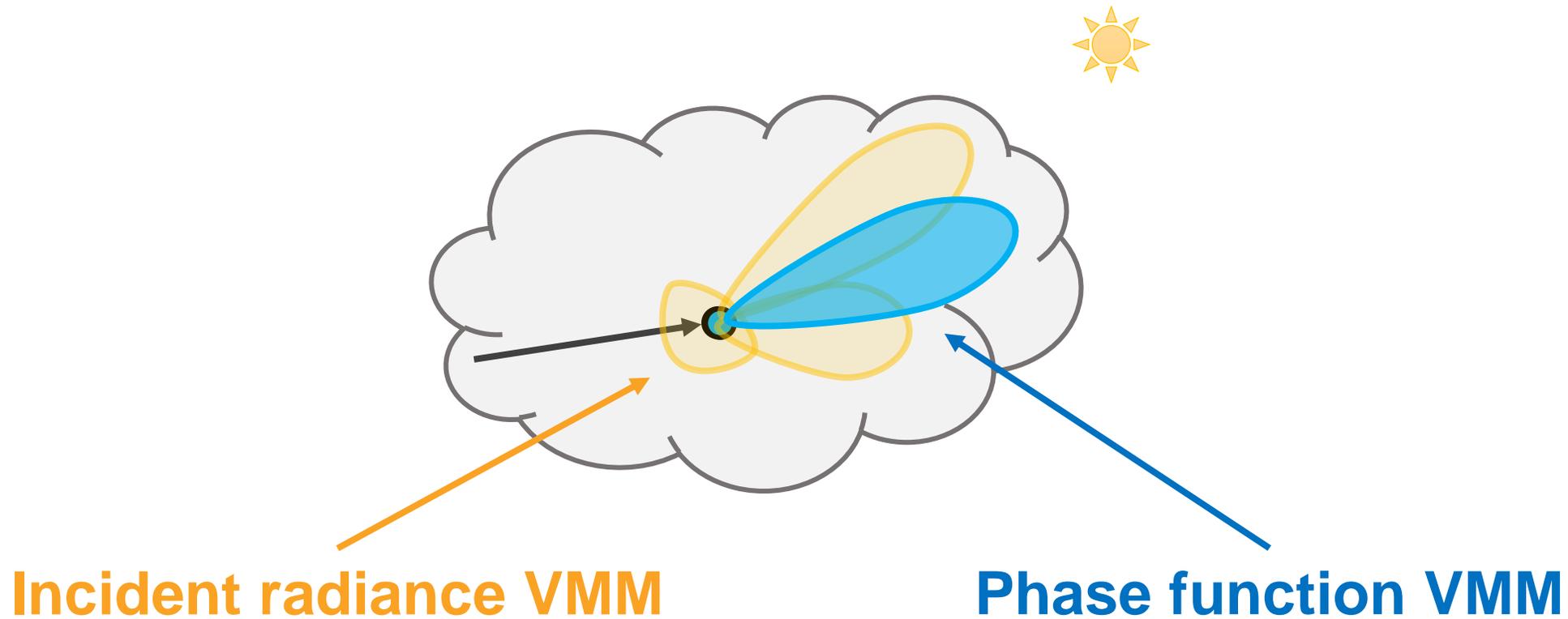
$$\tilde{p}_{\omega}^{zv}(\dots) \propto \tilde{f}(\dots) \cdot \tilde{L}(\dots)$$

OUR GUIDED PRODUCT DIRECTIONAL SAMPLING

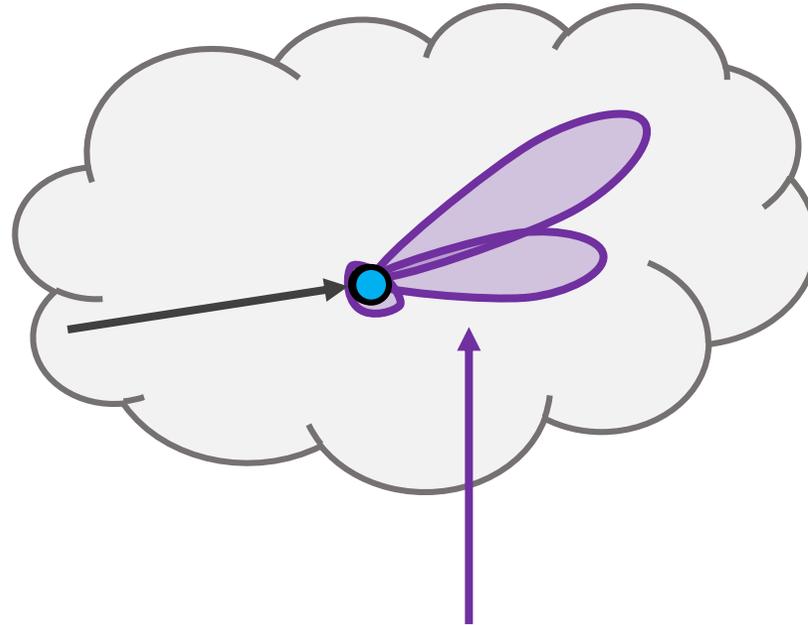


Incident radiance VMM

OUR GUIDED PRODUCT DIRECTIONAL SAMPLING

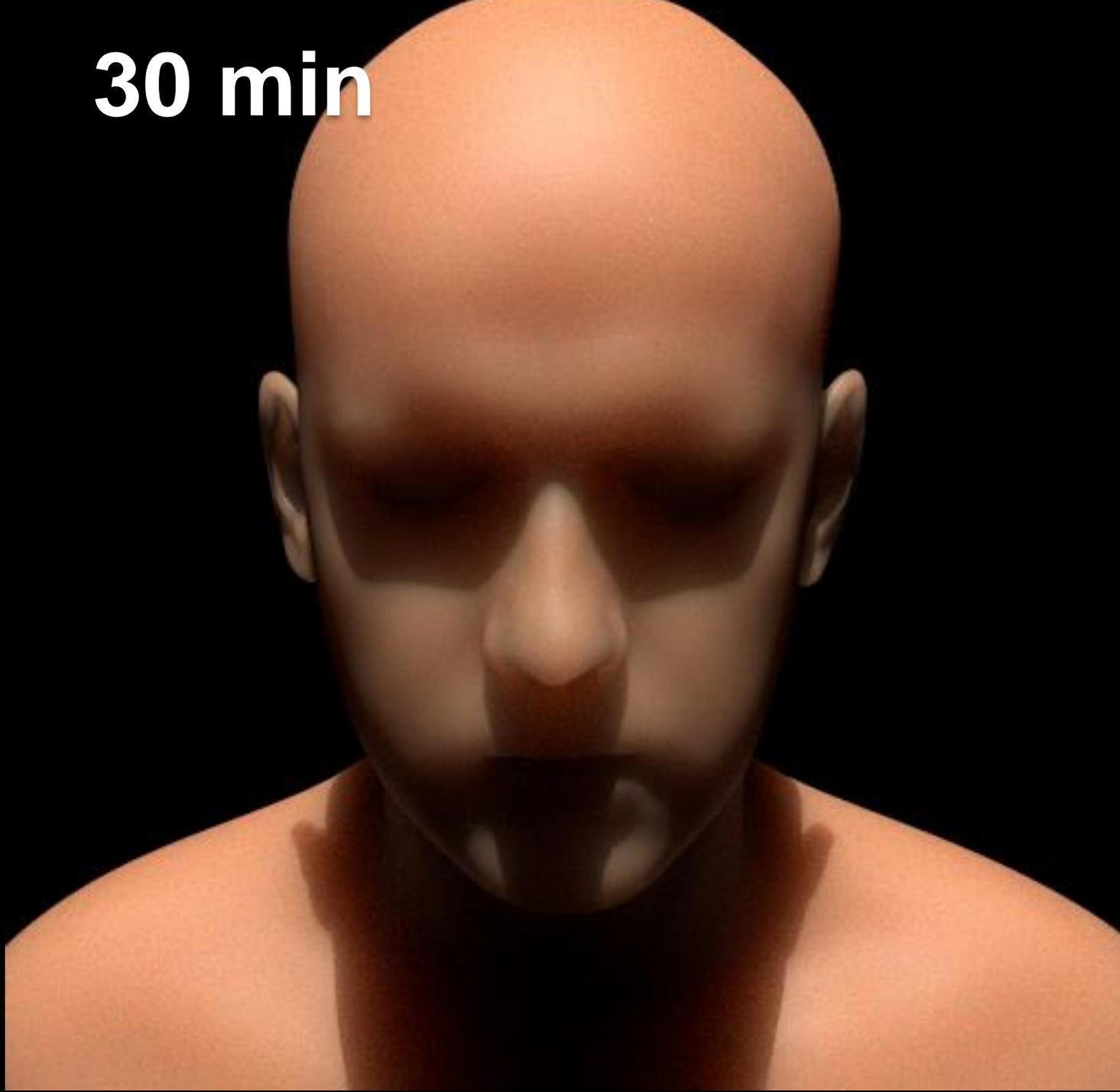


OUR GUIDED PRODUCT DIRECTIONAL SAMPLING



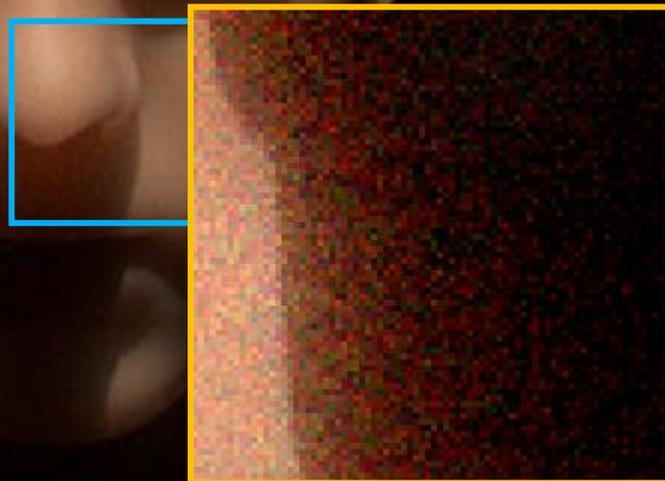
Product sampling VMM

30 min



30 min

No guiding

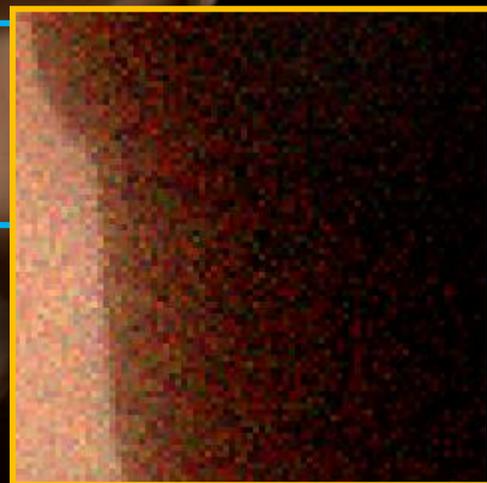


Spp: 2212
reIMSE: 0.376

30 min

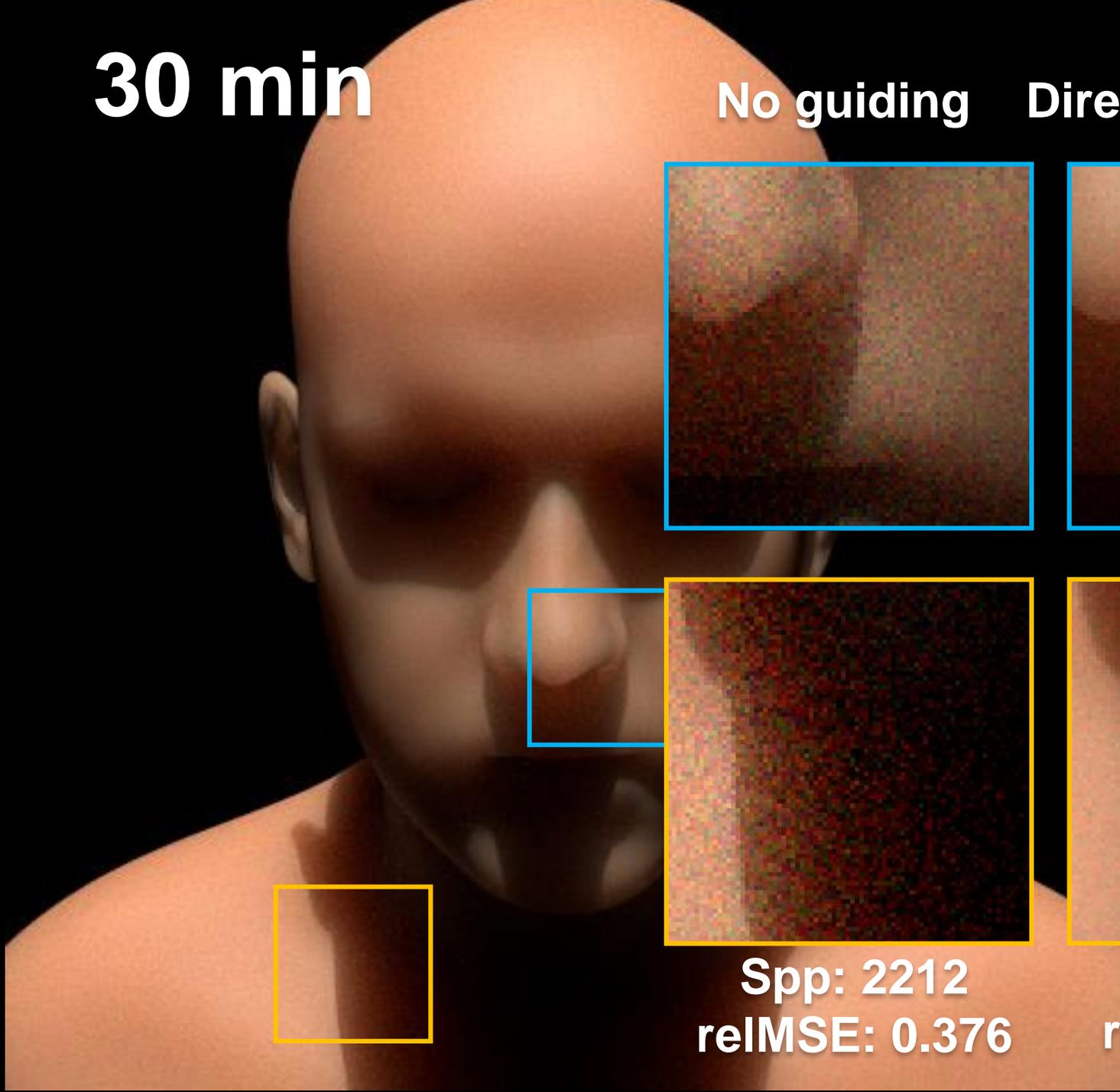
No guiding

Directional guiding



Spp: 2212
relMSE: 0.376

Spp: 1756
relMSE: 0.048



30 min

No guiding

Directional guiding

Dist + Direct



Spp: 2212
reIMSE: 0.376

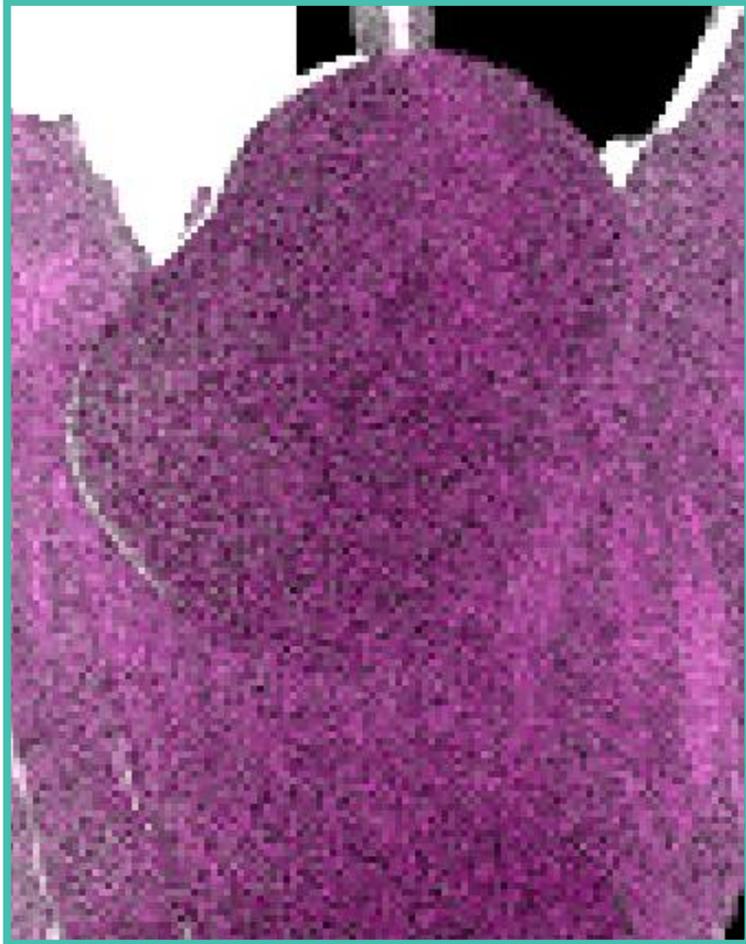
Spp: 1756
reIMSE: 0.048

Spp: 1228
reIMSE: 0.034



IMPORTANCE OF THE PRODUCT FOR DENSE ANISOTROPIC MEDIA

**No Guiding
(256 spp)**



**Product Guiding
(256 spp)**



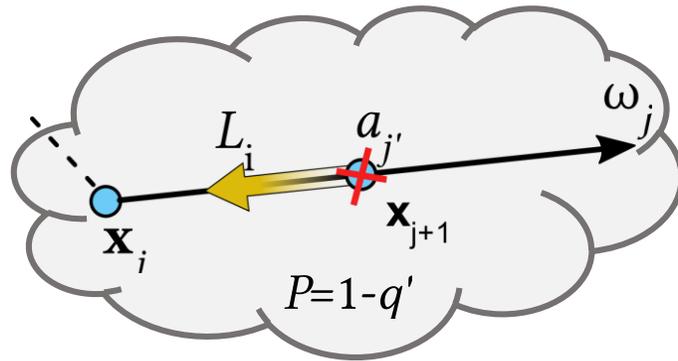
**Illum Guiding
(256 spp)**



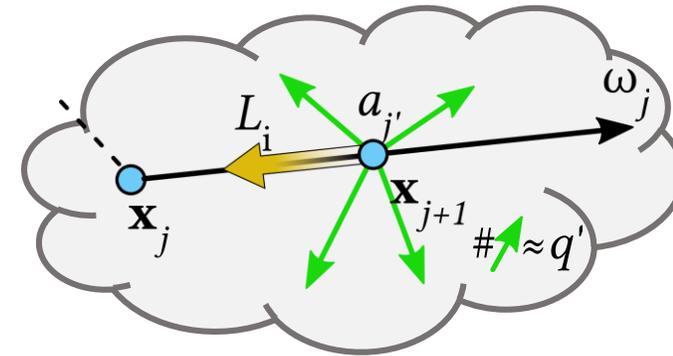


GUIDED RUSSIAN ROULETTE AND SPLITTING

Directional

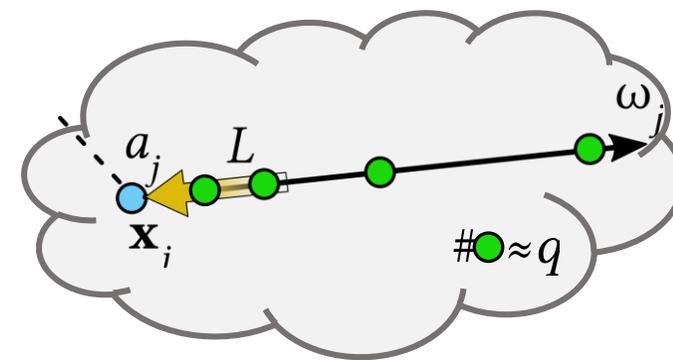
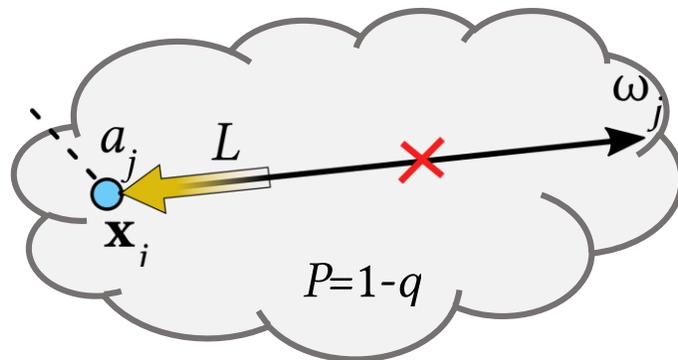


4a. Termination



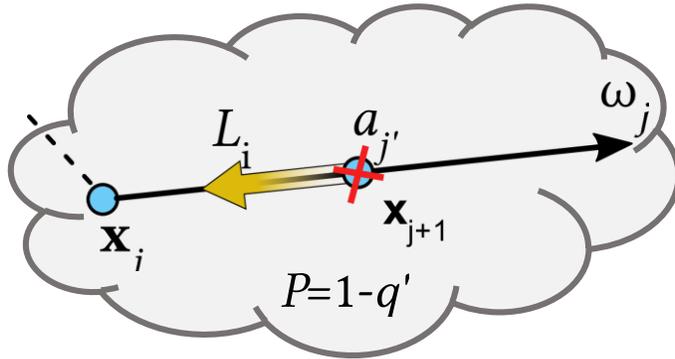
4b. Splitting

Distance

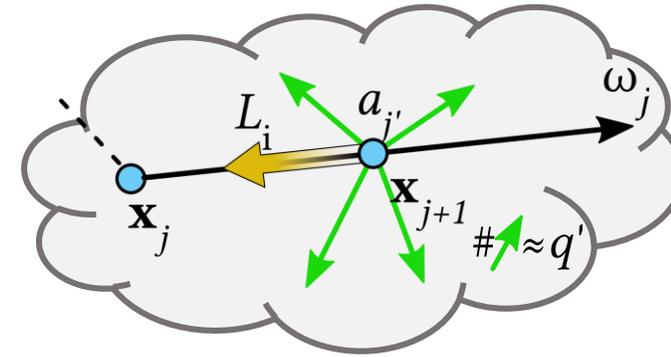




GUIDED RUSSIAN ROULETTE AND SPLITTING



4a. Termination



4b. Splitting

- Post-sampling compensation strategies:
 - Identify, if we **did** a **sub-optimal** sampling decision
 - Terminate: to increase performance
 - Split: bound/reduce sample variance

GUIDED RUSSIAN ROULETTE AND SPLITTING



survival prob /
splitting factor → $q = \frac{E[X]}{I}$

Path contribution

Reference solution

- Path contribution: $E[X]$
 - The expected contribution if we continue the path

- Reference solution: I
 - the final pixel value

GUIDED RUSSIAN ROULETTE AND SPLITTING



survival prob /
splitting factor → $q = \frac{E[X]}{I} = 1$ **Zero-Variance
Estimator**

Path contribution

Reference solution

- Path contribution: $E[X]$
 - The expected contribution if we continue the path

- Reference solution: I
 - the final pixel value

GUIDED RUSSIAN ROULETTE AND SPLITTING



survival prob /
splitting factor \rightarrow $q = \frac{E[X]}{I}$

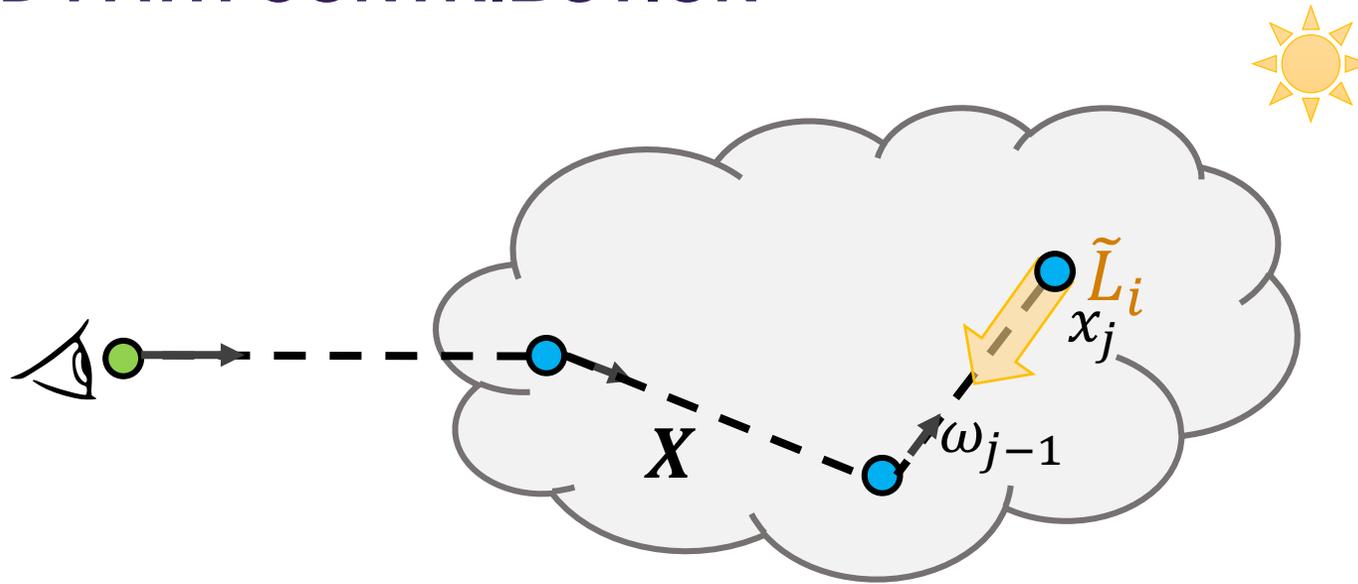
Path contribution \swarrow

Reference solution \nwarrow

- If $q \leq 1$: Russian Roulette
 - Terminates low contributing paths
 - Survival probability: q
- If $q > 1$: Splitting
 - Splits an under sampled paths with a potential high contribution (q times)



ESTIMATED PATH CONTRIBUTION



Path throughput: $f(\mathbf{X})/p(\mathbf{X})$

In-scattered radiance estimate

$$E[\mathbf{X}] = a'(\mathbf{X}) \cdot \tilde{L}_i(x_j, \omega_{j-1})$$

- See course notes or paper for more details

GUIDED RUSSIAN ROULETTE AND SPLITTING: PIXEL ESTIMATE



- Ray marched cache to integrate: $T \cdot \sigma_s \cdot \tilde{L}_i$

45 min

No RR



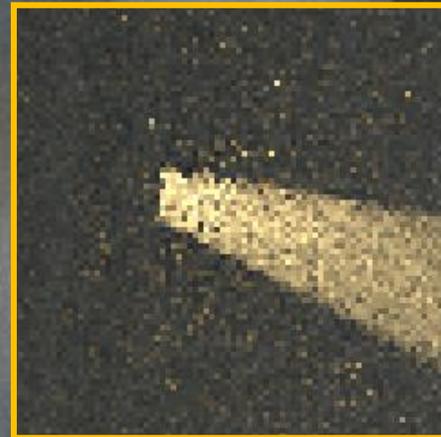
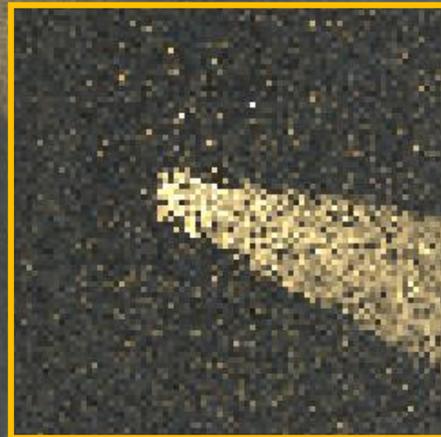
Spp: 468
reIMSE: 0.454



45 min

No RR

Guided RR



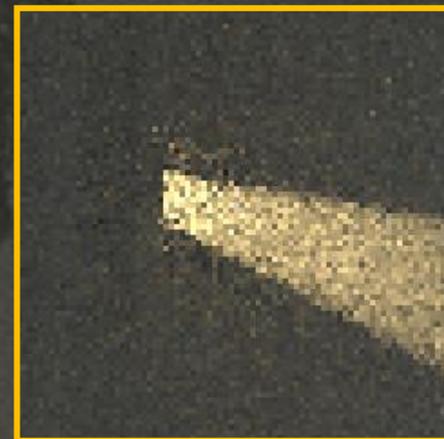
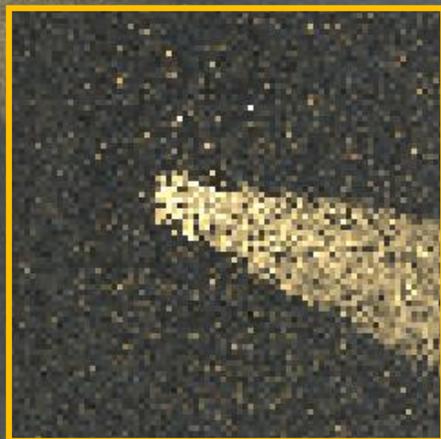
Spp: 468
relMSE: 0.454

Spp: 1500
relMSE: 0.174

45 min

No RR

Guided RR + Guided splitting



Spp: 468
relMSE: 0.454

Spp: 1500
relMSE: 0.174

Spp: 1340
relMSE: 0.066

Guided RR



Spp: 1500
reIMSE: 0.174

+ Guided splitting



Spp: 1340
reIMSE: 0.066

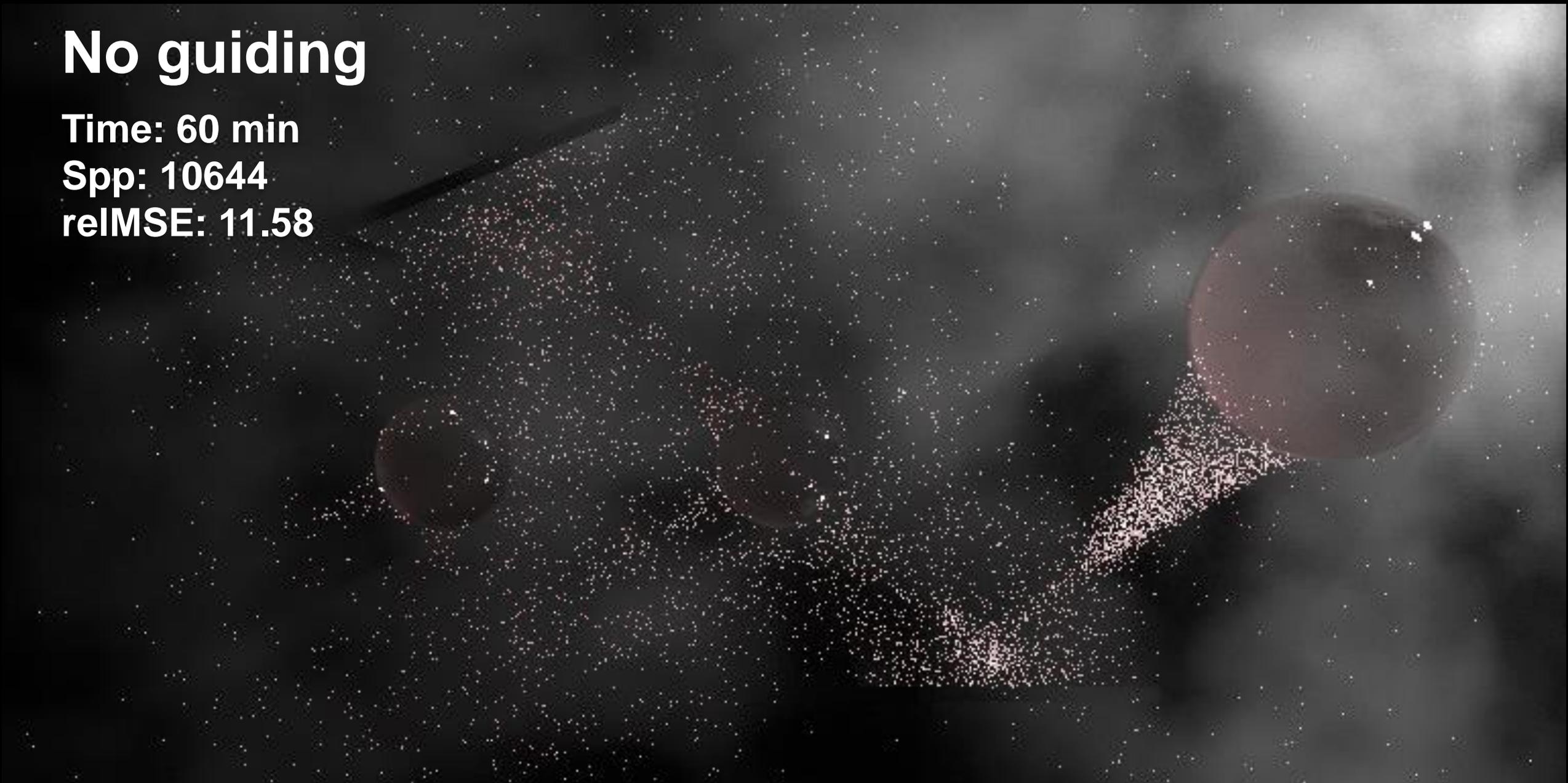


No guiding

Time: 60 min

Spp: 10644

relMSE: 11.58



Distance guiding

Time: 60 min

Spp: 4624

relMSE: 3.520



Distance + directional guiding

Time: 60 min

Spp: 4448

relMSE: 0.468



Distance + directional guiding + GRRS

Time: 60 min

Spp: 3796

relMSE: 0.321





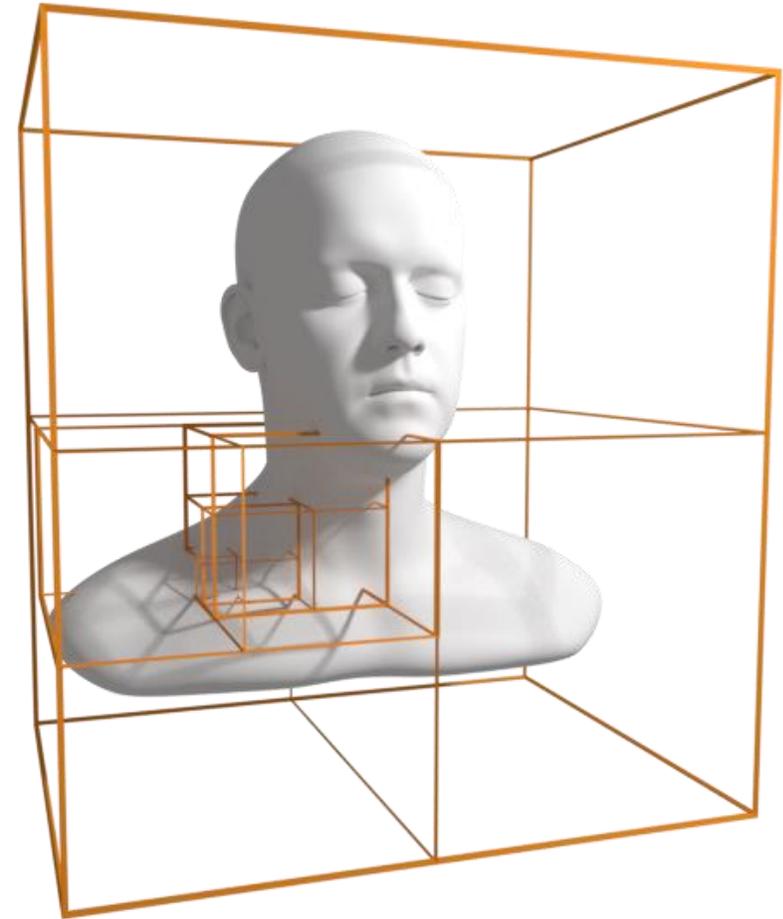
OPEN PROBLEMS AND LIMITATIONS

Open Challenges to make it bullet proof



SPATIAL CACHE STRUCTURE

- Naive approach to define the resolution:
 - Heuristic based on sample numbers
 - Takes time or many samples to model/separate fine features (e.g. thin shafts or caustics) (PPG by [Mueller2017] has the same problem)
 - Influences the performance of some sampling methods (e.g. distance)
- **Ideal structure should adjust to the light transport characteristics**



INSUFFICIENT CACHE SIZES



- Shared problem with other 3D caching based guiding approaches (e.g. [Vorba2014], [Mueller2017], ...)
- By merging the samples of a spatial cache we blur the directional distribution
- Can lead to incorrect estimates of L and L_i



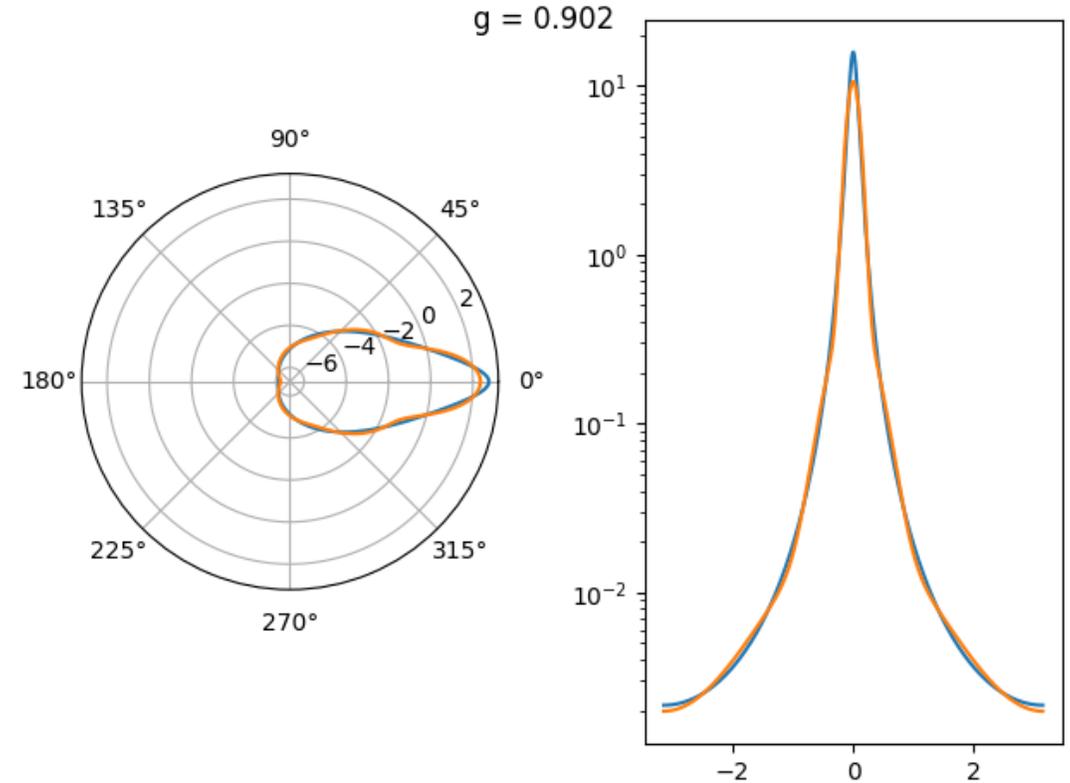
PRODUCTION CHALLENGES

How can we get good guiding estimates?



VMM PHASE FUNCTION FITTING: PRE-PROCESSING STEP

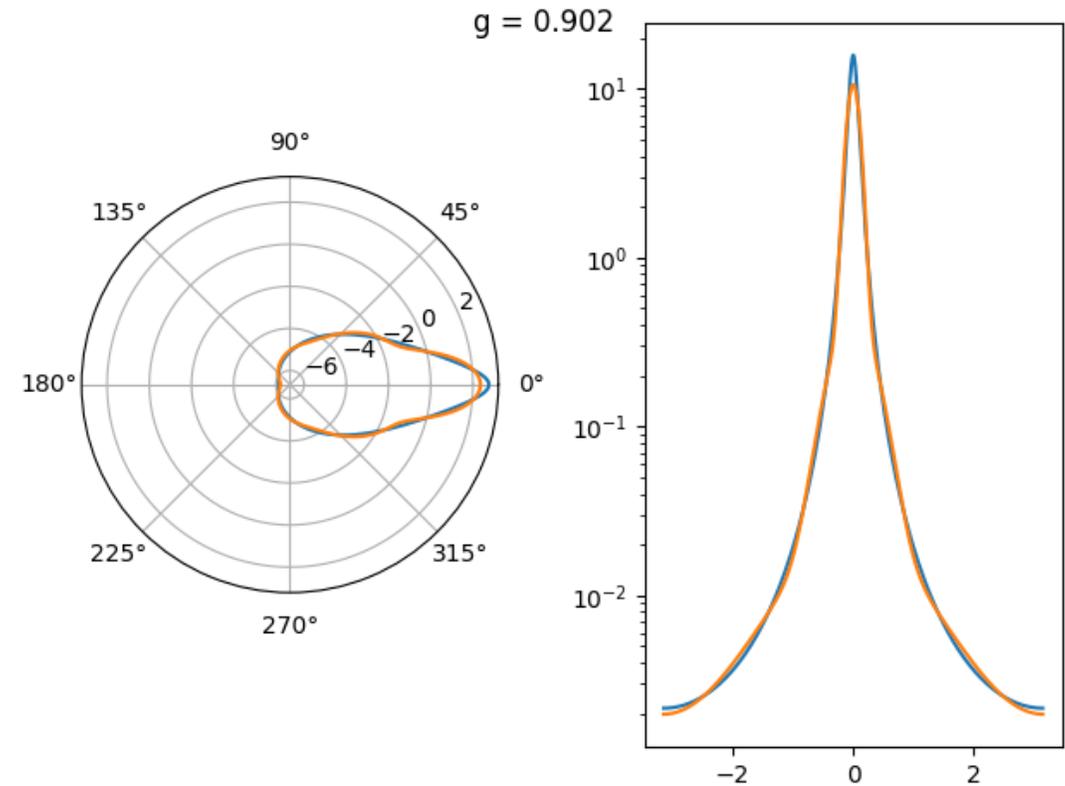
- Pre-processing step:
 - Fitting a VMM for each phase function
- Fitting up to $K = 4$ components
 - Details in the course notes
- **Open Challenge:**
 - Procedural phase functions or procedural mixtures?
 - How to deal with changing mean cosines (roughening/mollification)?





VMM PHASE FUNCTION FITTING: PRE-PROCESSING STEP

- Pre-processing step:
 - Fitting a VMM for each phase function
- Fitting up to $K = 4$ components
 - Details in the course notes
- **Open Challenge:**
 - **Procedural phase functions or procedural mixtures?**
 - **How to deal with changing mean cosines (roughening/mollification)?**



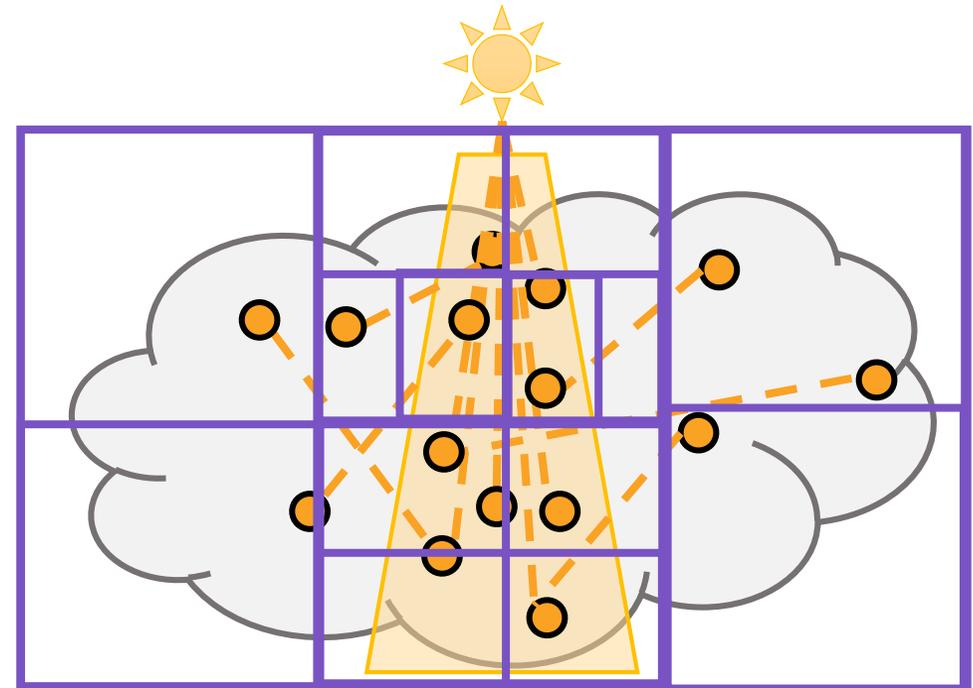


FITTING/LEARNING THE INCIDENT RADIANCE MIXTURES

Photon-based pre-processing ([Herholz2019][Vorba2014])

Pros:

- Photons directly represents the light transport
- Spatial distribution corresponds to important features (light shaft)
- Number of traced photons can be fixed
- No additional fitting during rendering



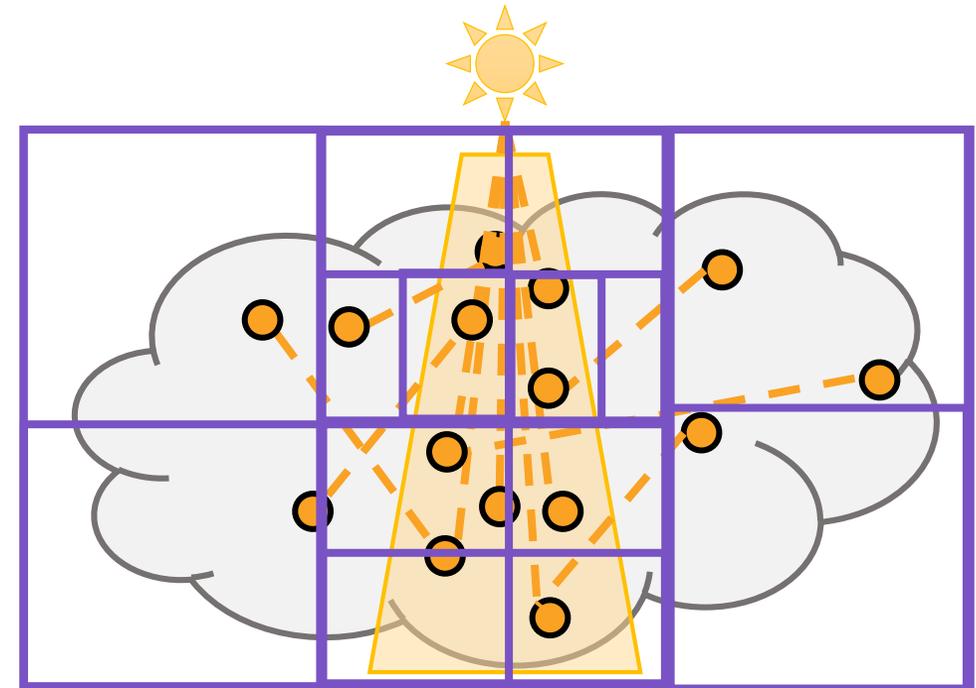
FITTING/LEARNING THE INCIDENT RADIANCE MIXTURES



Photon-based pre-processing ([Herholz2019][Vorba2014])

Cons:

- Pre-processing step:
 - Long time to first render iteration
- Complex scenes need bidirectional learning:
 - Ping-Pong style [Vorba2014]
- It is not the ideal solution for artists in production



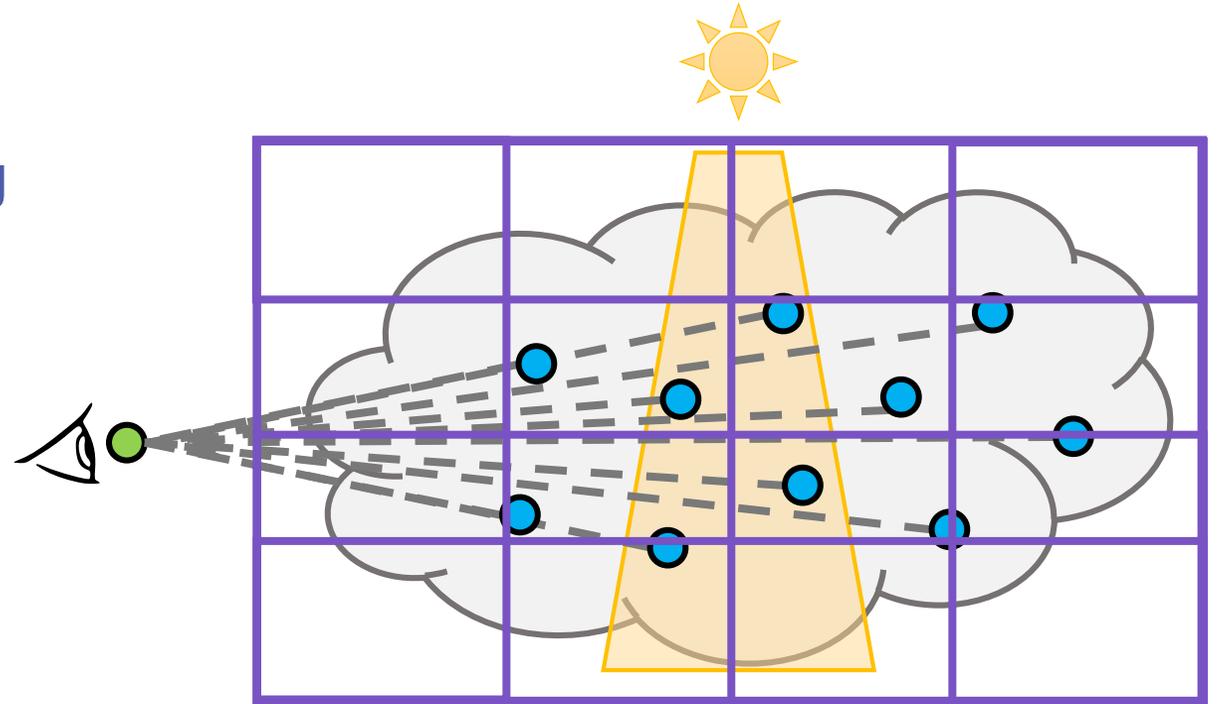


FITTING/LEARNING THE INCIDENT RADIANCE MIXTURES

Progressive learning (PPG-style [Mueller2017])

Pros:

- **First experiments show promising results**
- No pre-processing
- Refines spatial and directional distributions in each iteration
- Sample data gets more reliable



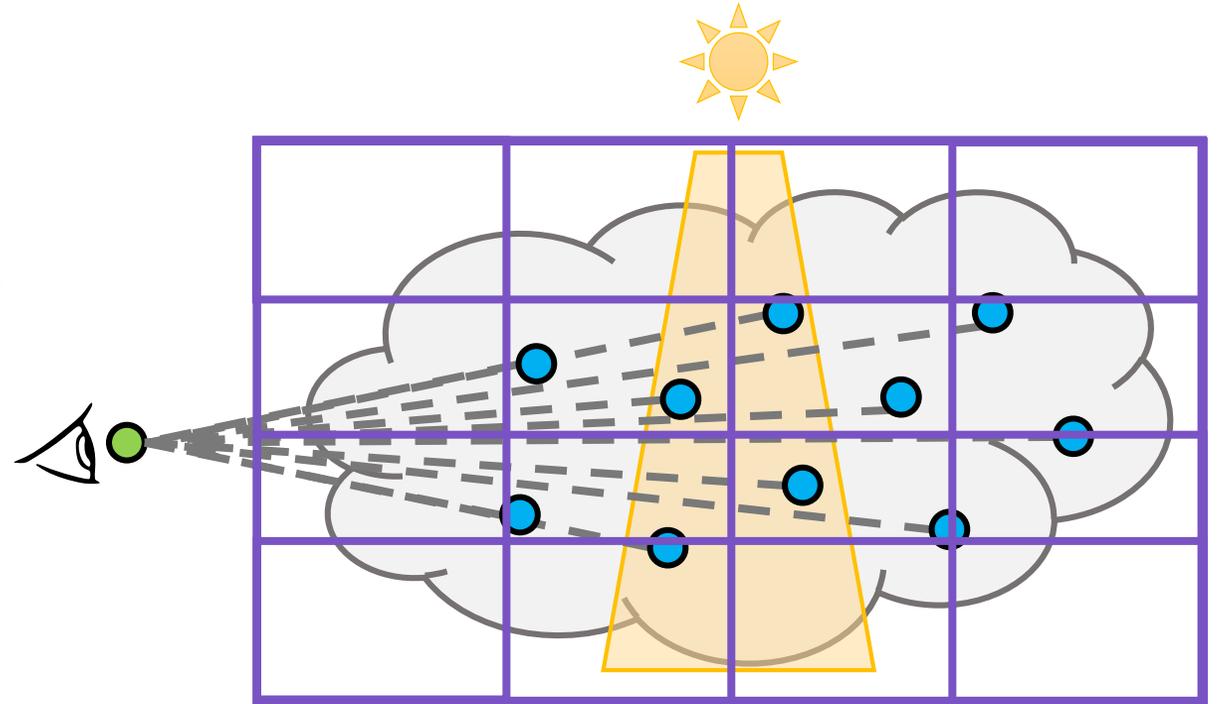


FITTING/LEARNING THE INCIDENT RADIANCE MIXTURES

Progressive learning (PPG-style [Mueller2017])

Cons (open challenges):

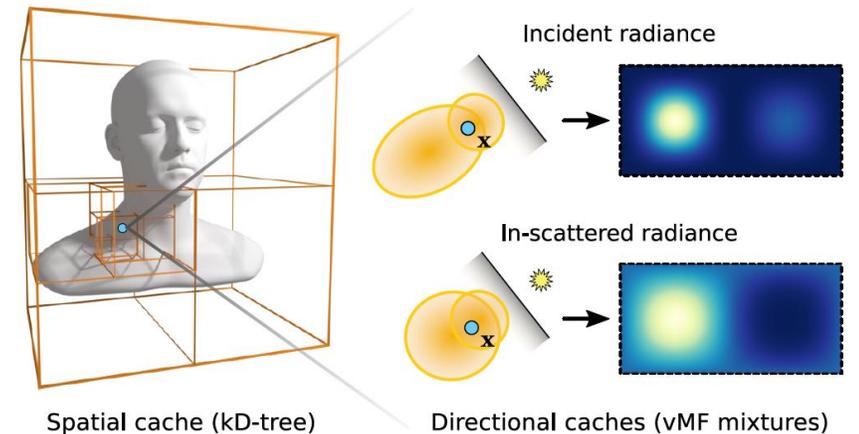
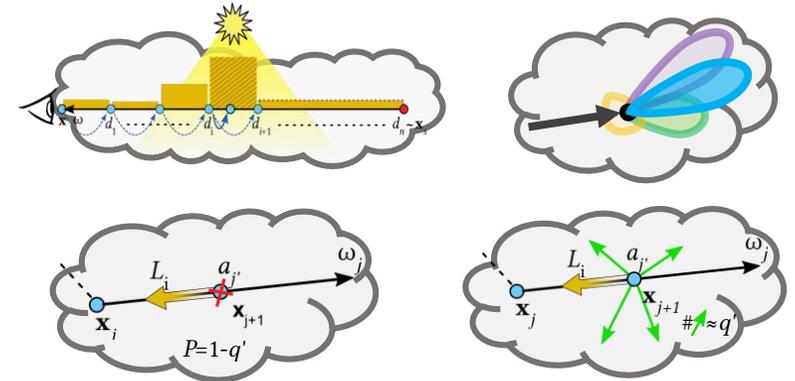
- Sample count grows exponential:
 - Memory and fitting time increases
 - **Shorter update rates ?**
 - **Online fitting for mixtures?**
- Spatial structure adapts slowly to LT:
 - Important for distance sampling





CONCLUSION

- Even approximate local sampling decisions lead to a good approximation of the globally optimal guiding distribution (and thus significantly reducing MC variance)
- Converges to optimal ZV estimator in the hypothetical limit (i.e., if the approximations were perfect)
- Solely based one guiding structure for all decisions (incident radiance VMMs)





THANK YOU



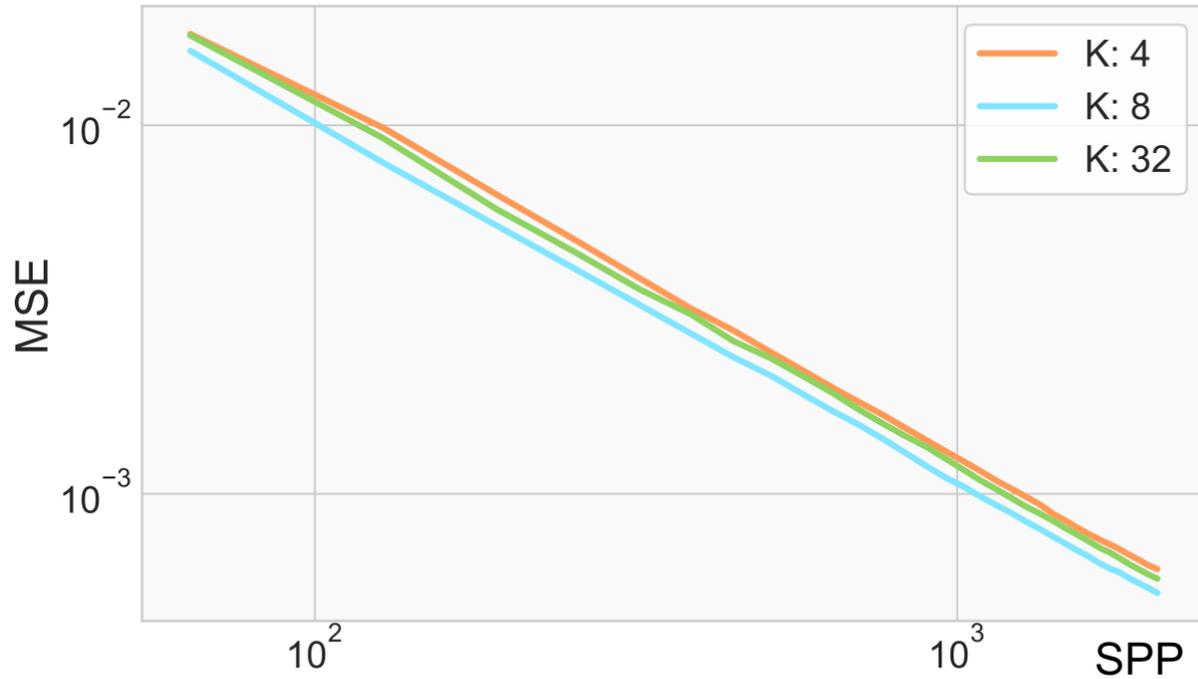
REFERENCES

- [Fong2017]: “Production volume rendering”
- [Novak2018]: “Monte Carlo methods for volumetric light transport simulation”
- [Kulla2012]: “Importance sampling techniques for path tracing in participating media”
- [Georgiev2012]: “importance sampling of low-order volumetric scattering”
- [Krivanek2014]: “A zero-variance-based sampling scheme for Monte Carlo subsurface scattering”
- [Meng2016]: “Improving the Dwivedi sampling scheme”
- [Vorba2014]: “Online learning of parametric mixture models for light transport simulation”
- [Vorba2016]: “Adjoint-driven Russian roulette and splitting in light transport simulation”
- [Herholz2016]: “Product importance sampling for light transport path guiding”
- [Koerner2016]: “Subdivision next-event estimation for path-traced subsurface scattering”
- [Mueller2017]: “Practical path guiding for efficient light-transport simulation”
- [Hoogenboom2008]: “Zero-varianceMonte Carlo schemes revisited”
- [Pegoraro2008]:” Sequential Monte Carlo integration in low-anisotropy participating media”
- [Bashford2012]: “A significance cache for accelerating global illumination”

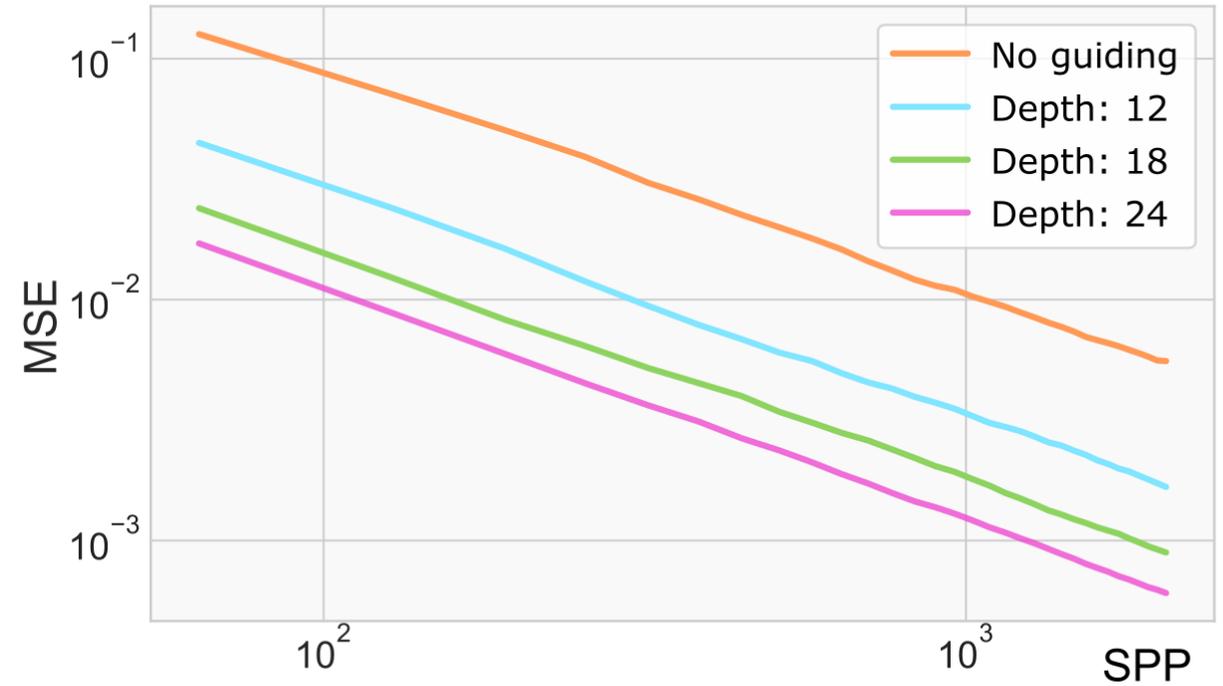


ADJOINT ESTIMATE ACCURACY

Directional resolution

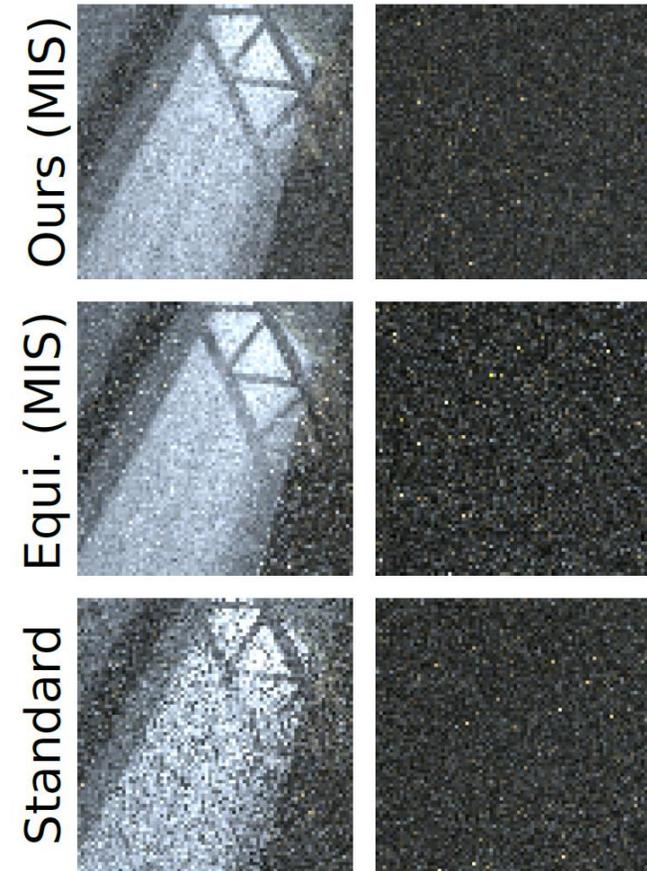
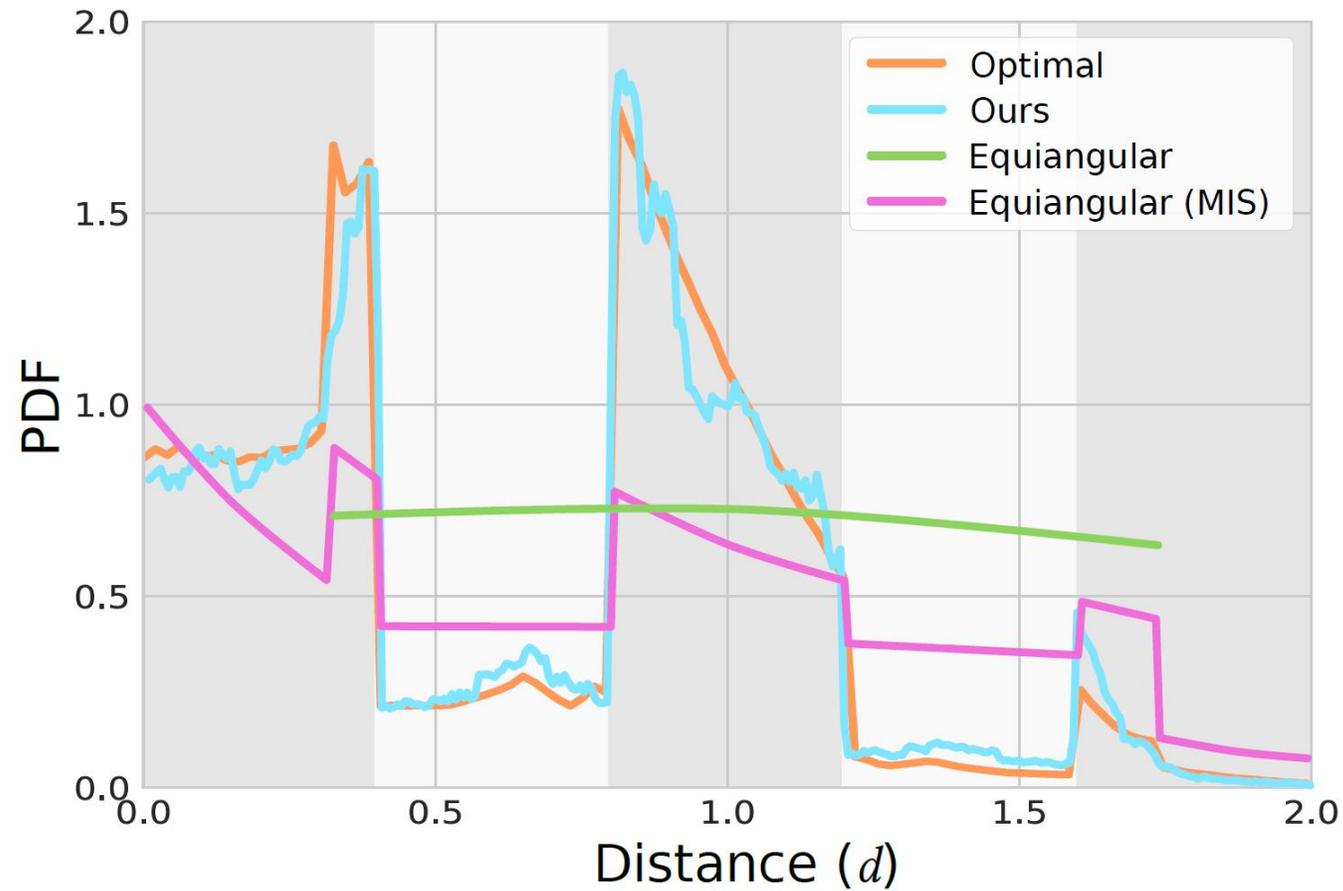


Spatial resolution

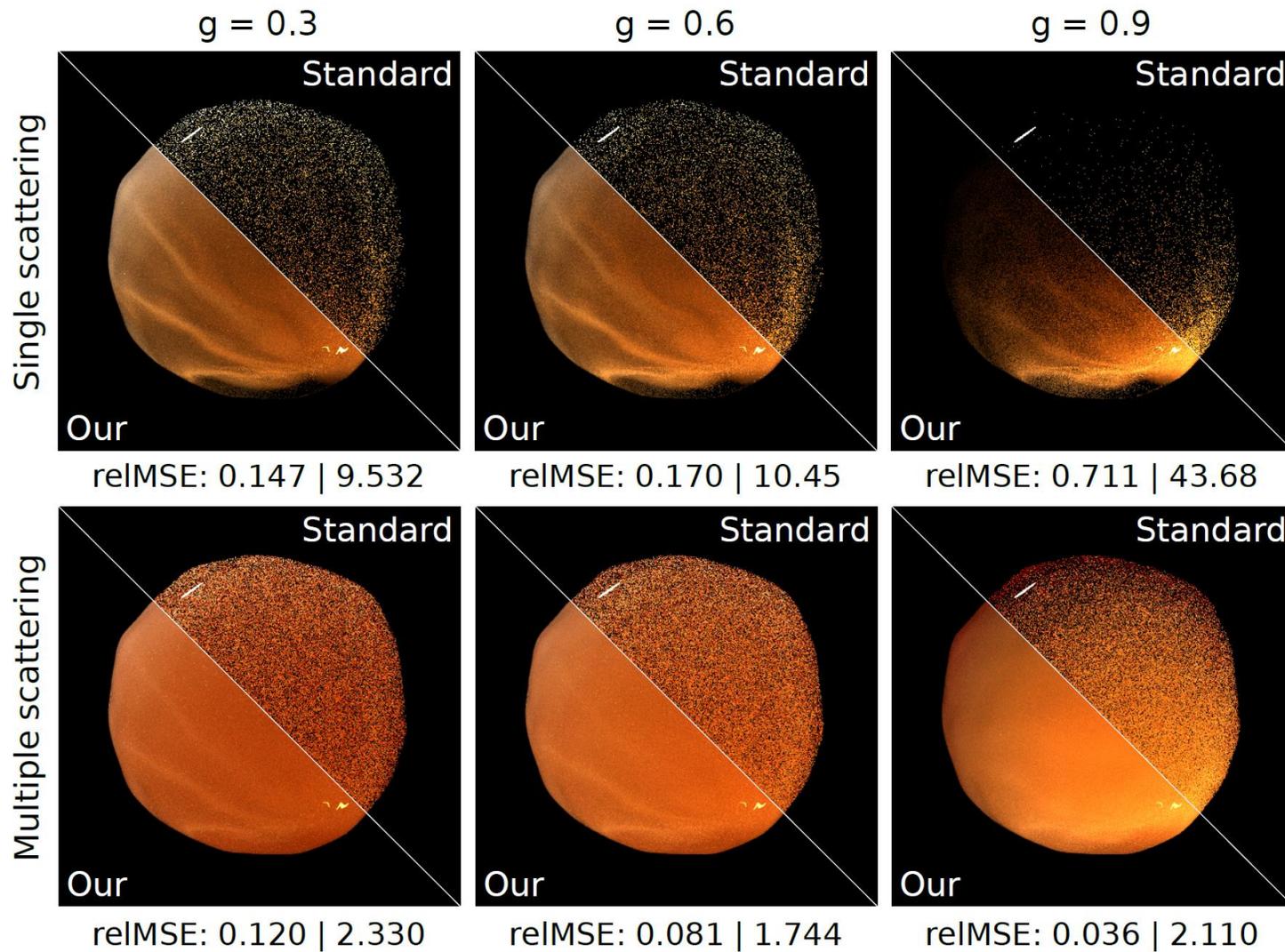




COMPARING AGAINST EQUIANGULAR SAMPLING

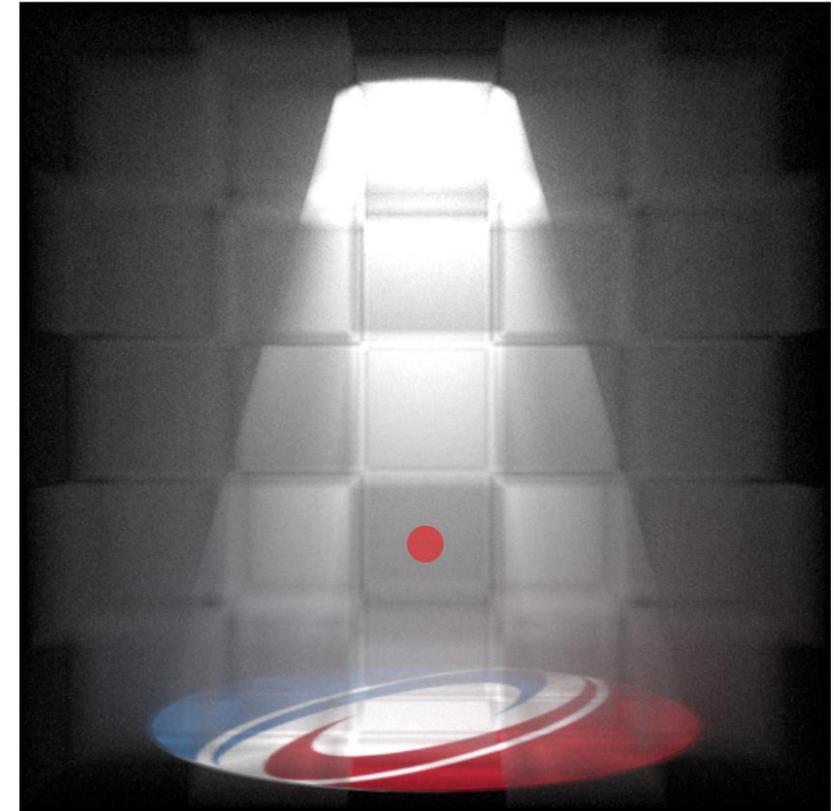
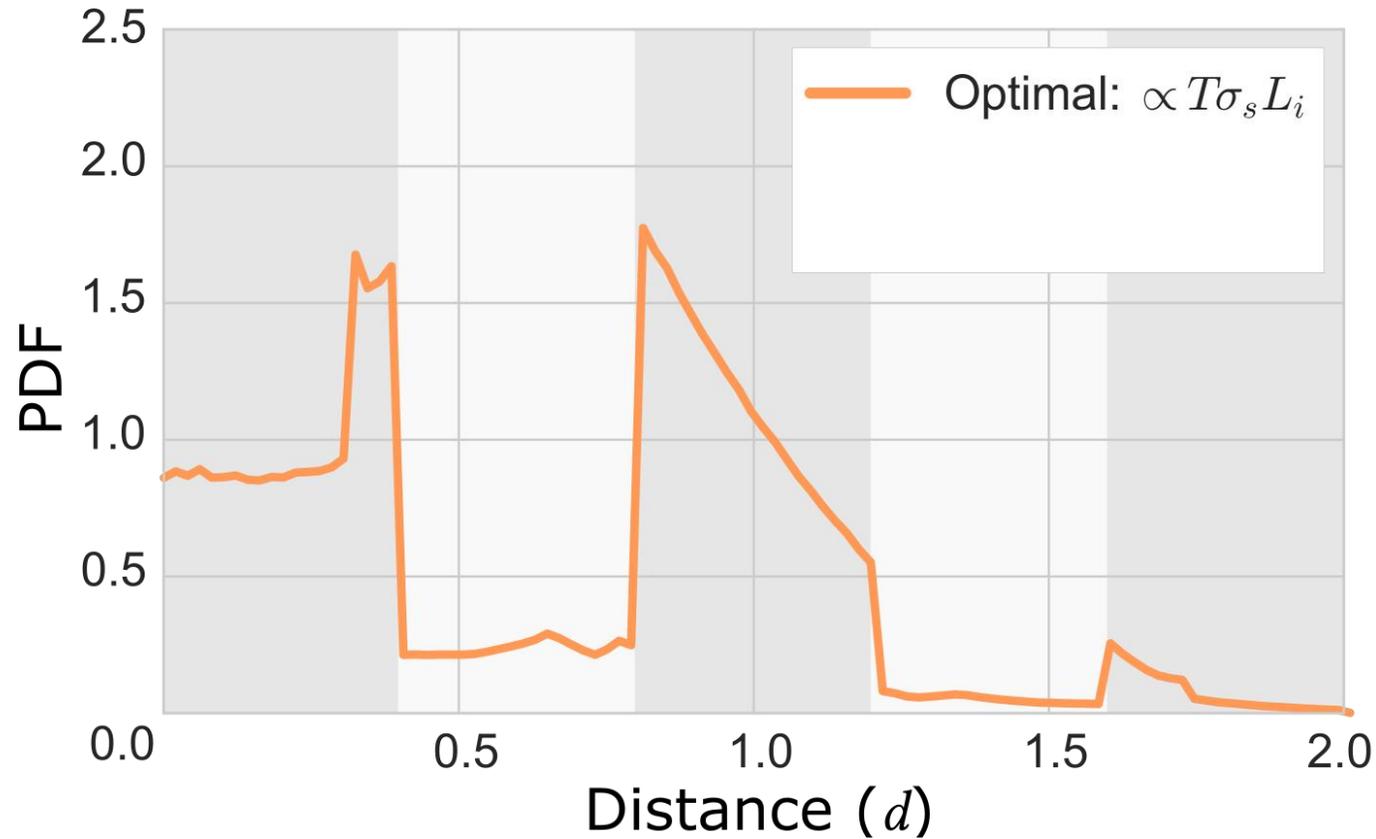


BUMPY SPHERE



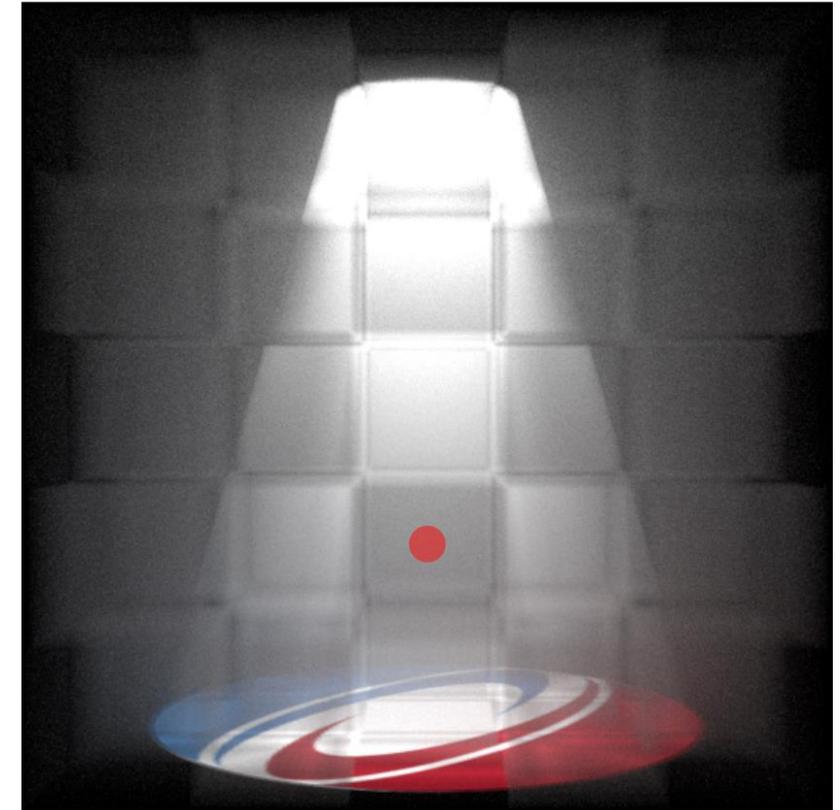
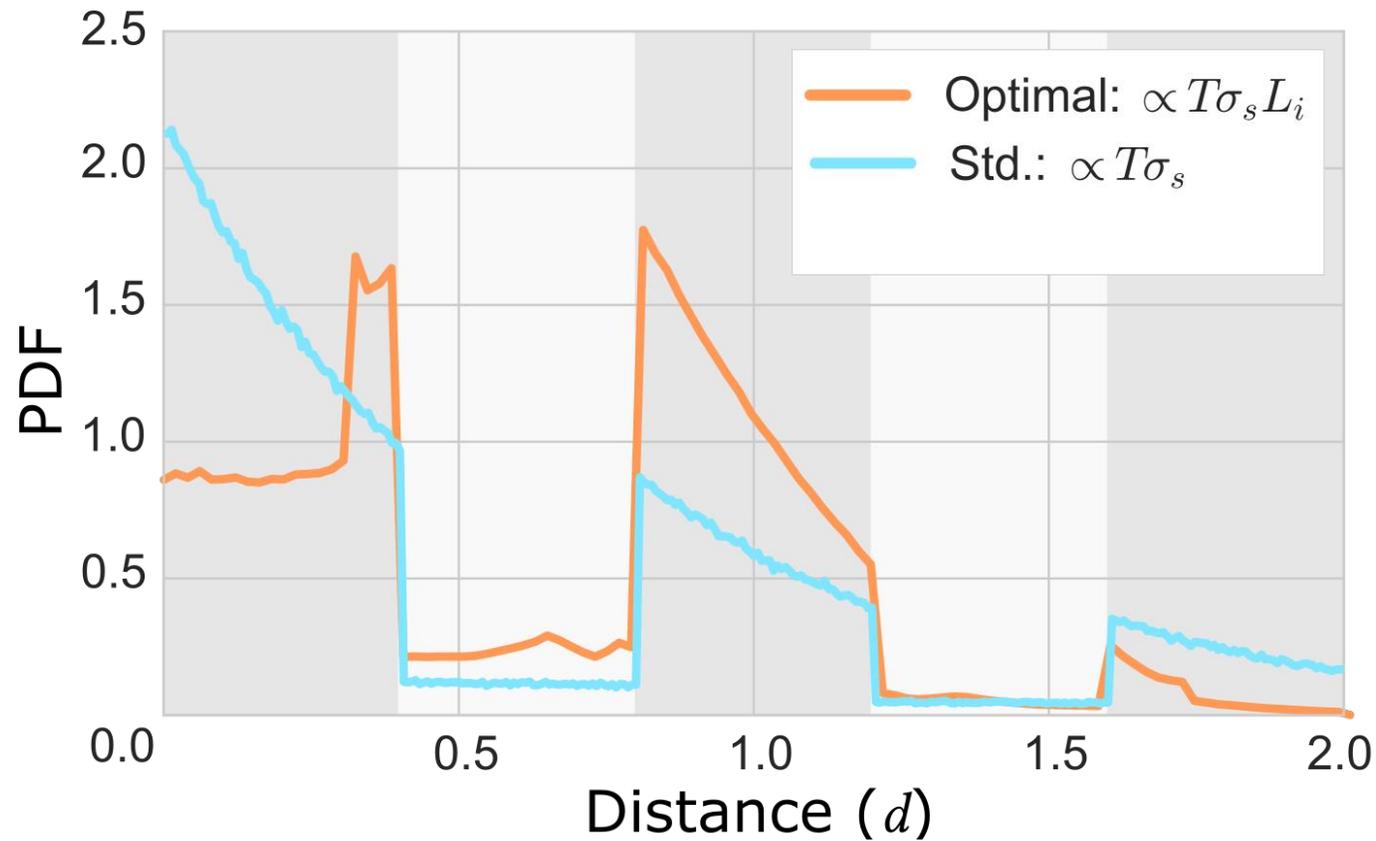


INCREMENTAL GUIDED DISTANCE SAMPLING



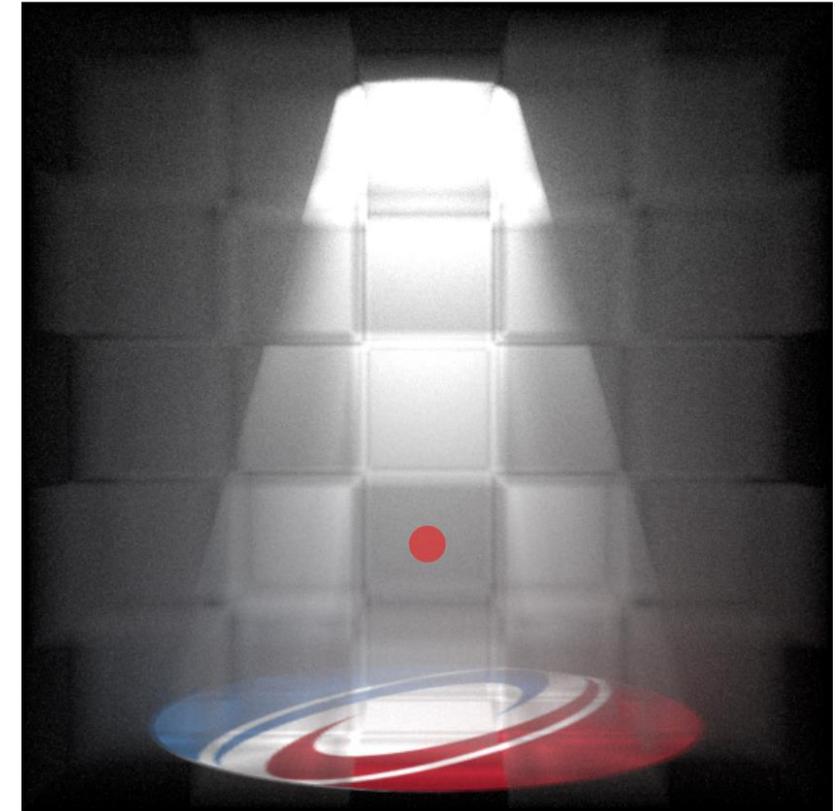
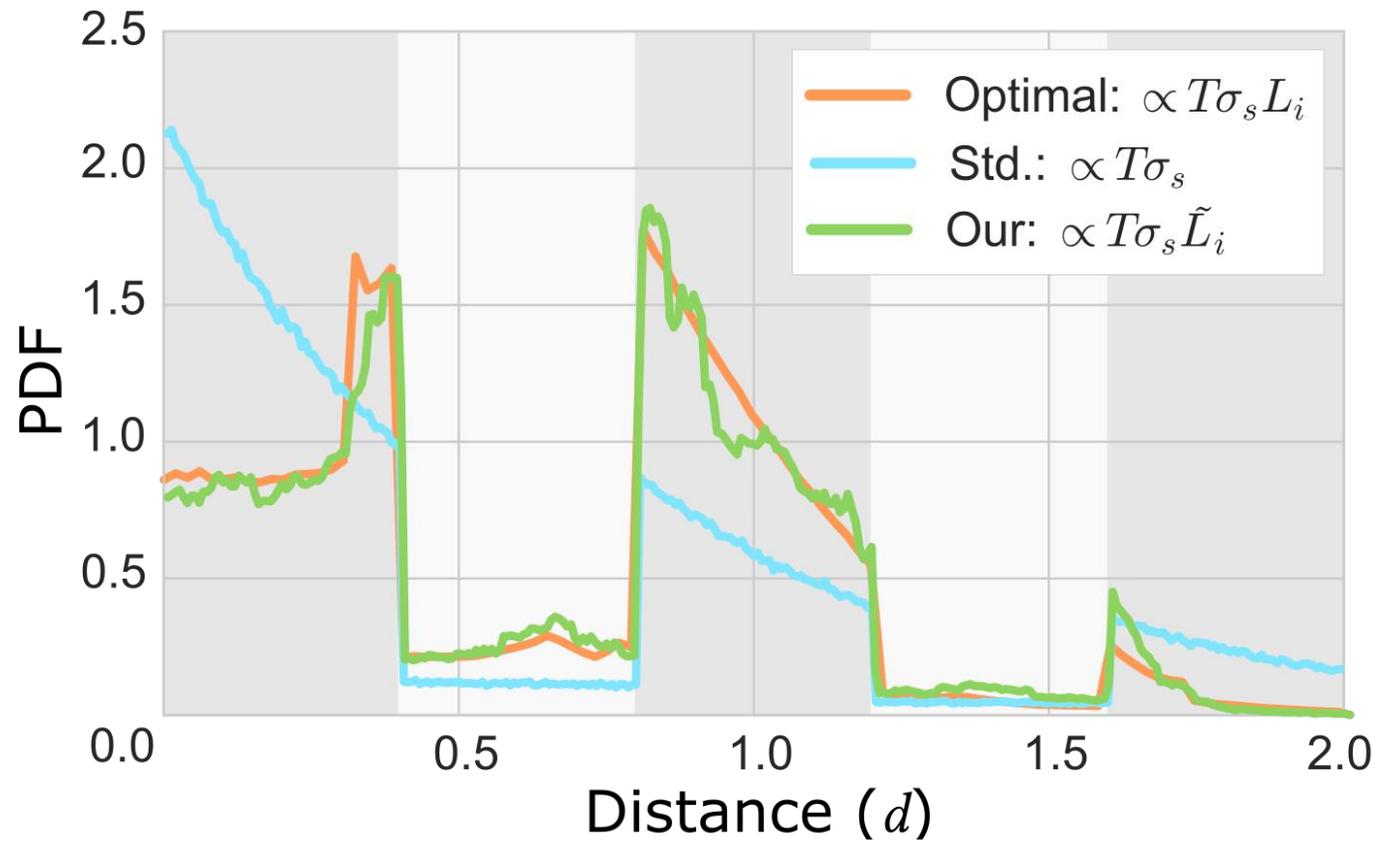


INCREMENTAL GUIDED DISTANCE SAMPLING



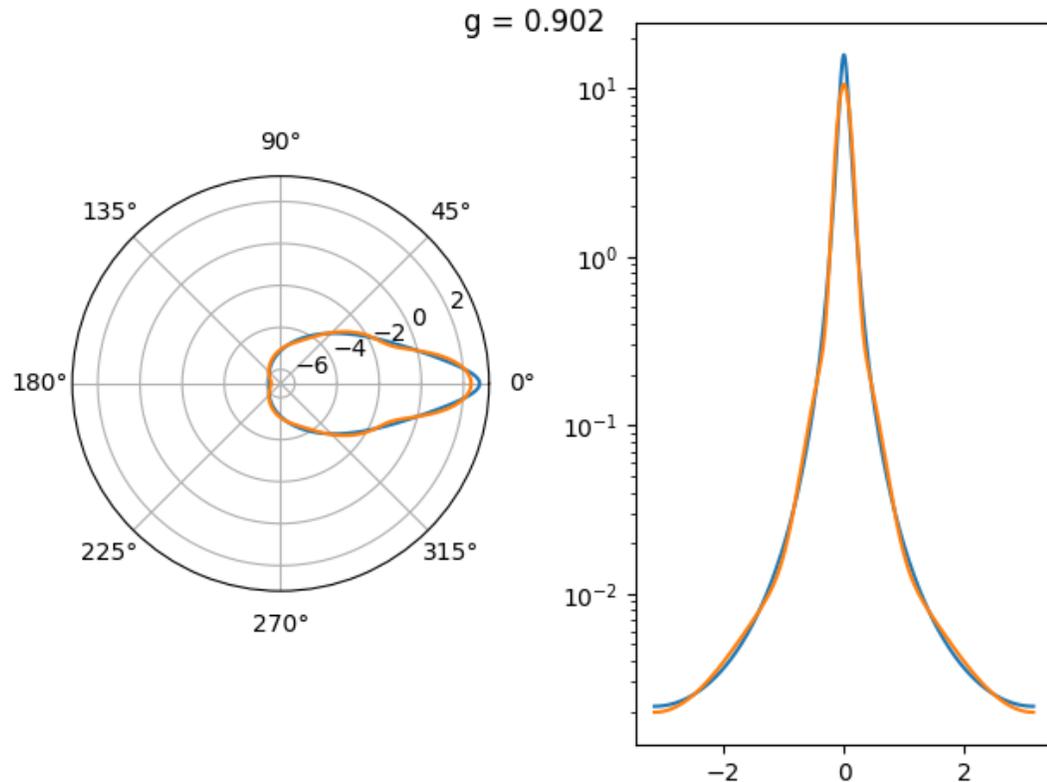


INCREMENTAL GUIDED DISTANCE SAMPLING





VMM PHASE FUNCTION FITTING: PRE-PROCESSING STEP



- Using up to $K = 4$ components
- Optimization Problem:

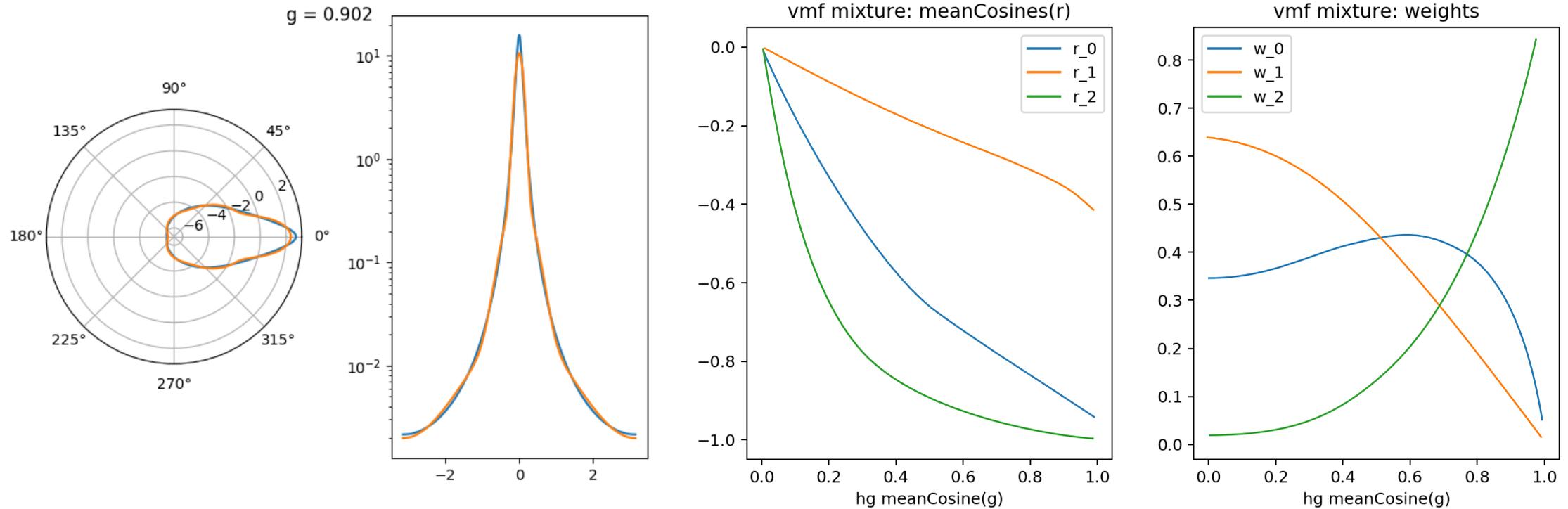
$$\arg \min_{\Theta_f} \sum_{n=1}^N [\mathcal{L}_{\log}(f(\omega_n, \dots), V(\omega_n, \Theta_f))]^2$$

$$\mathcal{L}_{\log}(d, m) = \log(d + \epsilon) - \log(m + \epsilon)$$

$$\epsilon = (1e - 4) \cdot \max(d_1, \dots, d_n)$$



VMM PHASE FUNCTION FITTING



- Manifold representation of the VMM parameters for an HG phase function model for different mean cosines