Advanced 3D computer graphics for movies and games (NPGR010)

– Light reflection, BRDF

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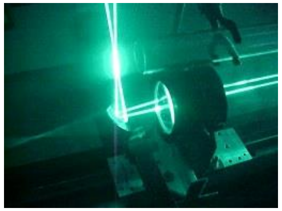
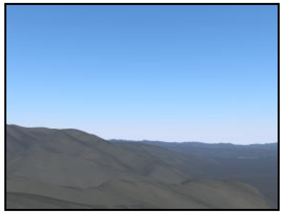
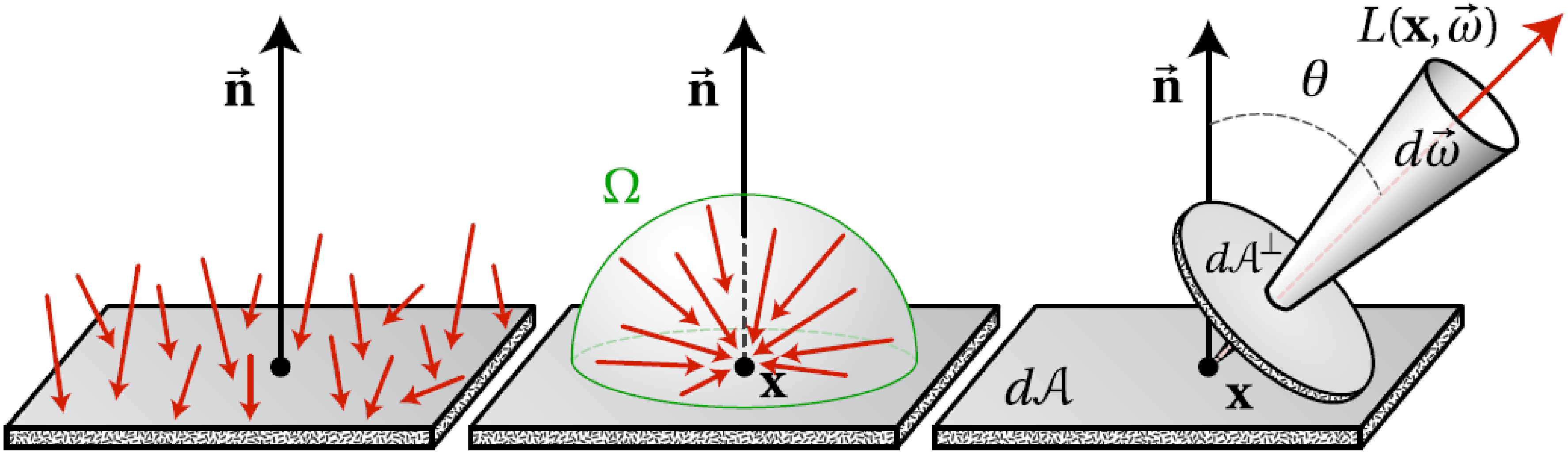
Slides of prof. Jaroslav Křivánek, minor edits by Jiří Vorba

Recap – Basic radiometric quantities



(a) 3D Flux (b) 3D Irradiance (c) 3D Radiance

Image: Wojciech Jarosz

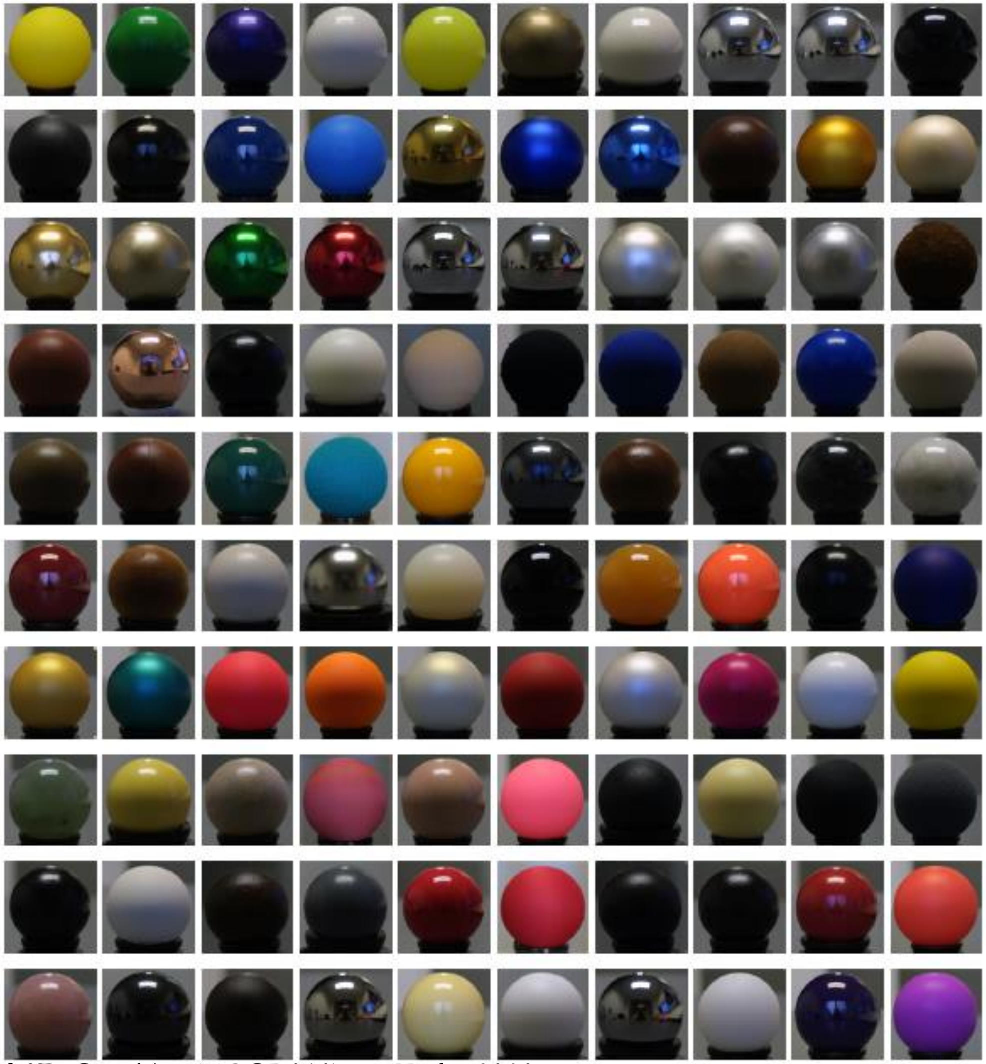
Interaction of light with a surface

- Absorption
- Reflection
- Transmission / refraction

- Reflective properties of materials determine
  - the relation of **reflected** radiance L_r to **incoming** radiance L_i , and therefore
  - the **appearance** of the object: color, glossiness, etc.

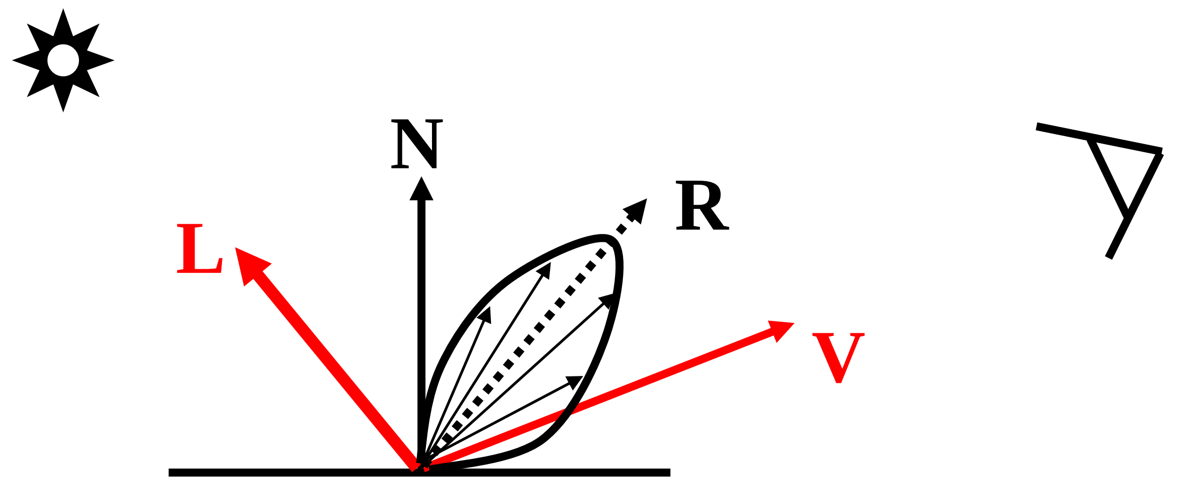
Interaction of light with a surface

- Same illumination
- Different materials



Source: MERL BRDF database

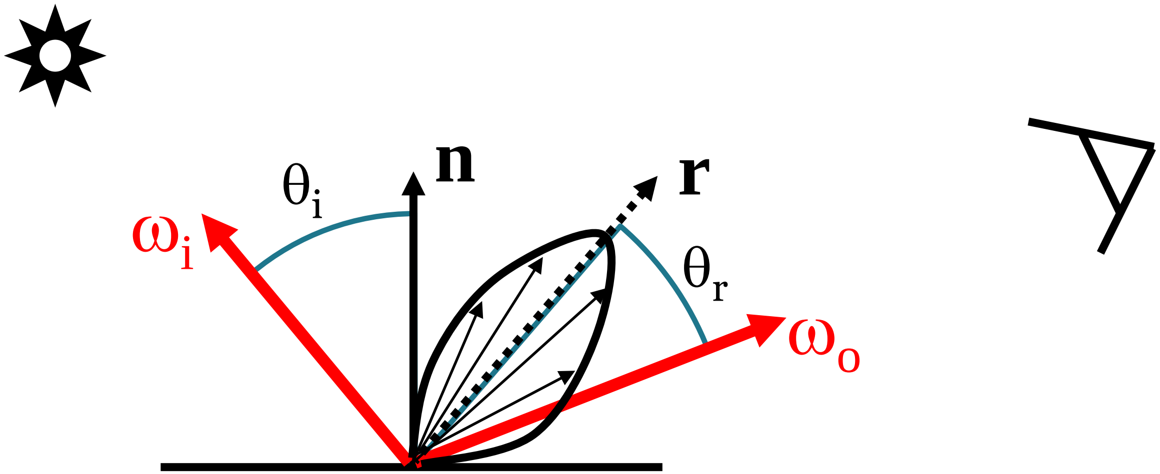
Recall the Phong shading model



$$C = I\left(k_d(N \cdot L) + k_s(V \cdot R)^n\right)$$

$$R = 2(N \cdot L)N - L$$

I) Adopt radiometric notation

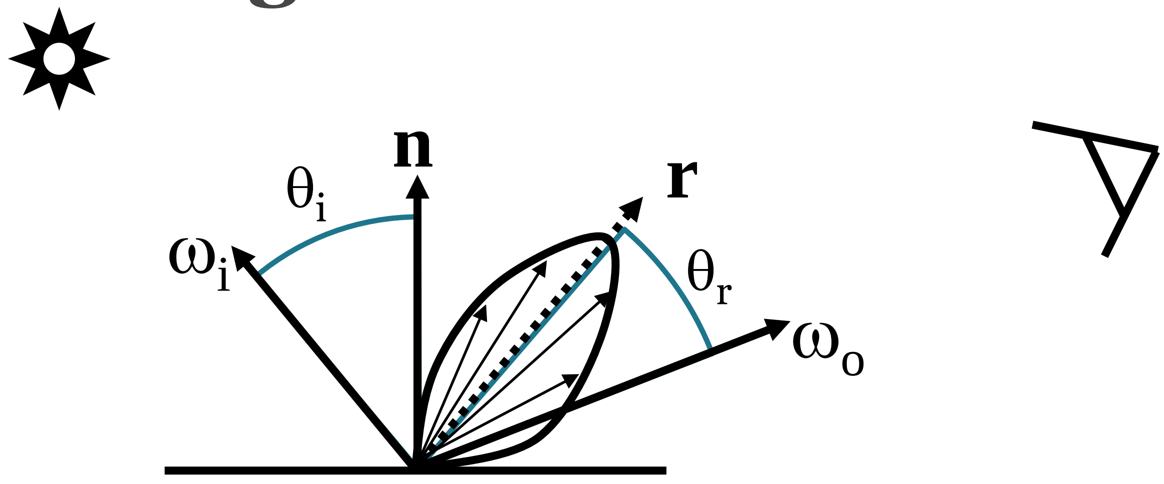


$$L_o(\omega_o) = L_i(\omega_i)\left(k_d \cos\theta_i + k_s \cos^n \theta_r\right)$$

$$\cos\theta_r = \omega_o \cdot \mathbf{r} \qquad \mathbf{r} = 2(\mathbf{n}\cdot\omega_i)\mathbf{n} - \omega_i$$

Exact same thing as on the previous slide – just using physically-based notation.

BRDF corresponding to the original Phong shading model



BRDF: $$f_r = \frac{L_o}{L_i \cos\theta_i}$$

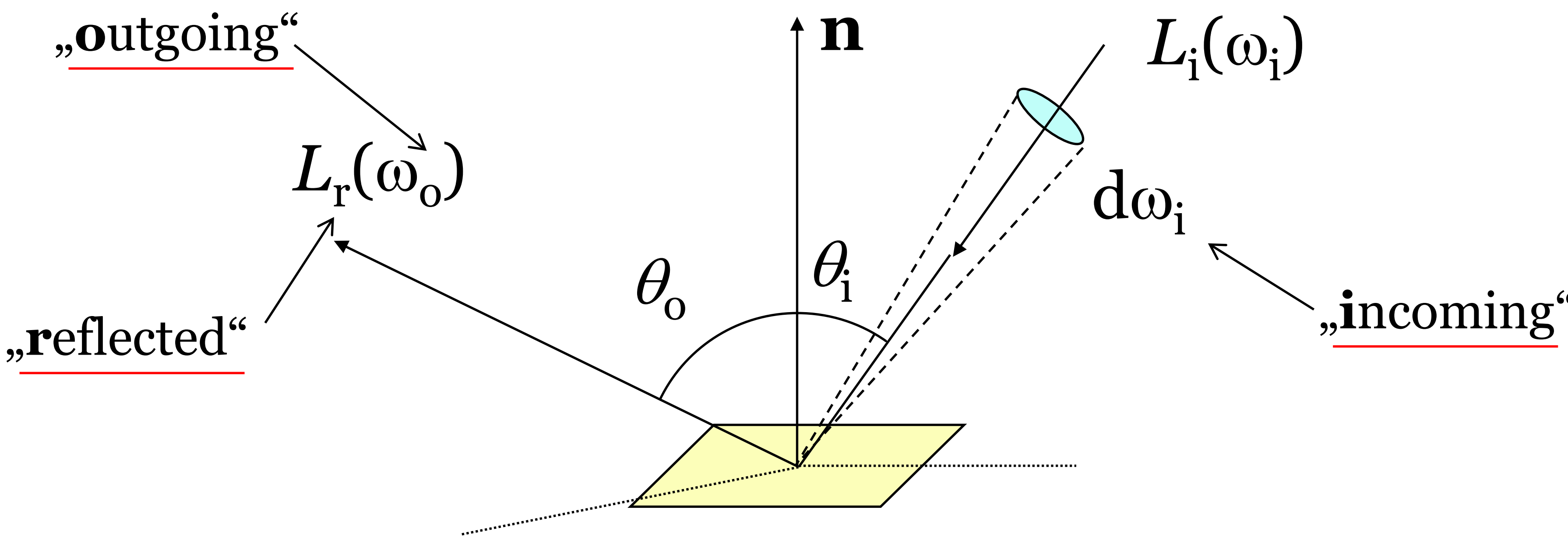
General definition of a BRDF

$$f_r^{\,Phong\ Orig} = k_d + k_s \frac{\cos^n \theta_r}{\cos\theta_i}$$

Application of this definition to the Phong shading formula.

BRDF – Formal definition

- **B**idirectional **R**eflectance **D**istribution **F**unction



$$f_r(\omega_i \rightarrow \omega_o) = \frac{dL_r(\omega_o)}{L_i(\omega_i) \cdot \cos\theta_i \cdot d\omega_i} \quad [sr^{-1}]$$

BRDF

- Mathematical model of the reflection properties of a surface

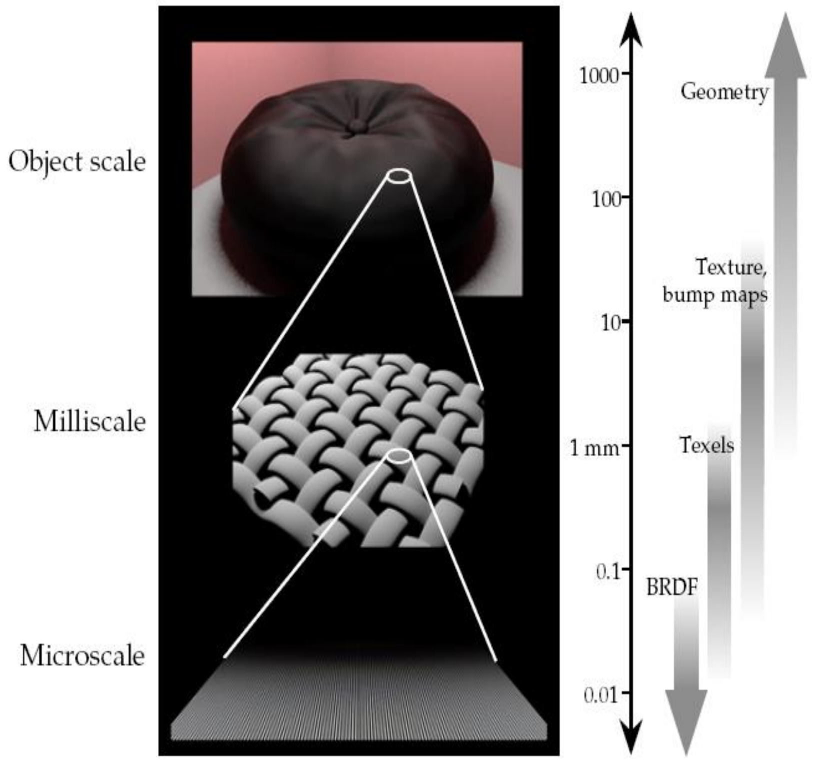
- Intuition
  - **Value of a** BRDF = **probability density**, describing the event that a light energy "packet", or "photon", coming from direction ω_i gets reflected to the direction ω_o.

- Range:

$$f_r(\omega_i \to \omega_o) \in [0, \infty)$$

BRDF

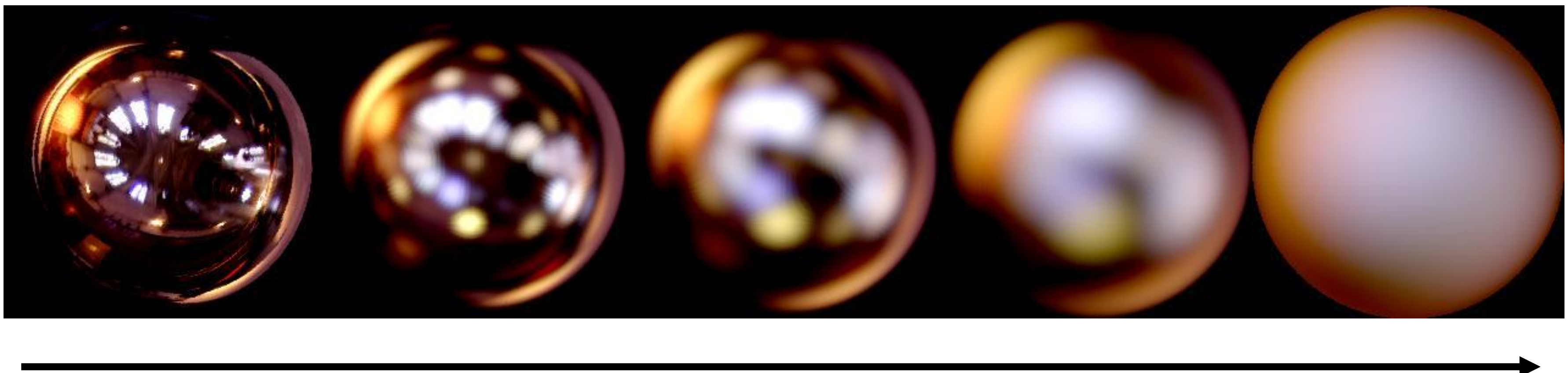
- The BRDF is a **model of the bulk behavior of light** on the microstructure when viewed from distance



Westin et al. Predicting Reflectance Functions from Complex Surfaces, SIGGRAPH 1992.

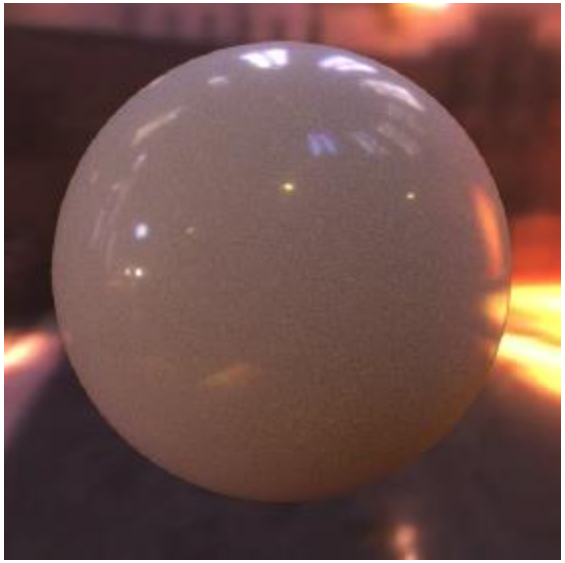
Surface roughness and blurred reflections

- The rougher the blurrier

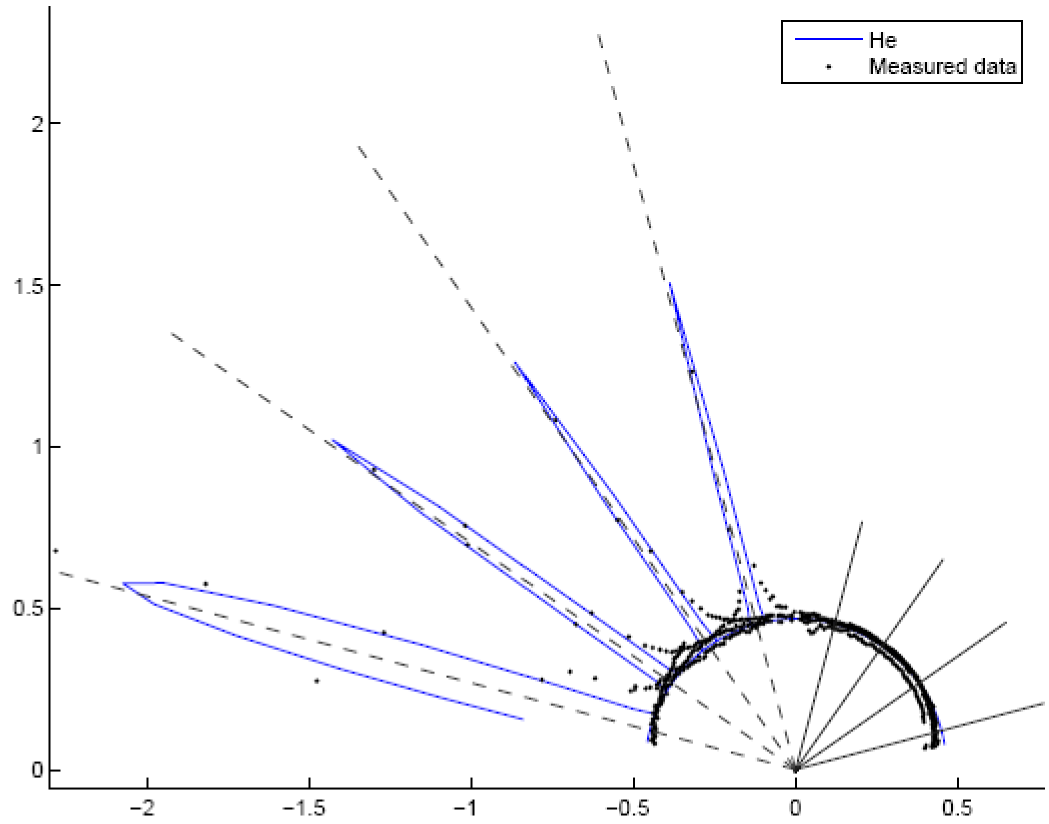


Microscopic surface roughness

Surface appearance and the BRDF



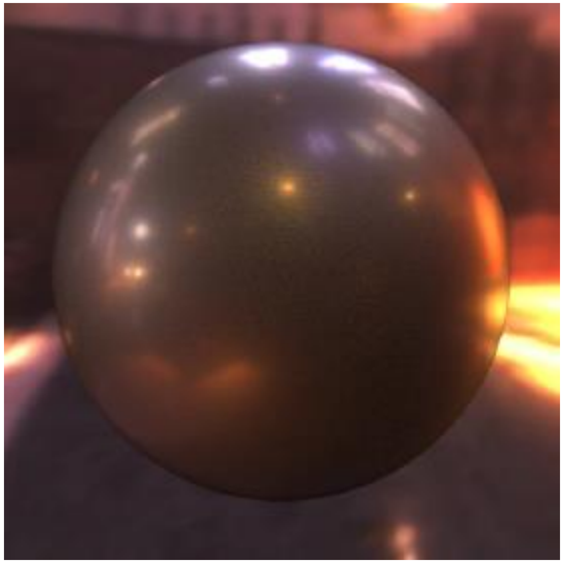
Appearance



BRDF lobe
(for four different viewing directions)

Souce: Ngan et al. Experimental analysis of BRDF models, http://people.csail.mit.edu/addy/research/brdf/

Surface appearance and the BRDF



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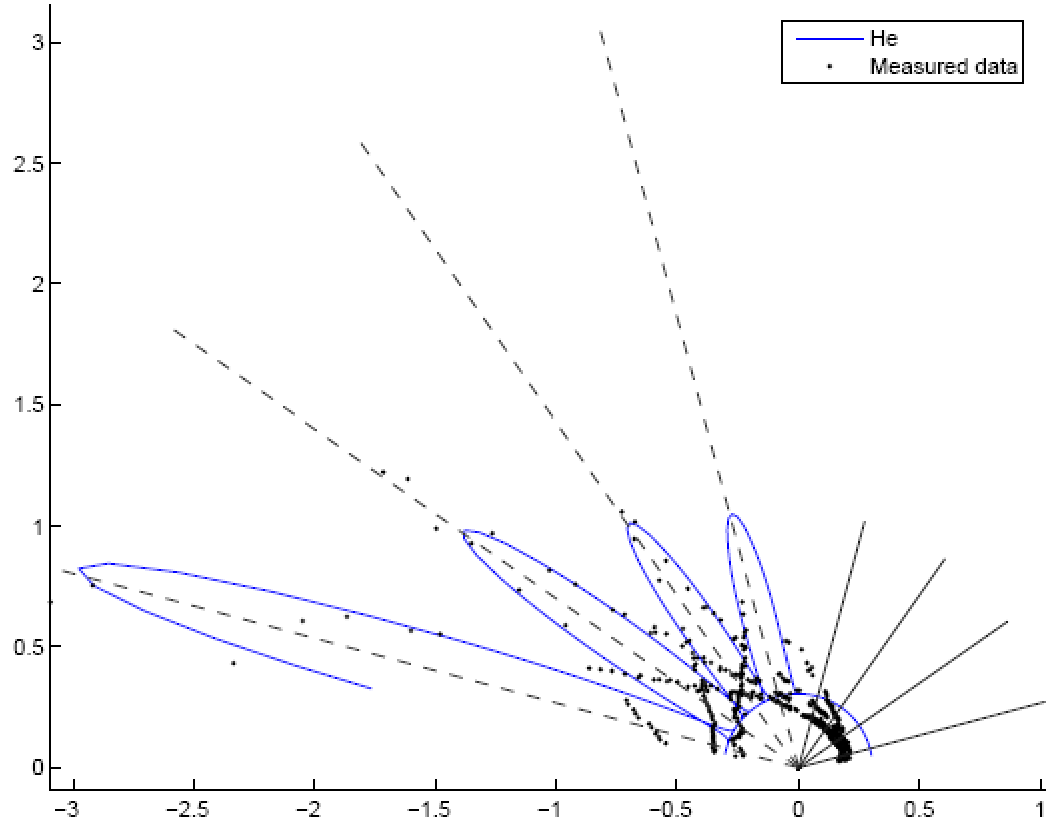

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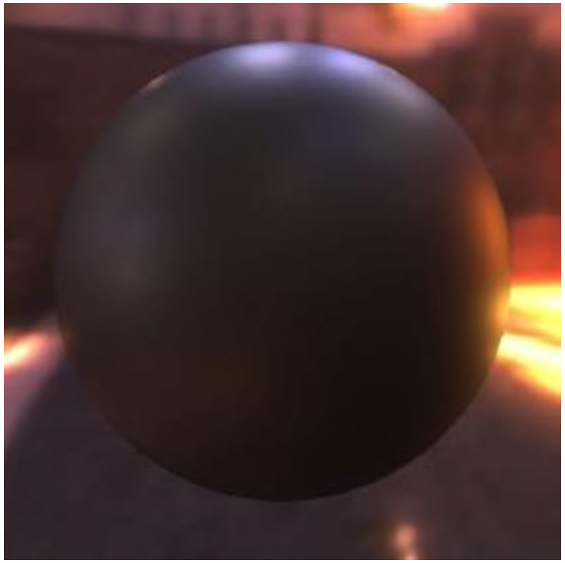

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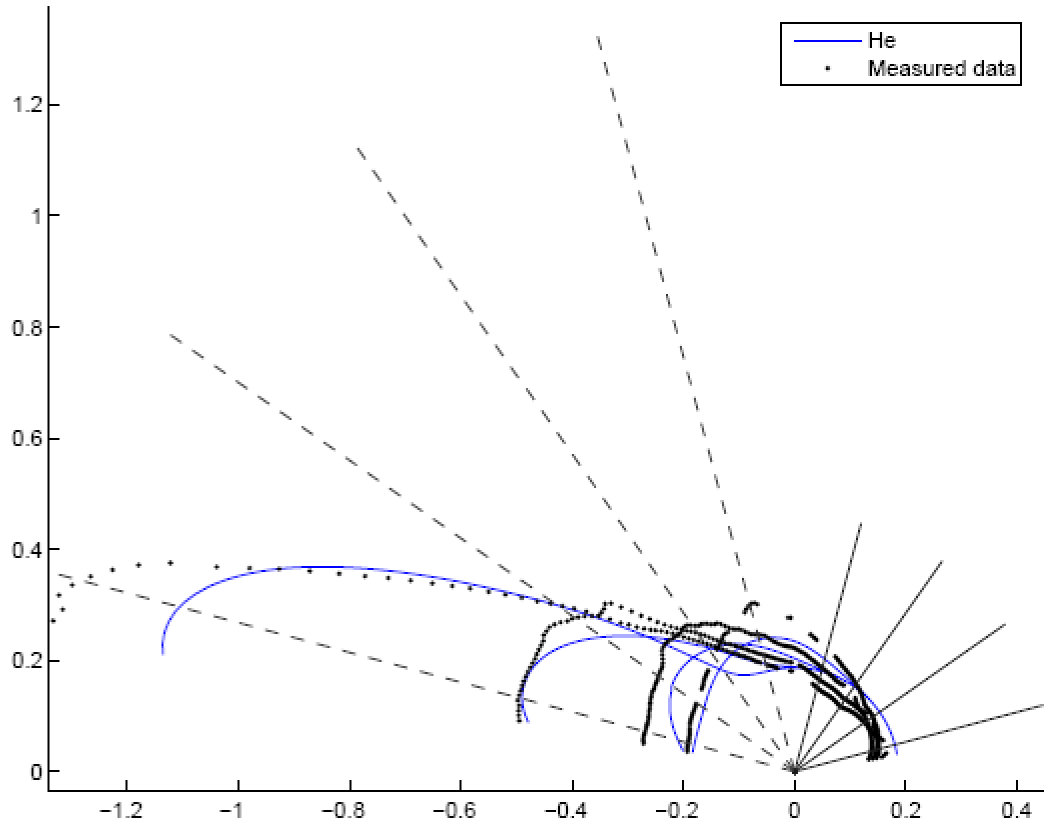

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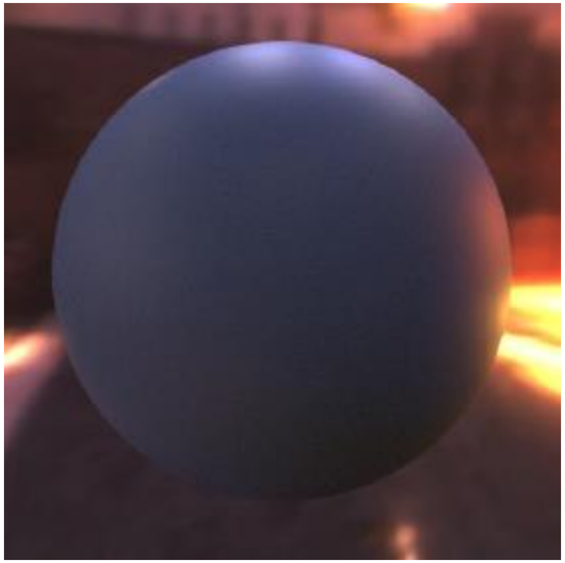

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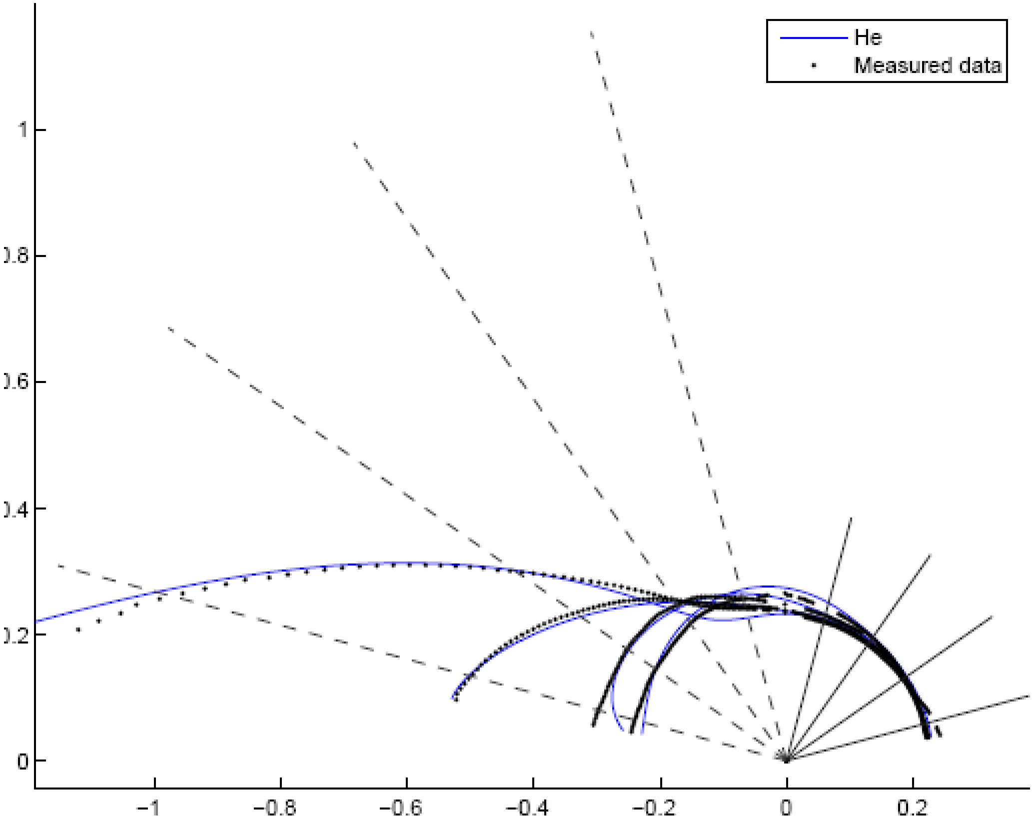

BRDF lobe
(for four different viewing directions)

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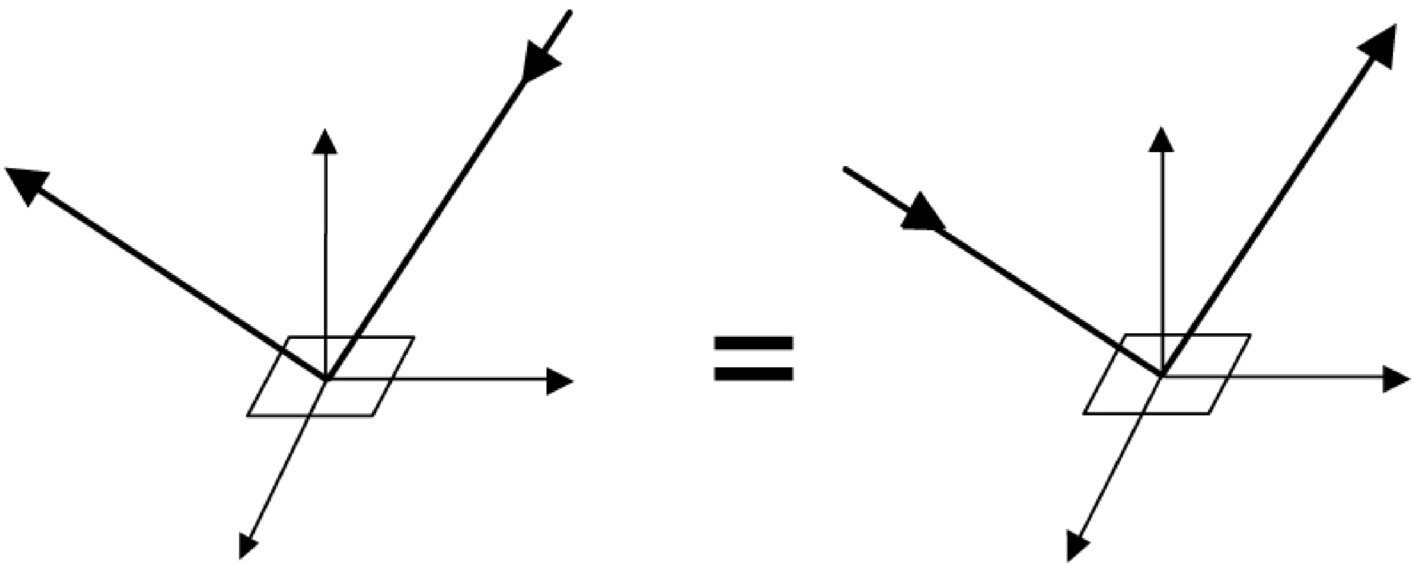
BRDF properties

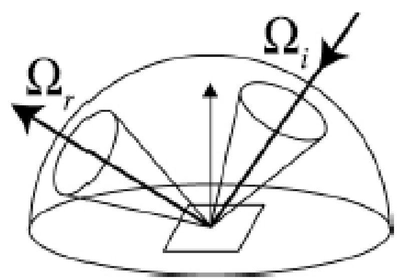
- **Helmholz reciprocity** (always holds in nature, a physically-plausible BRDF model must follow it)

$$f_r(\omega_i \to \omega_o) = f_r(\omega_o \to \omega_i)$$



BRDF properties

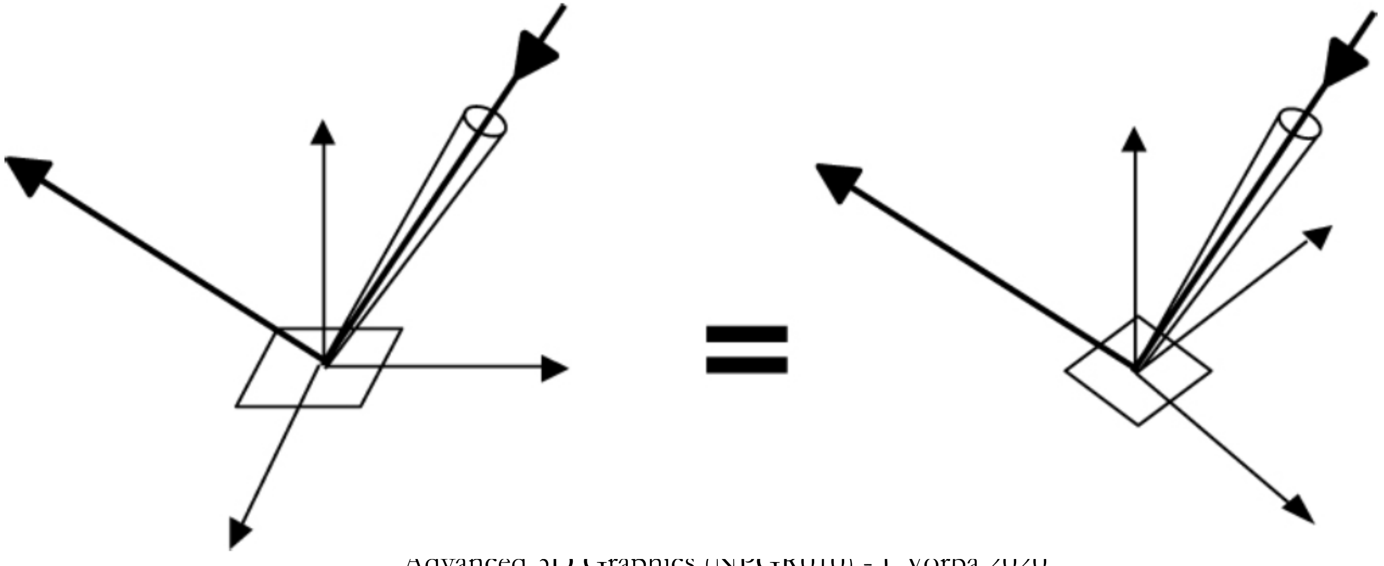
- **Energy conservation**
  - A patch of surface cannot reflect more light energy than it receives



BRDF (an)isotropy

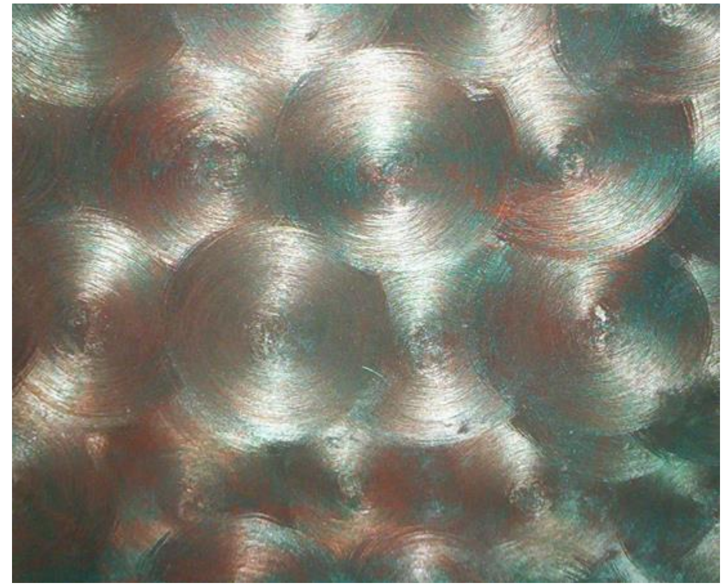
- **Isotropic BRDF** = invariant to a rotation around surface normal

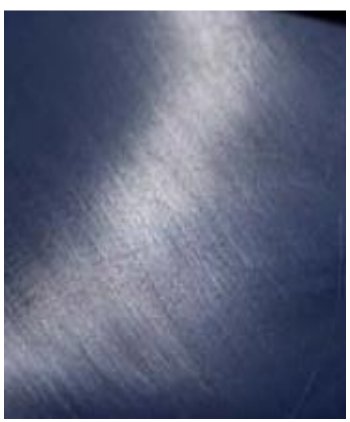
$$
\begin{aligned}
f_r(\theta_i, \phi_i ; \theta_o, \phi_o) &= f_r(\theta_i, \phi_i + \phi ; \theta_o, \phi_o + \phi) \\
&= f_r(\theta_i,\ \theta_o,\ \phi_o - \phi_i)
\end{aligned}
$$

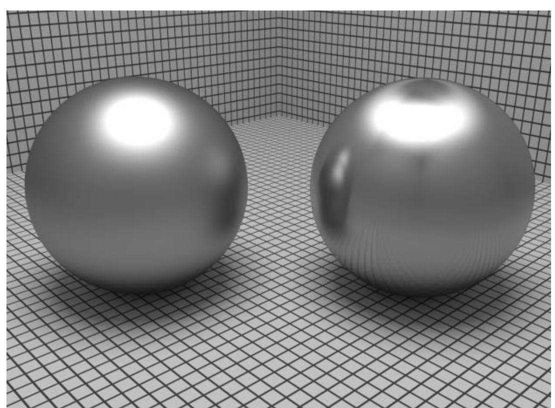


Surfaces with anisotropic BRDF

Figure 9: Anisotropic Aluminum Wheel



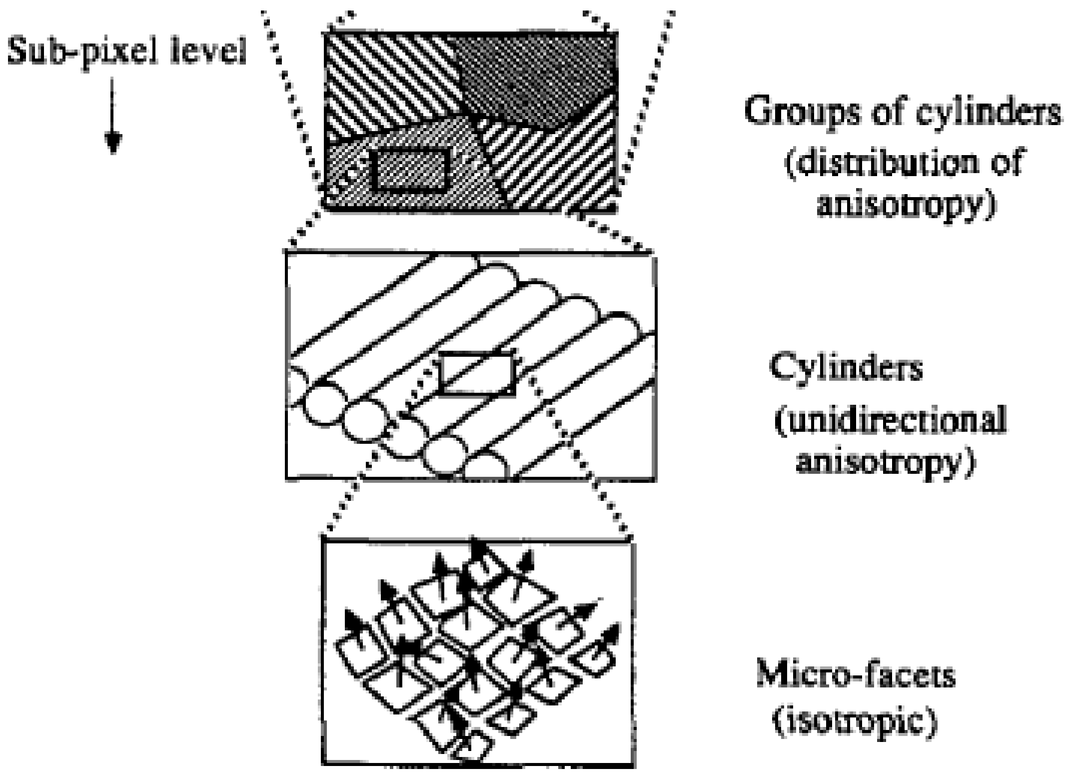




fibers

Anisotropic BRDF

- Different microscopic roughness in different directions (brushed metals, fabrics, …)



Isotropic vs. anisotropic BRDF

- **Isotropic** BRDFs have only 3 degrees of freedom
  - Instead of ϕ_i and ϕ_o it is enough to consider only $\Delta\phi = \phi_i - \phi_o$
  - But this is not enough to describe an anisotropic BRDF

- Description of an **anisotropic** BRDF
  - ϕ_i and ϕ_o are expressed in a **local coordinate frame** (U, V, N)
    - U … tangent – e.g. the direction of brushing
    - V … binormal
    - N … surface normal … the Z axis of the local coordinate frame

Reflection equation

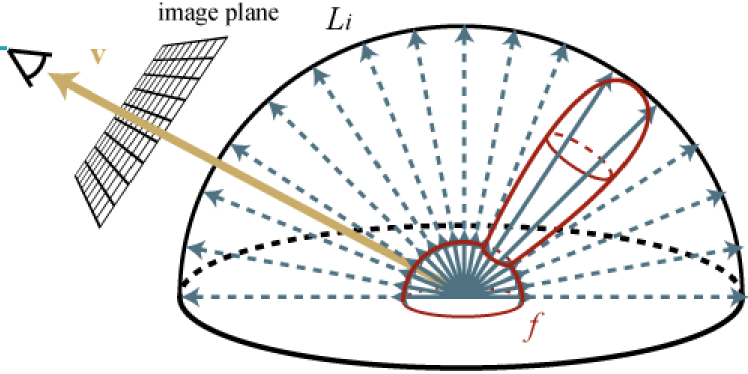
- A.k.a. reflectance equation, illumination integral, OVTIGRE ("outgoing, vacuum, time-invariant, gray radiance equation")

- "How much **total** light gets reflected in the direction ω_o?"

- From the definition of the BRDF, we have

$$\mathrm{d}L_{\mathrm{r}}(\omega_{\mathrm{o}}) = f_r(\omega_{\mathrm{i}} \to \omega_{\mathrm{o}}) \cdot L_{\mathrm{i}}(\omega_{\mathrm{i}}) \cdot \cos\theta_{\mathrm{i}} \, \mathrm{d}\omega_i$$

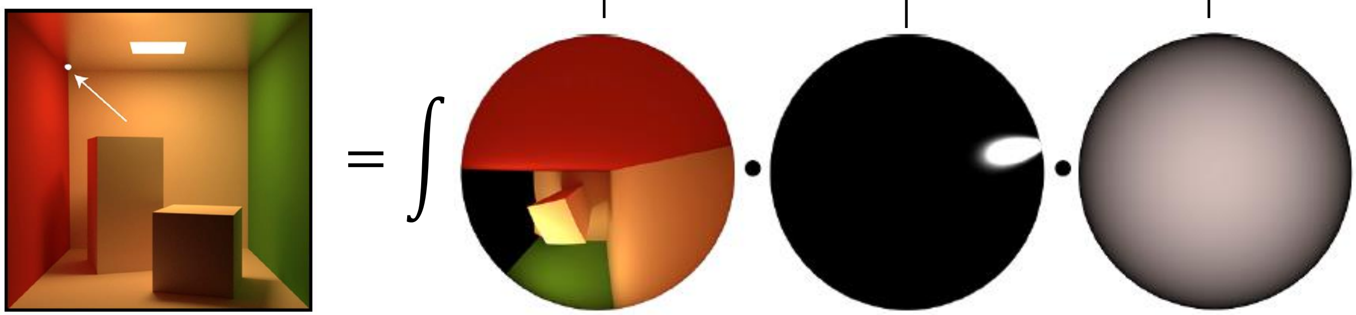
Reflection equation



- Total reflected radiance: integrate contributions of incident radiance, weighted by the BRDF, over the hemisphere

$$L_{\mathrm{r}}(\omega_{\mathrm{o}}) = \int_{H(\mathbf{x})} L_{\mathrm{i}}(\omega_{\mathrm{i}}) \cdot f_r(\omega_{\mathrm{i}} \to \omega_{\mathrm{o}}) \cdot \cos\theta_{\mathrm{i}} \, \mathrm{d}\omega_{\mathrm{i}}$$

upper hemisphere over x

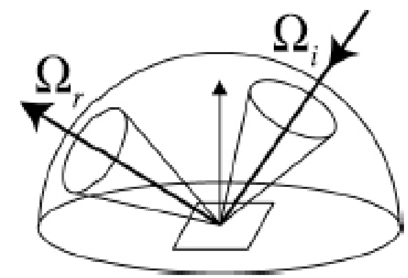


Reflection equation

- Evaluating the reflectance equation renders images!!!
  - Direct illumination
    - Environment maps
    - Area light sources
    - etc.

Energy conservation – More rigorous

- Reflected flux per unit area (i.e. radiosity *B*) cannot be larger than the incoming flux per unit surface area (i.e. irradiance *E*).



$$\frac{B}{E} = \frac{\int L_r(\omega_o)\cos\theta_o \, d\omega_o}{\int L_i(\omega_i)\cos\theta_i \, d\omega_i} =$$

$$= \frac{\int \left[\int f_r(\omega_\mathrm{i} \to \omega_\mathrm{o}) L_i(\omega_i)\cos\theta_i \, d\omega_i \right] \cos\theta_o \, d\omega_o}{\int L_i(\omega_i)\cos\theta_i \, d\omega_i} =$$

$$\le 1$$

Reflectance

- Ratio of the **incoming** and **outgoing flux**
  - A.k.a. „albedo“ (used mostly for diffuse reflection)
- **Hemispherical-hemispherical** reflectance
  - See the “Energy conservation” slide
- **Hemispherical-directional** reflectance
  - The amount of light that gets reflected in direction ω_o when illuminated by the unit, uniform incoming radiance.

$$\rho(\omega_o) = a(\omega_o) = \int_{H(\mathbf{x})} f_r(\omega_i \to \omega_o) \cos\theta_i \, d\omega_i$$

Hemispherical-directional reflectance

- Nonnegative
- Less than or equal to 1 (energy conservation)

$$\rho(\omega_o) \in [0, 1]$$

- Equal to **directional-hemispherical reflectance**
  - What is the percentage of the energy coming from the incoming direction ω_i that gets reflected (to any direction)?“
  - Equality follows from the Helmholz reciprocity

Albedo

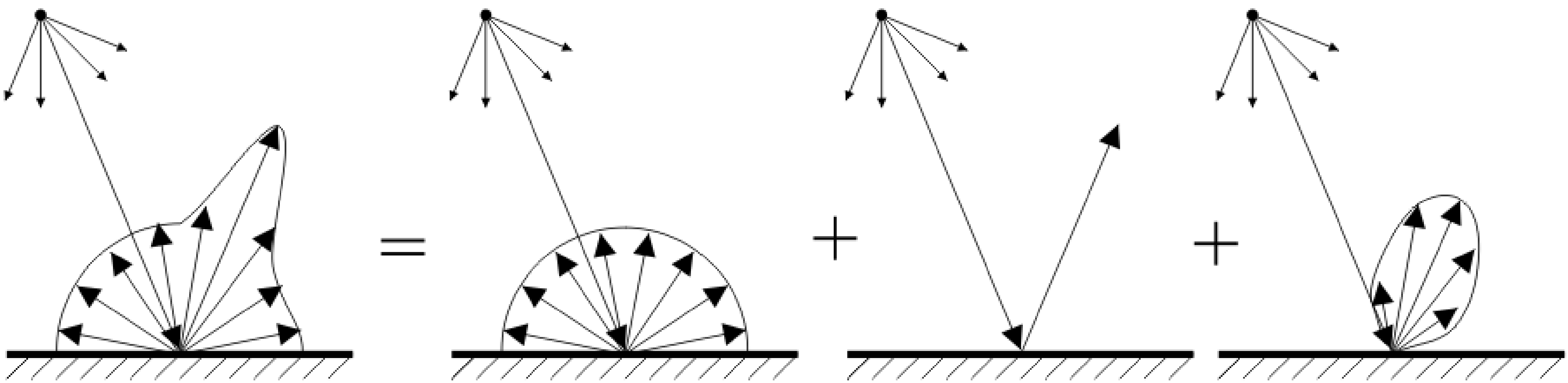
- fraction of light reflected from a diffuse surface
  - usually refers to an average across the visible spectrum

- examples
  - clouds 80%
  - fresh snow 80%
  - old snow 40%
  - grass 30%
  - soil 15%
  - rivers 7%
  - ocean 3%

equality explains "whiteout" in blizzards (clouds, fresh snow)

not including mirror reflections of the sun (rivers, ocean)

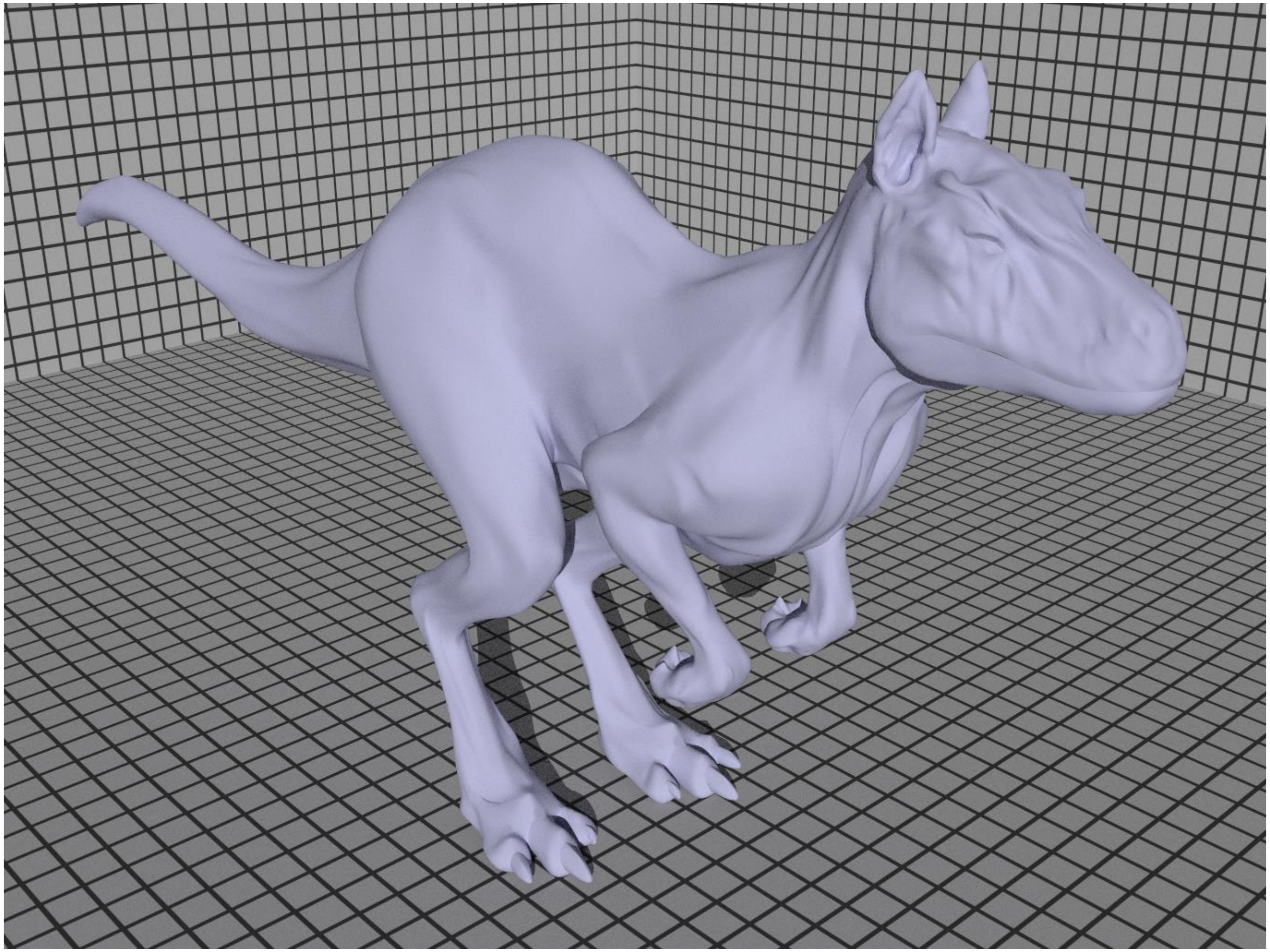
BRDF components



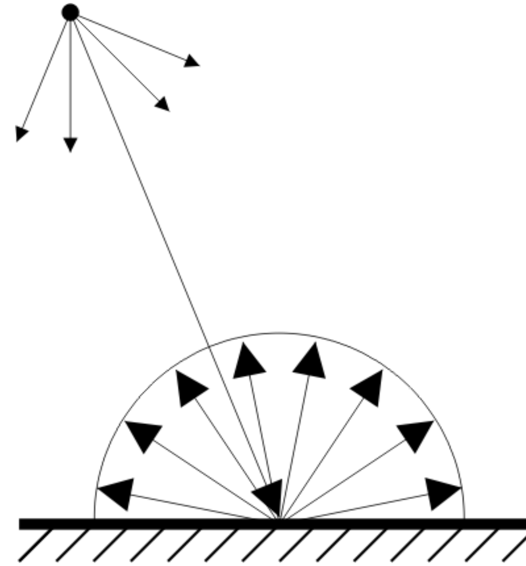
General BRDF = Ideal diffuse (Lambertian) + Ideal specular + Glossy, directional diffuse

Ideal diffuse reflection

Ideal diffuse reflection



Ideal diffuse reflection

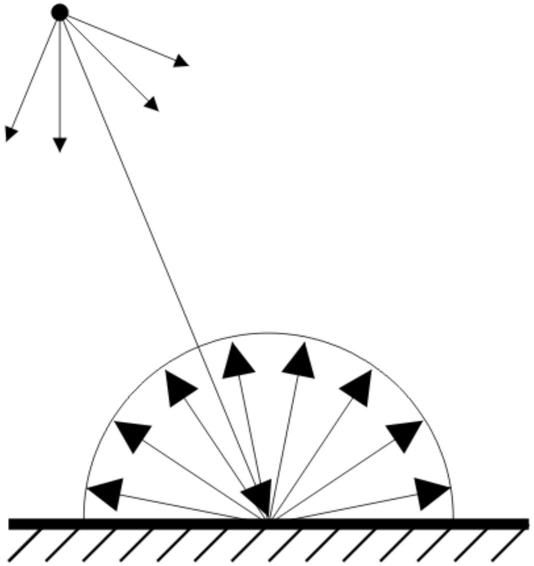


- A.k.a. Lambertian reflection
  - Johann Heinrich Lambert, „Photometria“, 1760.

- Postulate: Light gets reflected to all directions with the same probability, irrespective of the direction it came from
- The corresponding BRDF is a constant function (independent of ω_i , ω_o)

$$f_{r,d}(\omega_i \to \omega_o) = f_{r,d}$$

Ideal diffuse reflection



- Reflection on a Lambertian surface:

$$L_o(\omega_o) = f_{r,d} \int_{H(\mathbf{x})} L_i(\omega_i) \cos\theta_i \, \mathrm{d}\omega_i$$
$$= f_{r,d} \, E$$

irradiance

- **View independent appearance**
  - Outgoing radiance L_o is independent of ω_o

- **Reflectance** (derive)

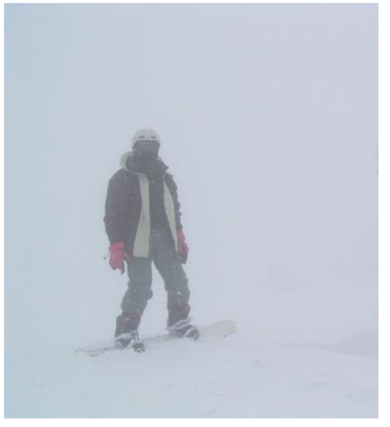
$$\rho_d = \pi \cdot f_{r,d}$$

Ideal diffuse reflection

- Mathematical idealization that does not exist in nature
- The actual behavior of natural materials deviates from the Lambertian assumption especially for grazing incidence angles

White-out conditions

- Under a covered sky we cannot tell the shape of a terrain covered by snow



- We do not have this problem close to a localized light source.

- **Why?**

White-out conditions

- We assume sky radiance independent of direction (covered sky)

$$L_{\mathrm{i}}(\mathbf{x}, \omega_{\mathrm{i}}) = L^{\mathrm{sky}}$$

- We also assume Lambertian reflection on snow

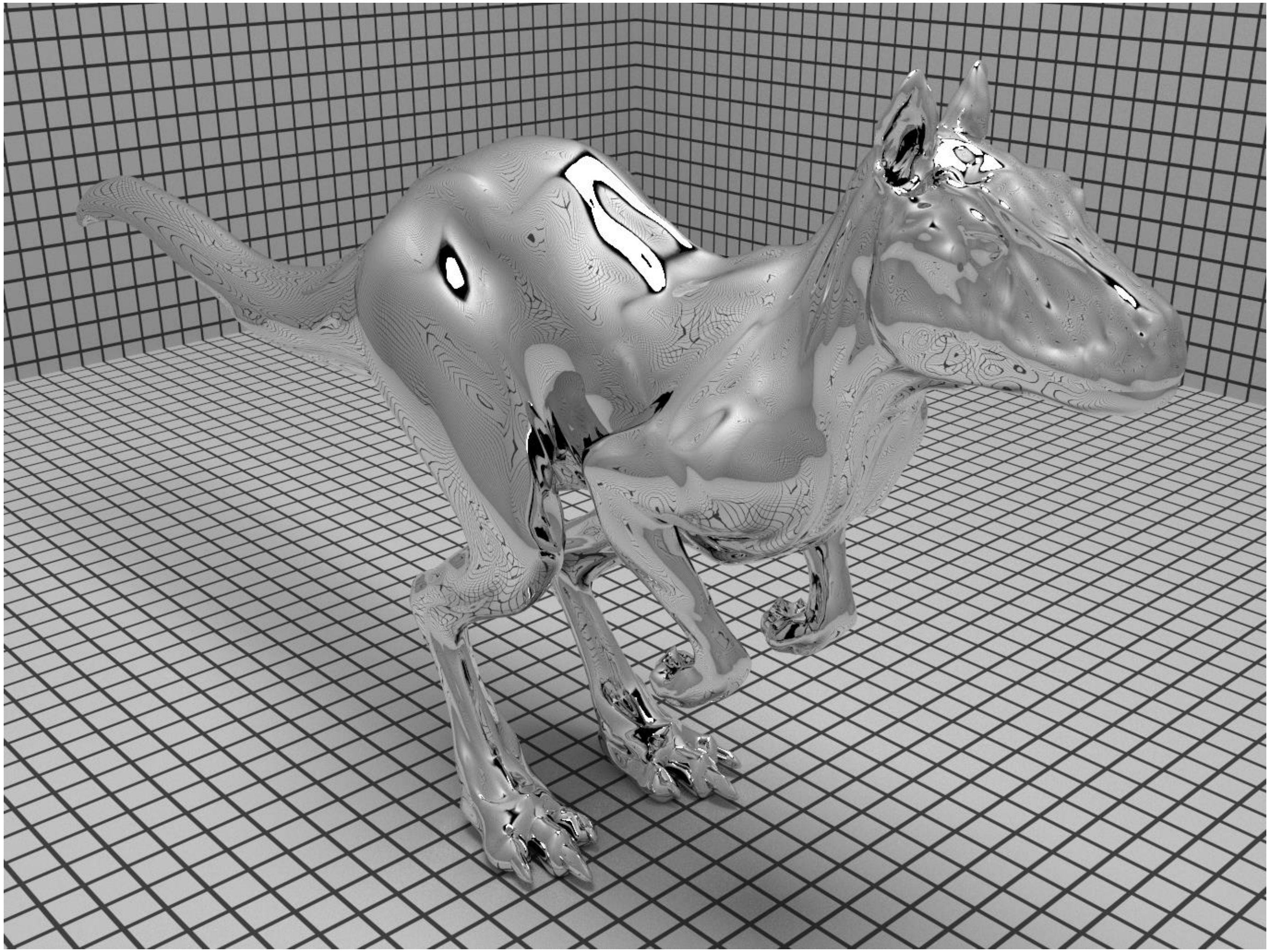
- Reflected radiance given by:

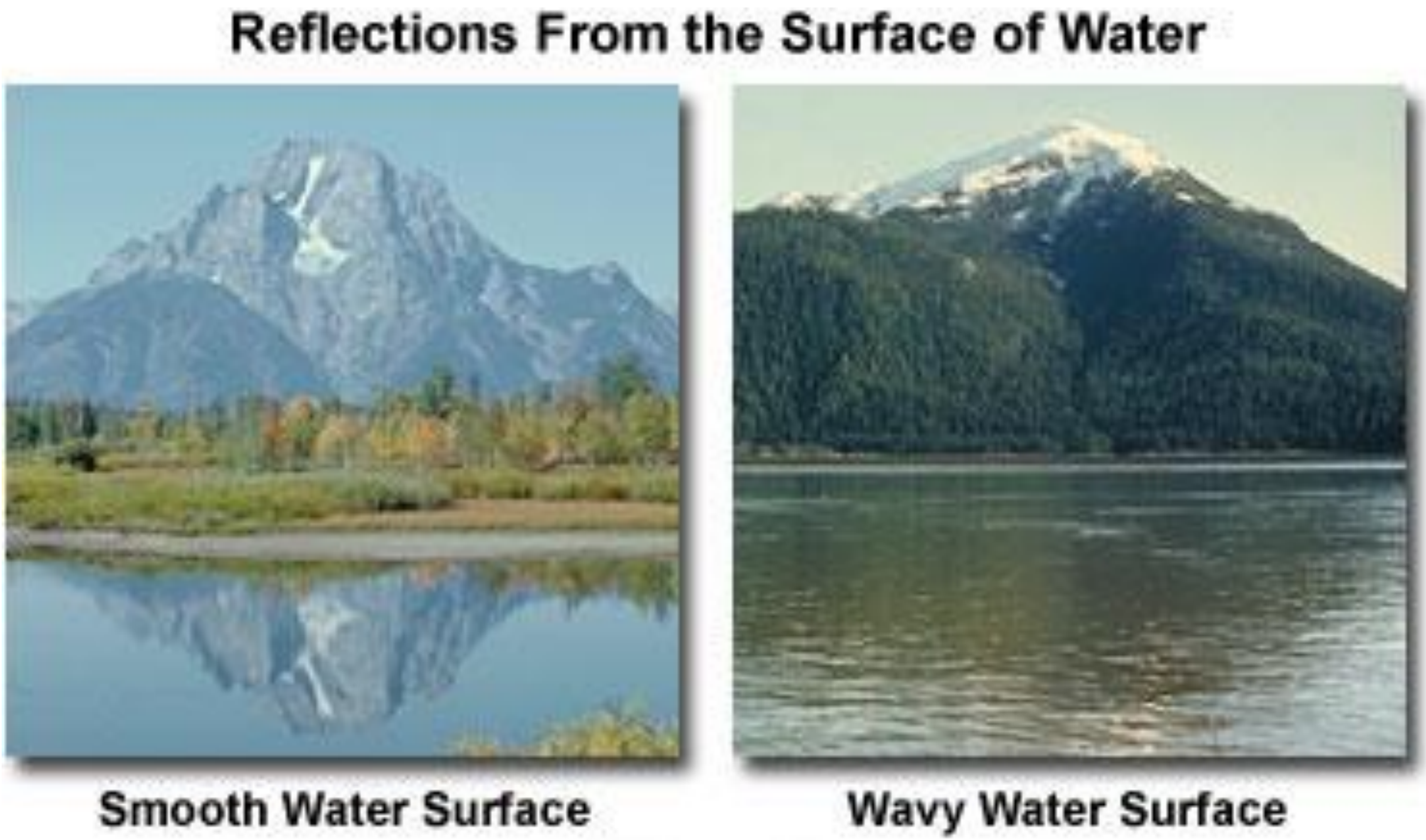
$$L_{\mathrm{o}}^{\mathrm{snow}} = \rho_d^{\mathrm{snow}} \cdot L_{\mathrm{i}}^{\mathrm{sky}}$$

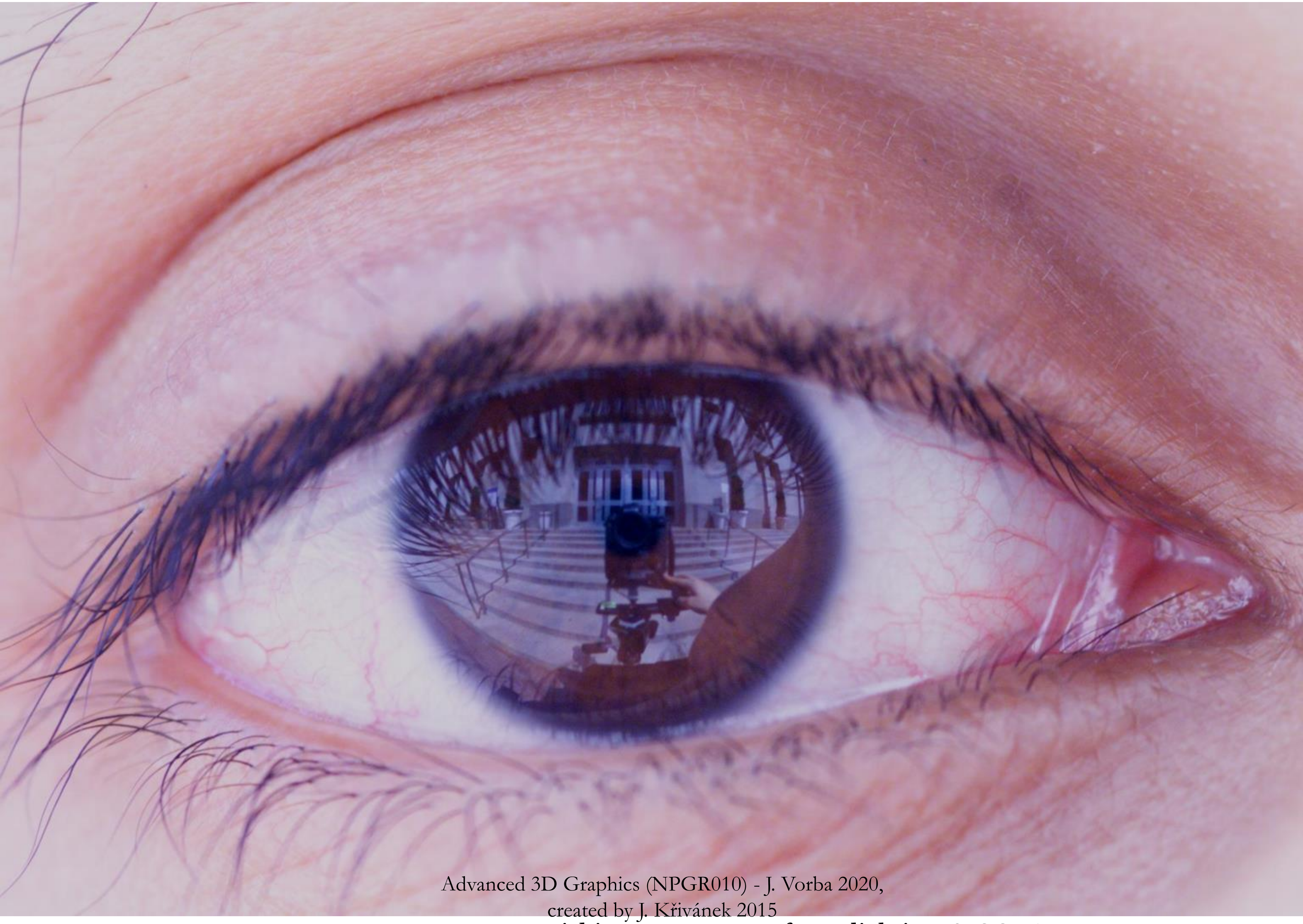
White-out!!!

Ideal mirror reflection

Ideal mirror reflection

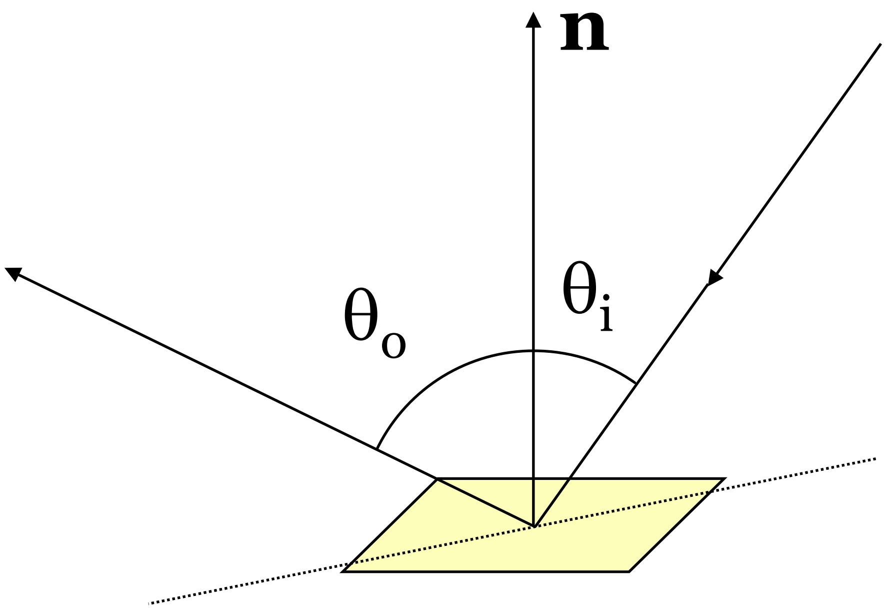


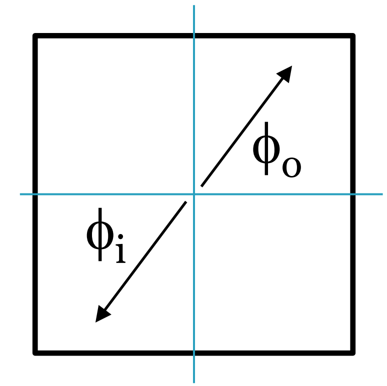

Reflections From the Surface of Water
Smooth Water Surface
Wavy Water Surface



Nishino, Nayar: Eyes for Relighting, SIGGRAPH 2004

The law of reflection





$$\theta_o = \theta_i$$

$$\phi_o = (\phi_i + \pi) \bmod 2\pi$$

- Direction of the reflected ray (derive the formula)

$$\omega_o = 2(\omega_i \cdot \mathbf{n})\mathbf{n} - \omega_i$$

Digression: Dirac delta distribution

- **Definition** (informal):

$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1.$$

- The following holds for any *f*:

$$\int_{-\infty}^{\infty} f(x)\delta(x)\, dx = f(0)$$

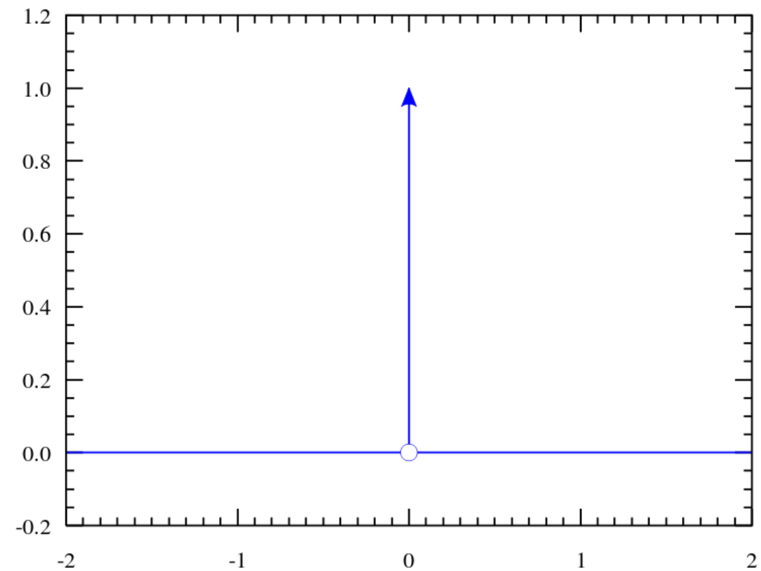
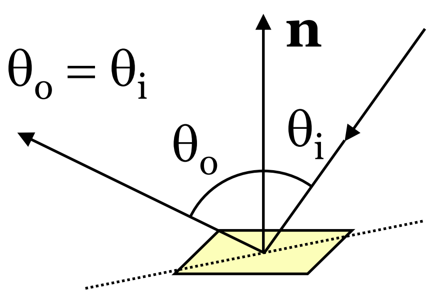


Image: Wikipedia

- Delta distribution is **not a function** (otherwise the integrals would = 0)

BRDF of the ideal mirror

- BRDF of the ideal mirror is a Dirac delta distribution



We want:

$$L_r(\theta_o, \varphi_o) = R(\theta_i)\, L_i(\theta_o, \varphi_o \pm \pi)$$

Fresnel reflectance (see below)

$$f_{r,m}(\theta_i, \varphi_i; \theta_o, \varphi_o) = R(\theta_i)\, \frac{\delta(\cos\theta_i - \cos\theta_o)\, \delta(\varphi_i - \varphi_o \pm \pi)}{\cos\theta_i}$$

BRDF of the ideal mirror

- BRDF of the ideal mirror is a Dirac delta distribution

- Verification:

$$L_r(\theta_o, \varphi_o) = \int f_{r,m}(.)\, L_i(.) \cos\theta_i \, d\omega_i$$

$$= \int R(\theta_i) \frac{\delta(\cos\theta_i - \cos\theta_o)\, \delta(\varphi_i - \varphi_o \pm \pi)}{\cos\theta_i} L_i(\theta_i, \varphi_i) \cos\theta_i \, d\omega_i$$

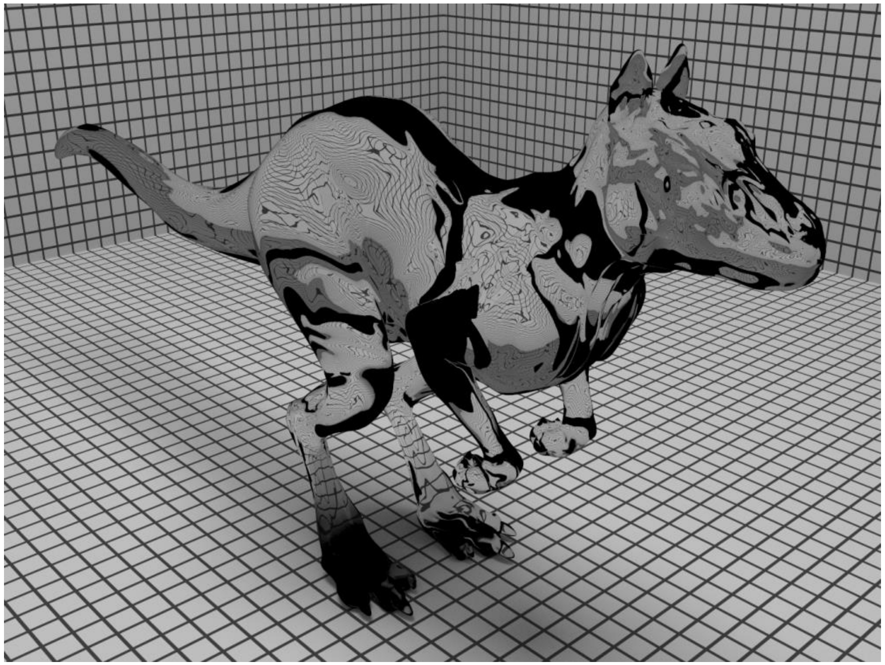
$$= R(\theta_i) L_i(\theta_r, \varphi_r \pm \pi)$$

Diego Velázquez, Venus at her Mirror, 1647

Q. Who is Venus looking at in the mirror?

Ideal refraction

Ideal refraction

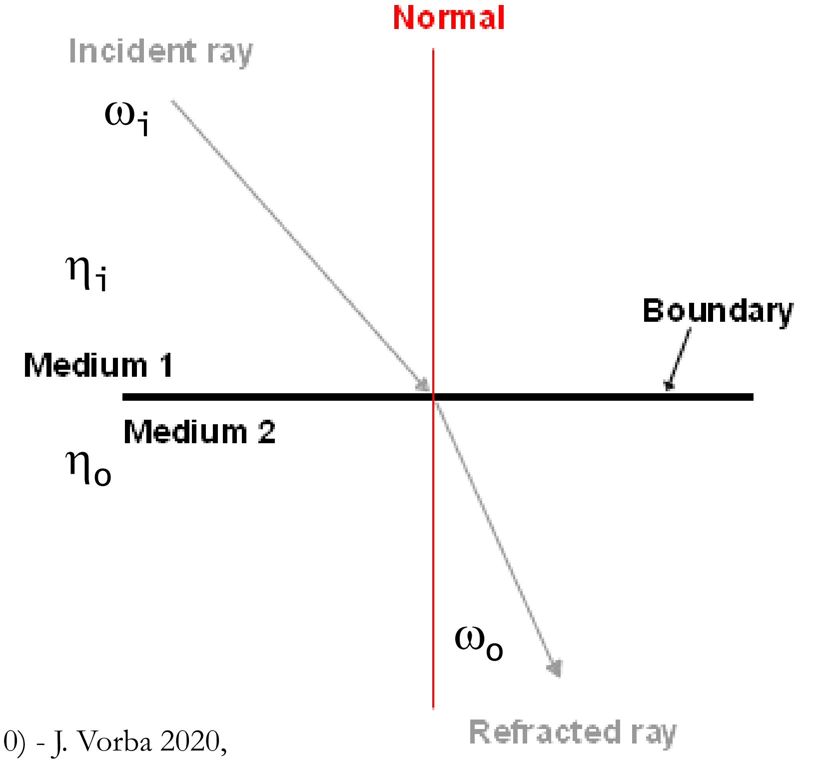


Ideal refraction

- **Index of refraction η**
  - "change of light speed"
  - Water 1.33, glass 1.6, diamond 2.4
  - Often depends on the wavelength

- **Snell's law**

$$\eta_i \sin\theta_i = \eta_o \sin\theta_o$$



Ideal refraction

- **Direction of the refracted ray**:

$$\omega_{\mathrm{o}} = -\eta_{\mathrm{io}}\,\omega_{\mathrm{i}} - \left(\eta_{\mathrm{io}}\cos\theta_{\mathrm{i}} + \underbrace{\sqrt{1-\eta_{\mathrm{io}}^2\left(1-\cos^2\theta_{\mathrm{i}}\right)}}\right)\mathbf{n}$$

$$\eta_{\mathrm{io}} = \frac{\eta_{\mathrm{i}}}{\eta_{\mathrm{o}}}$$

if < 0, **total internal reflection**

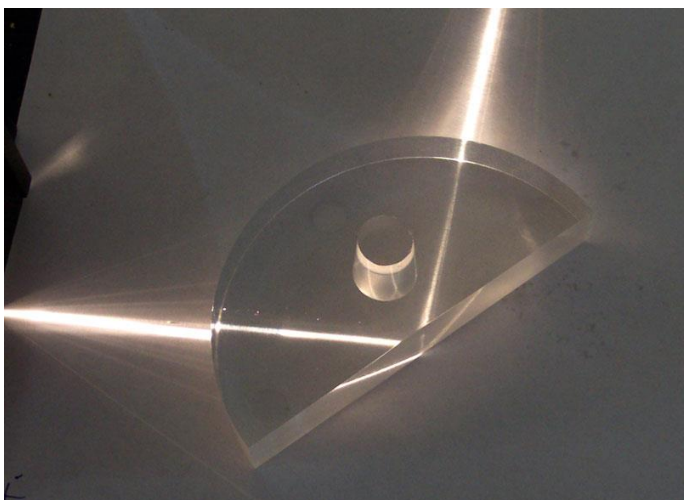


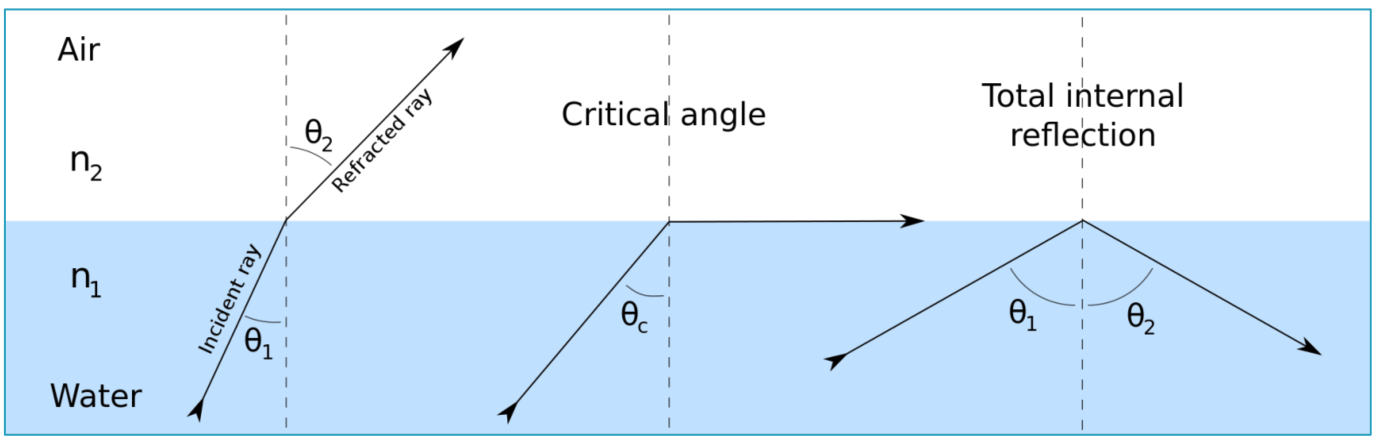
Image: wikipedia

Critical angle:

$$\theta_{\mathrm{i,c}} = \arcsin\left(\frac{\eta_{\mathrm{o}}}{\eta_{\mathrm{i}}}\right)$$

Snell’s window

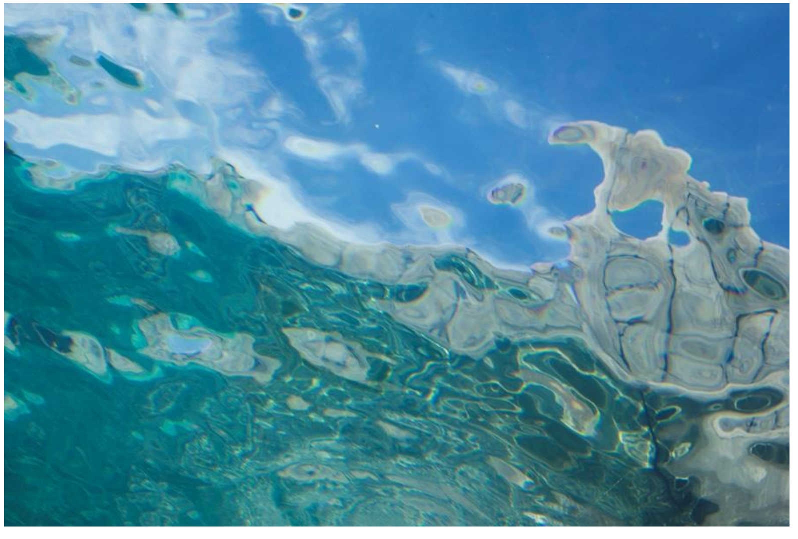
View straight up from underwater. The above-water hemisphere is visible, compressed (as by a circular fisheye lens) into a circle (Snell's window) bounded by the critical angle. Everything outside the critical-angle circle is reflected from below the water.



https://en.wikipedia.org/wiki/Snell%27s_window

Snell’s window

A diver viewed from below who appears inside of Snell's window.



The edge of Snell's window, in this case the boundary between reflected bottom (teal) and refracted sky and above-water structures (blue and gray)

https://en.wikipedia.org/wiki/Snell%27s_window

Ideal refraction

- **Change of radiance**
  - Follows from the conservation of energy (flux)
  - When going from an optically rarer to a denser medium, light energy gets “compressed” in directions => higher energy density => higher radiance

$$L_o = L_i \frac{\eta_o^2}{\eta_i^2}$$

BRDF of ideal refraction

- BRDF of the ideal refraction is a delta distribution:

Change of radiance

Fresnel transmittance

Snell's law

$$f_t(\theta_\mathrm{i},\varphi_\mathrm{i};\theta_\mathrm{o},\varphi_\mathrm{o}) = \frac{\eta_\mathrm{o}^2}{\eta_\mathrm{i}^2}(1-R(\theta_\mathrm{i}))\,\frac{\delta(\eta_\mathrm{i}\sin\theta_\mathrm{i}-\eta_\mathrm{o}\sin\theta_\mathrm{o})\,\delta(\varphi_\mathrm{i}-\varphi_\mathrm{o}\pm\pi)}{\cos\theta_\mathrm{i}}$$

Refracted ray stays in the incidence plane

Fresnel equations

Fresnel equations

- Read [frenel]
- Ratio of the transmitted and reflected light depends on the incident direction
  - From above – more transmission
  - From the side – more reflection
- Extremely important for realistic rendering of glass, water and other smooth dielectrics
- Not to be confused with Fresnel lenses (used in lighthouses)

Fresnel equations

<u>From the side</u>
- little transmission
- more reflection

Try for yourself!!!

<u>From above</u>
- little reflection
- more transmission

Fresnel equations

- **Dielectrics**

$$R_s = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2$$

$$R_p = \left| \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \right|^2$$

$$R = \frac{R_s + R_p}{2}$$

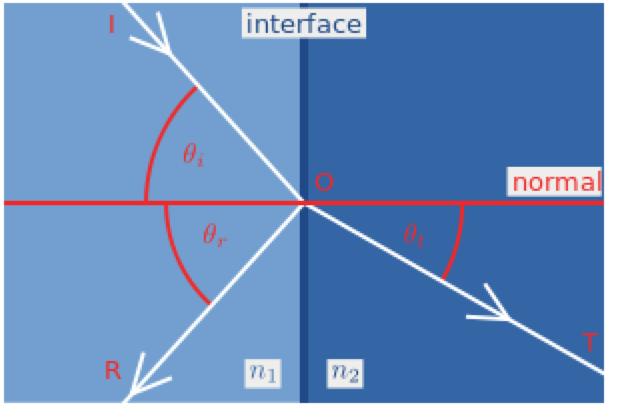
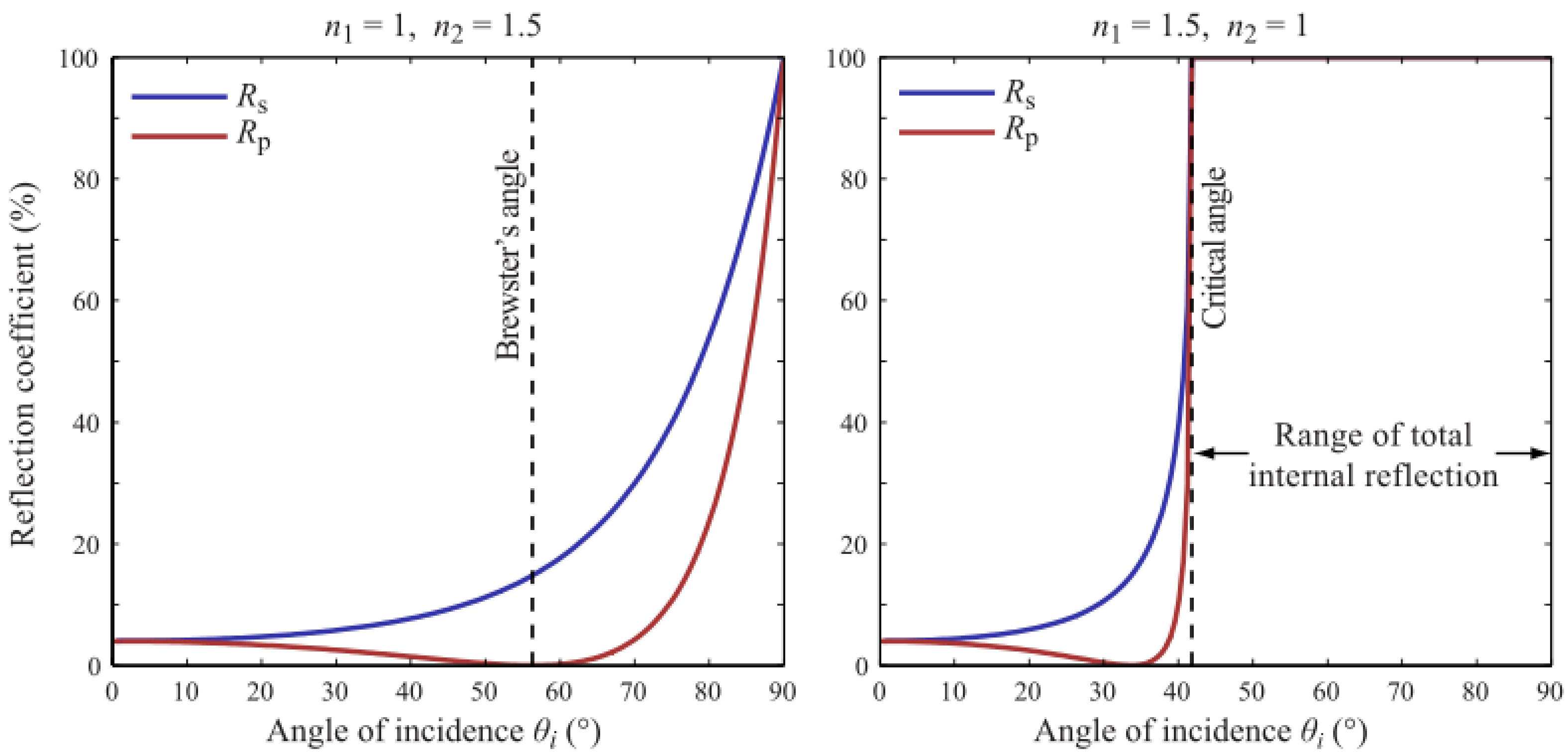


Image: Wikipedia

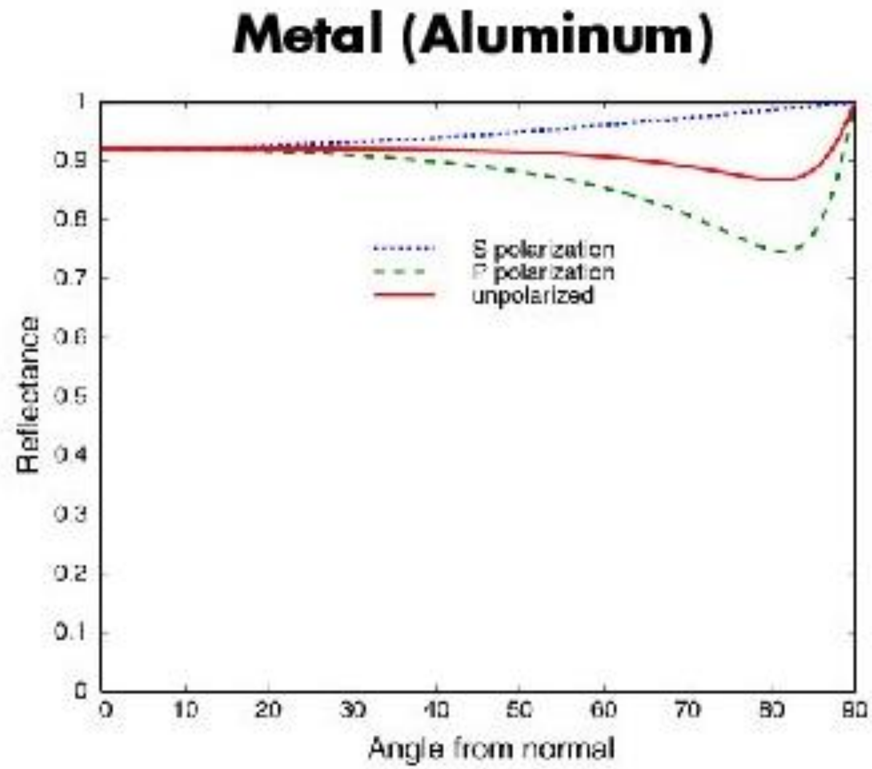
Fresnel equations

- **Dielectrics**



Fresnel equations

- **Metals**



Gold F(0)=0.82
Silver F(0)=0.95

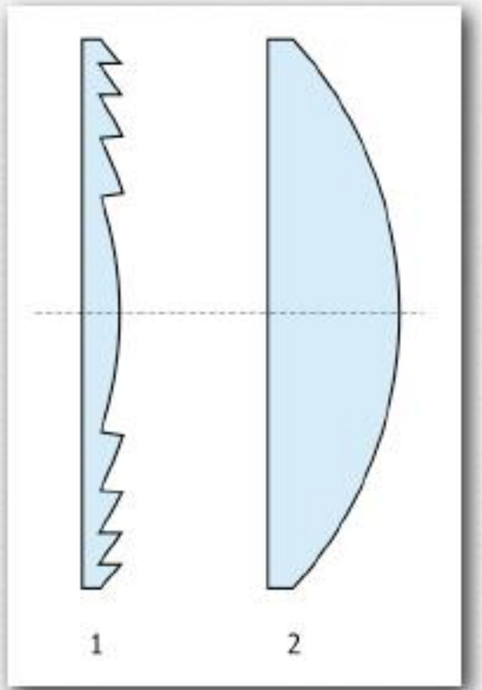
More on Fresnel equations in graphics

- https://seblagarde.wordpress.com/2013/04/29/memo-on-fresnel-equations/

Diggression

Fresnel Lens

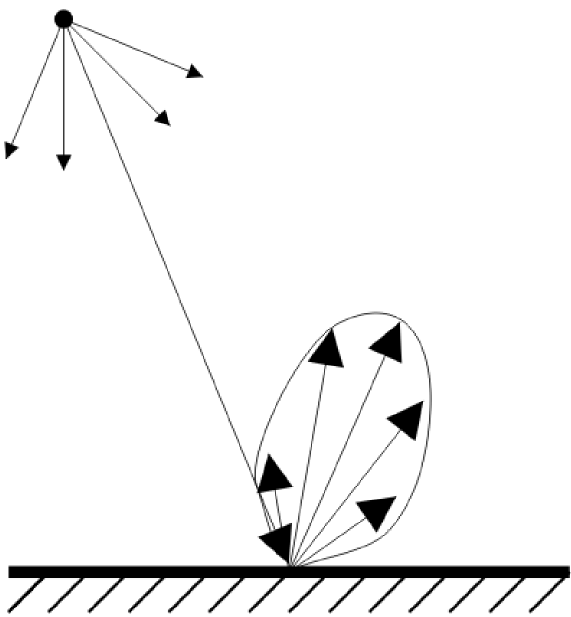
- same refractive power (focal length) as a much thicker lens
- good for focusing light, but not for making images



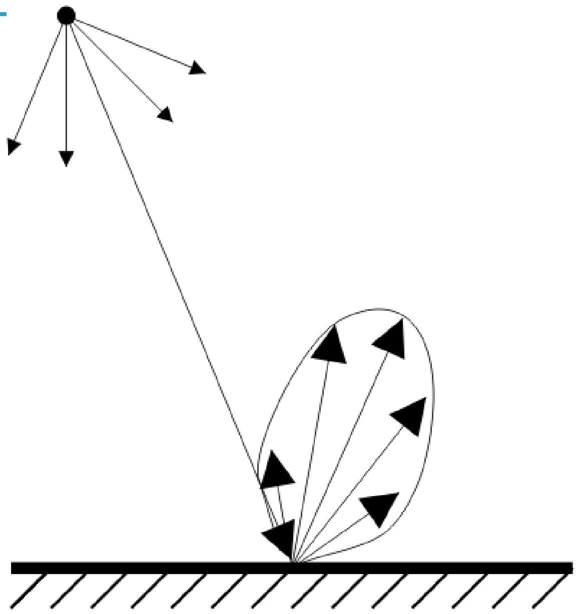
(wikipedia)

Tyler Westcott, Pigeon Point Lighthouse in light fog,
photographed during the annual relighting of its historical 1KW lantern, 2008
(Nikon D40, 30 seconds, ISO 200, not Photoshopped)

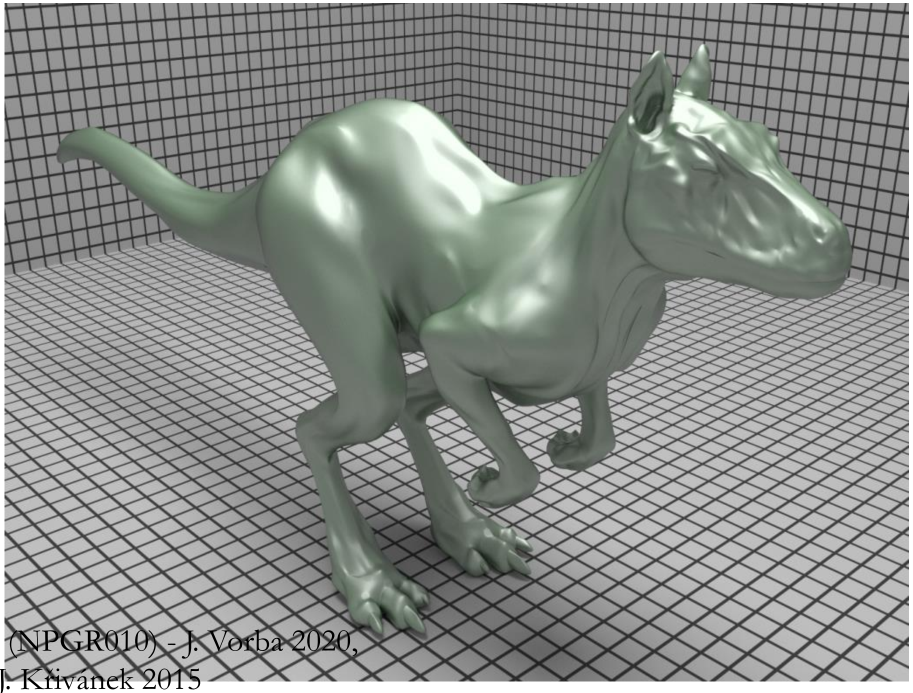
Glossy reflection



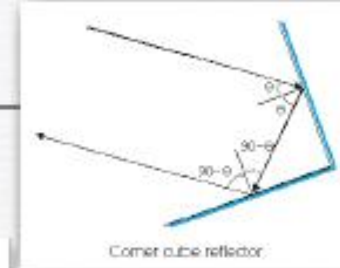
Glossy reflection

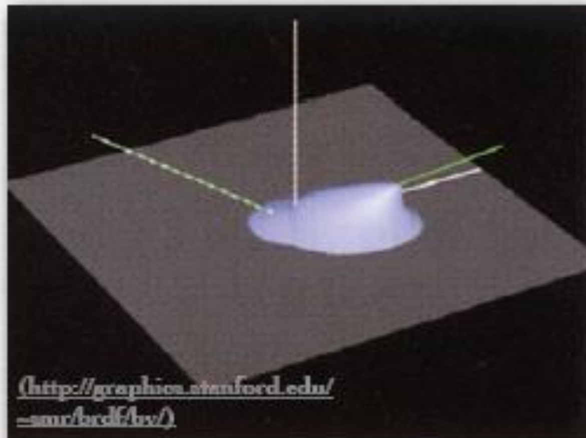


- Neither ideal diffuse nor ideal mirror
- All real materials in fact fall in this category



What unusual material property does this goniometric diagram depict?



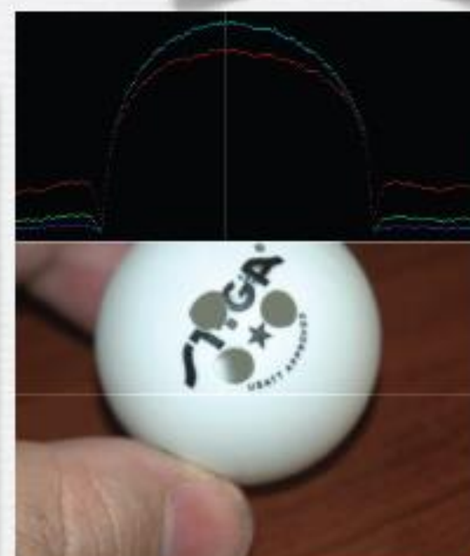

(http://graphics.stanford.edu/~smr/brdf/bv/)

- A. retroreflectivity
- the maria of the moon is retroreflective and gray
- a diffuse object, lit from the camera's viewpoint, falls off as cos θ

a full moon is roughly lit from the camera's viewpoint


(NASA)

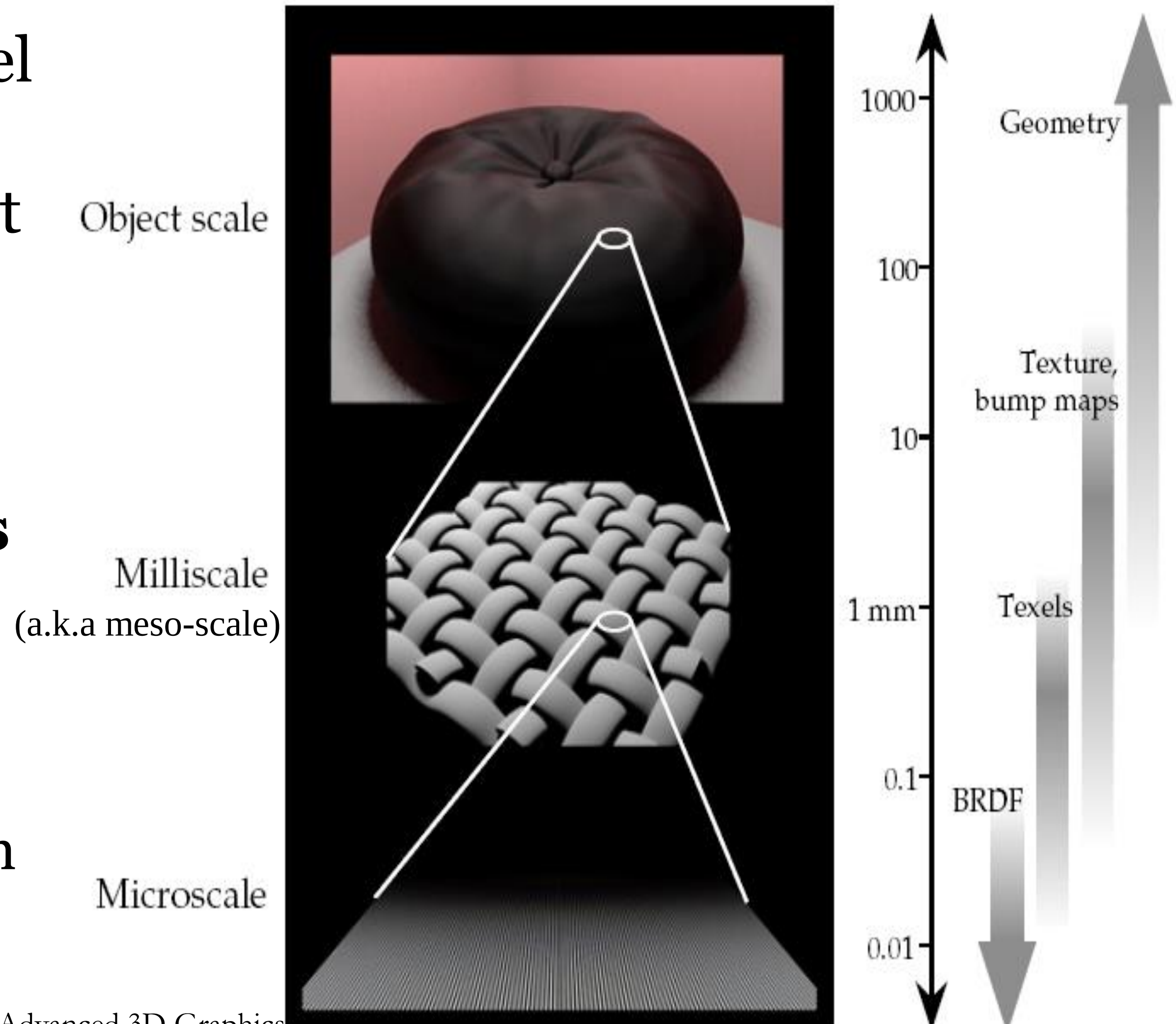
so is a flash photograph



BRDF models

BRDF modeling

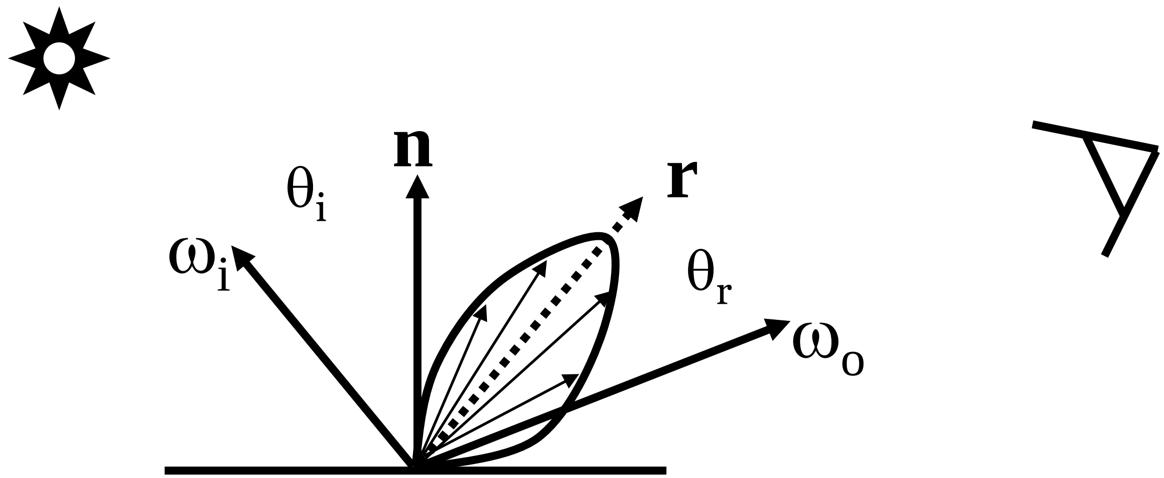
- BRDF is a model of the bulk behavior of light when viewing a surface from distance
- **BRDF models**
  - Empirical
  - Physically based
  - Approximation of measured data



Empirical BRDF models

- An arbitrary formula that takes ω_i and ω_o as arguments
- ω_i and ω_o are sometimes denoted L (**L**ight direction) a V (**V**iewing direction)
- Example: Phong model
- Arbitrary shading calculations (shaders)

BRDF corresponding to the original Phong shading model



$$f_r^{Phong\ Orig} = k_d + k_s \frac{\cos^n \theta_r}{\cos \theta_i}$$

Problems: breaks symmetry & energy conservation

Physically-plausible Phong BRDF

- Modification to ensure reciprocity (symmetry) and energy conservation

$$f_r^{\text{Phong modif}} = \frac{\rho_d}{\pi} + \frac{n+2}{2\pi}\rho_s \cos^n \theta_r$$

- Energy conserved when

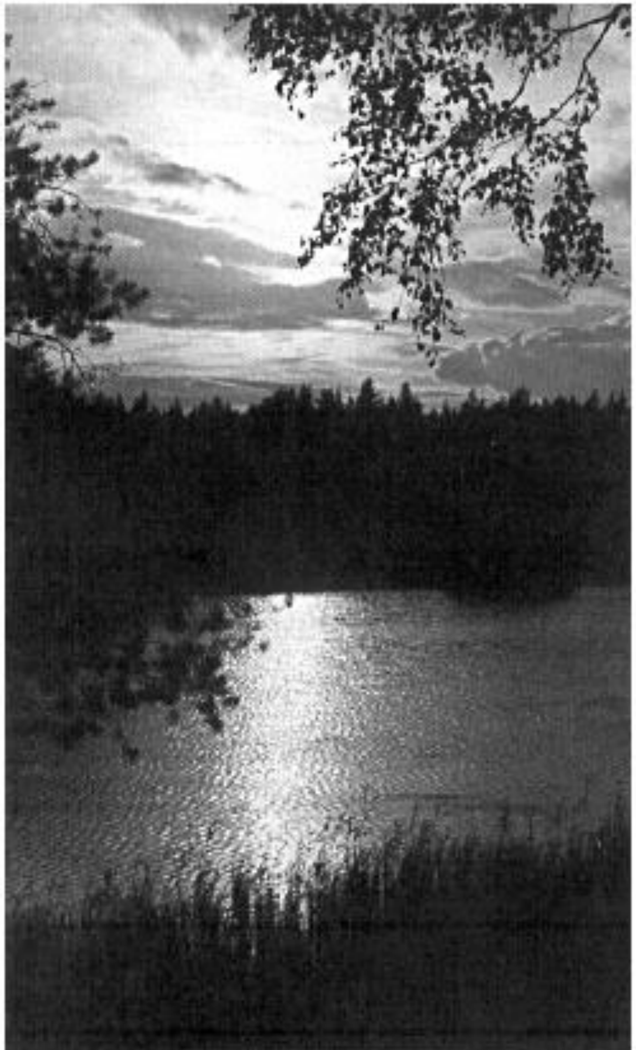
$$\rho_d + \rho_s \leq 1$$

- It is still an empirical formula (i.e. it does not follow from physical considerations), but at least it fulfills the basic properties of a BRDF

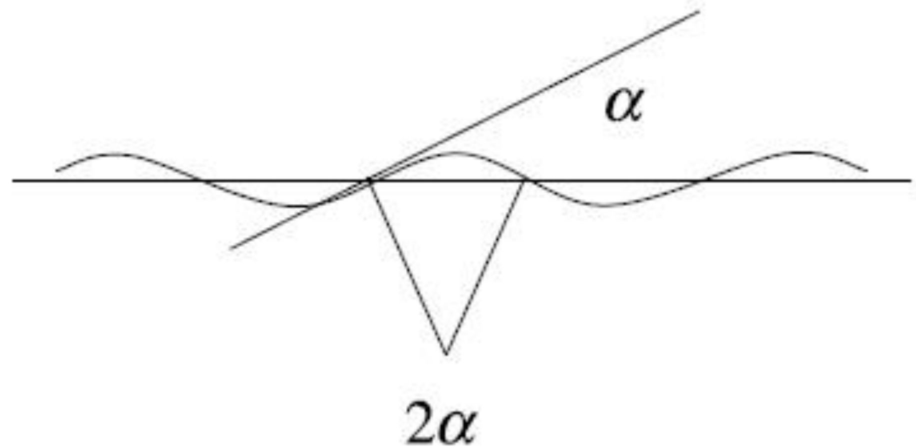
Physically-plausible BRDF models

- E.g. Torrance-Sparrow / Cook-Torrance model
- Based on the **microfacet theory**

Reflection of the Sun from the Sea



Minnaert, *Light and Color in the Outdoors*, p. 28

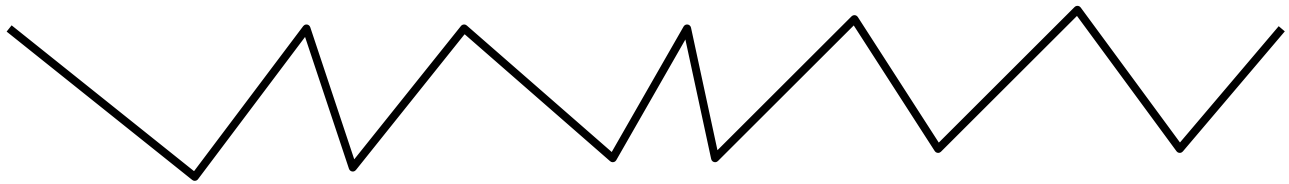


Microfacet BRDF

- Analytically derived
- Used for modeling rough surfaces (as the Phong model)
  - Corresponds more closely to reality than Phong
  - Derived from a physical model of the surface microgeometry (as opposed to “because it looks good”-approach used for the Phong model)

Microfacet BRDF

- Assumes that the macrosurface consists of randomly oriented microfacets



- We assume that each microfacet behaves as an ideal mirror.
- We consider 3 phenomena:

Microfacet BRDF

Microfacet theory [Cook et Torrance 1982]

A perfect mirror

- Reflection in a single direction
- Outgoing light visible surface normal aligned with the half vector
- Half Vector: $H = \frac{L+V}{\|L+V\|}$

Aggregation of micro-mirrors (micro-facets)

- Each micro-mirror have a micro-normal
- How many micro-mirror have their micro-normal aligned so that $H = N$?
- Statistical distribution: Normal Distribution Function (NDF)

Microfacet BRDF

Fresnel term

Geometry term
Models shadowing and masking

Microfacet distribution

$$f = \frac{F(\theta_i)G(\omega_i, \omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

Part of the macroscopic surface visible by the light source

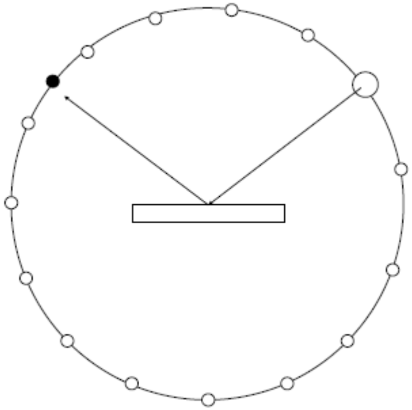
Part of the macroscopic surface visible by the viewer

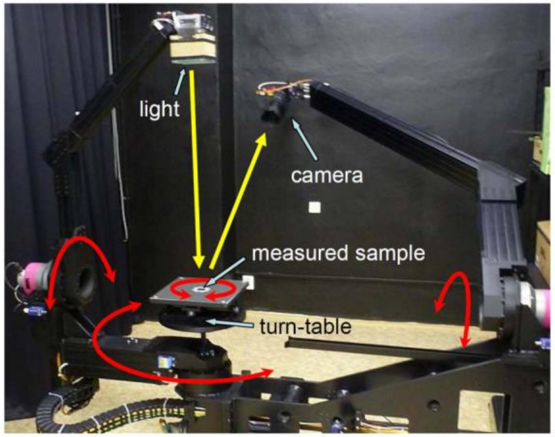
Approximation of measured data

- We can fit any BRDF model to the data
- Some BRDF models have been specifically designed for the purpose of fitting measured data, e.g. Ward BRDF, Lafortune BRDF
- **Nonlinear optimization** required to find the BRDF parameters

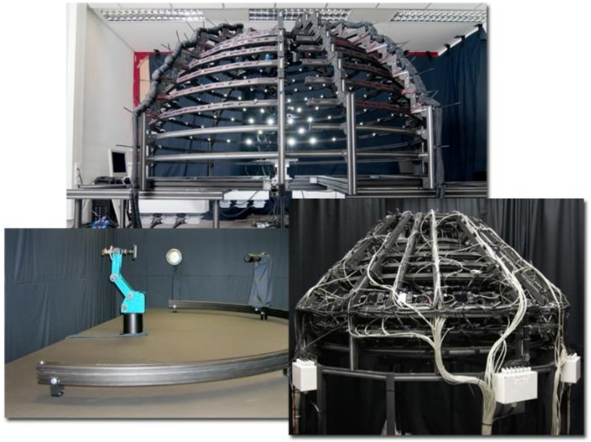
BRDF measurements – Gonio-reflectometer

Stanford





UTIA



University of Bonn

BRDF measurements – Gonio-reflectometer

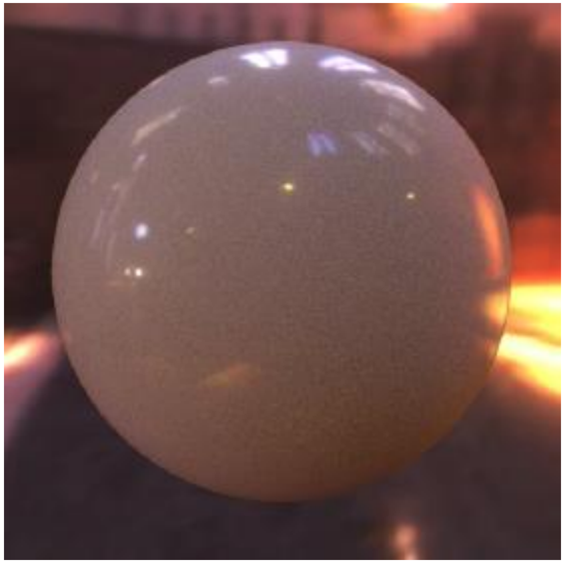
- Realistic graphics lab at EPFL
  - Probably the most advanced setup as of today
  - http://rgl.epfl.ch/pages/lab/pgII
- In Prague, CZ
  - The UTIA BTF database
    - http://btf.utia.cas.cz/
  - Czech Technical University, prof. Havran
    - https://dcgi.fel.cvut.cz/publications/2017/havran-sensors-lightdrum

Measured Material

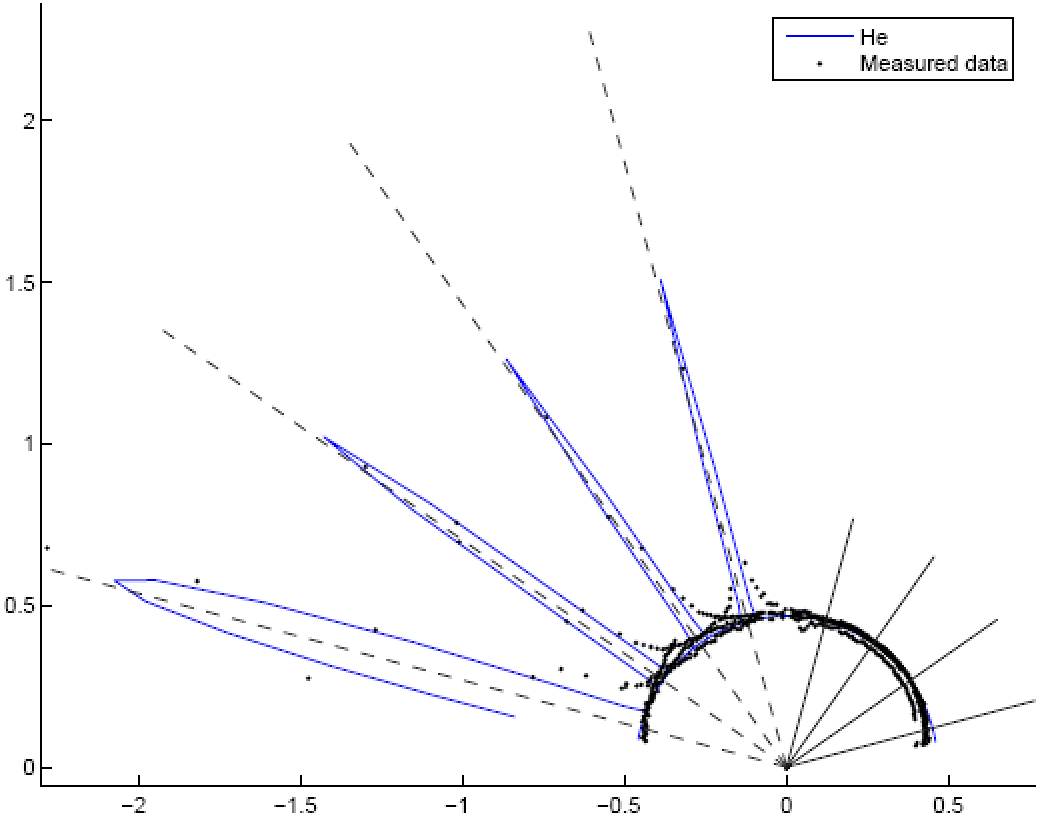
- Techniques for speeding measurements
  - Mirrors
  - Objects coated by the material:
    - Sphere [Matusik et al 2003]
    - Cylinders [Ngan et al 2005]

[Matusik et al 2003]

Surface appearance and the BRDF



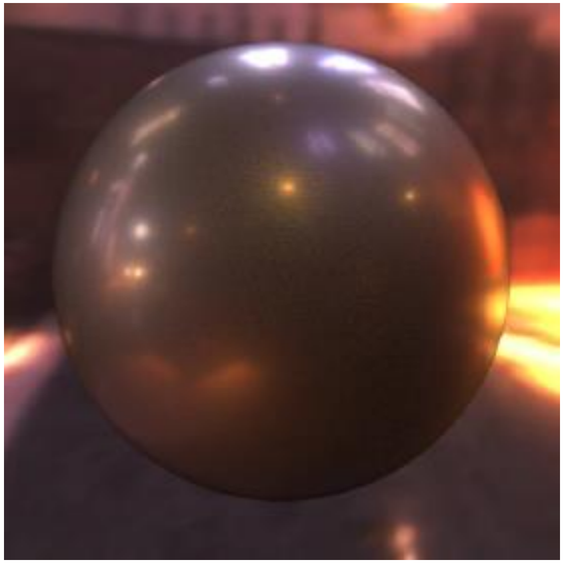
Appearance



BRDF lobe
(for four different viewing directions)

Souce: Ngan et al. Experimental analysis of BRDF models, http://people.csail.mit.edu/addy/research/brdf/

Surface appearance and the BRDF



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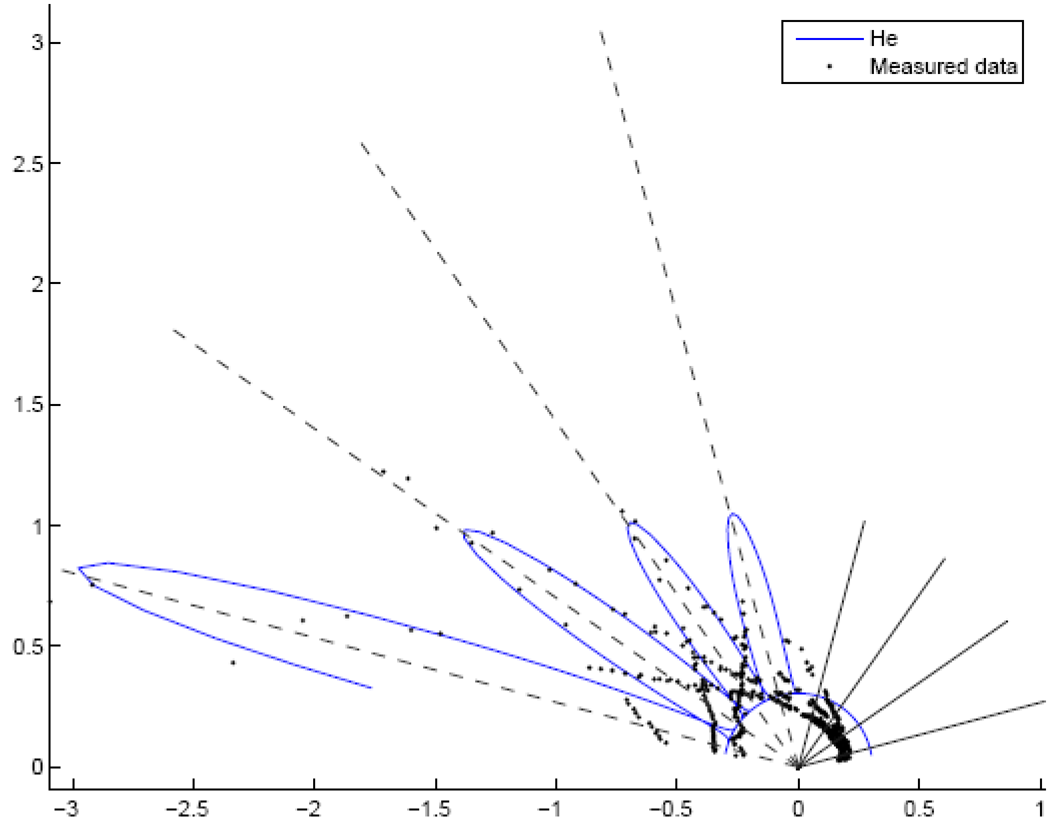

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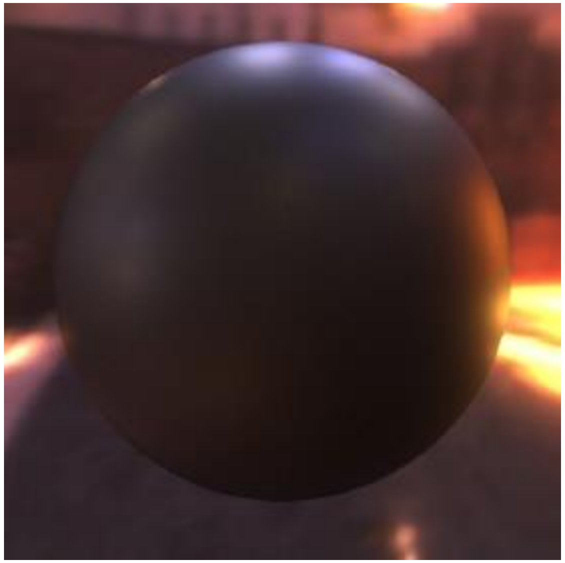

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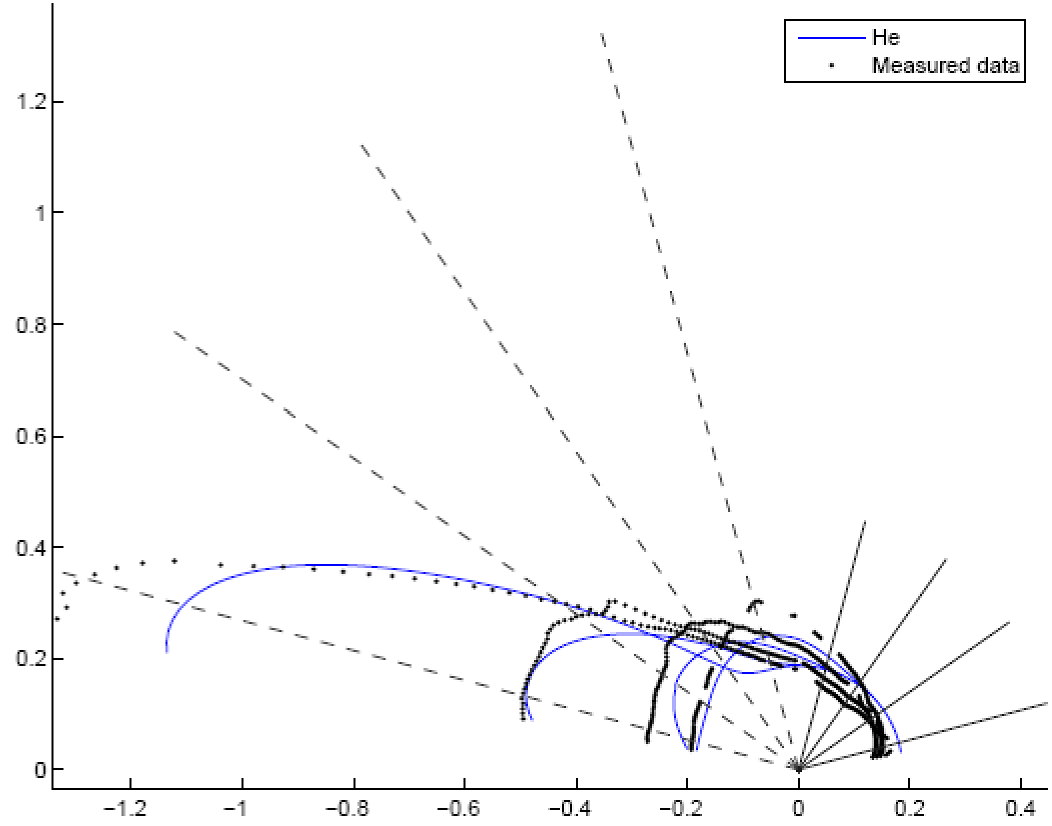

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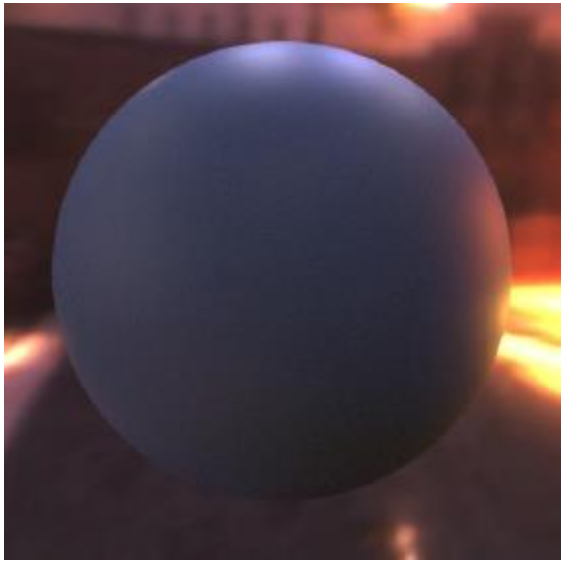
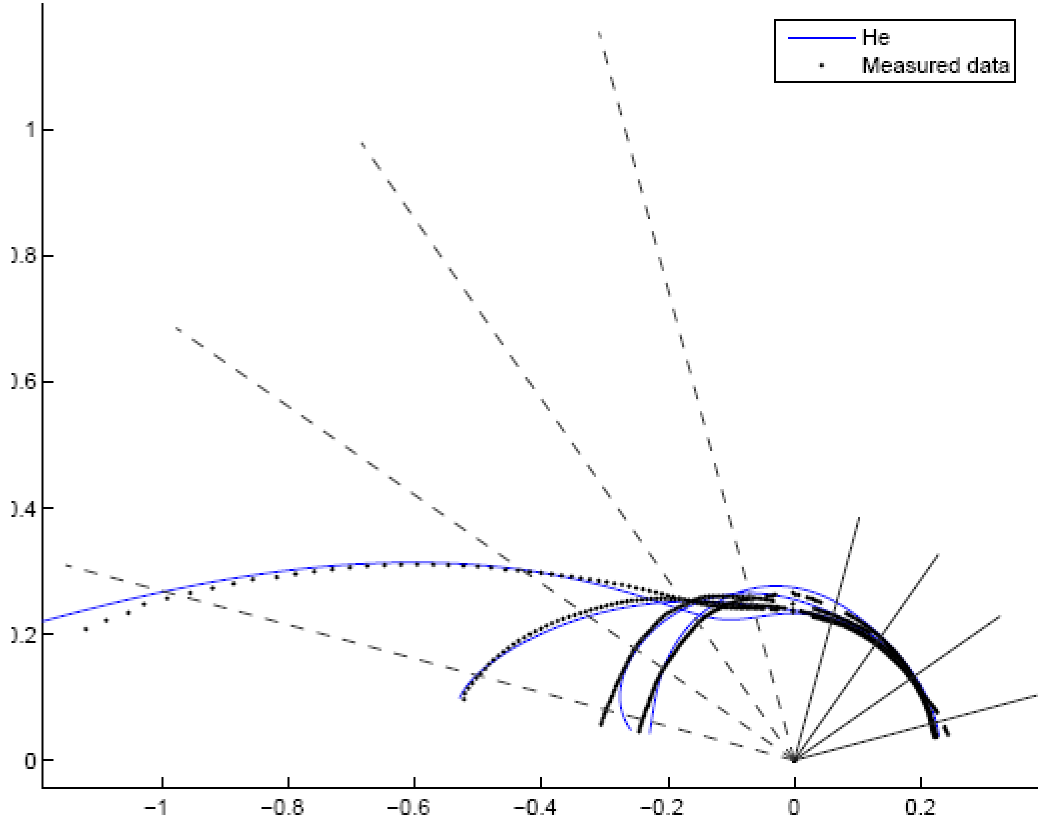


Appearance

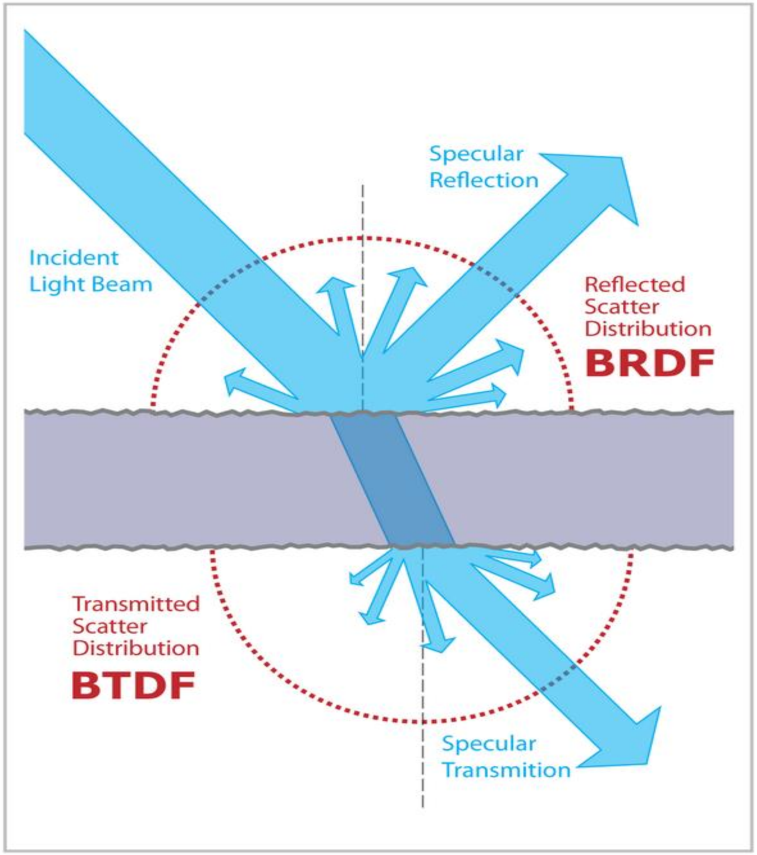


BRDF lobe
(for four different viewing directions)

Souce: Ngan et al. Experimental analysis of BRDF models, http://people.csail.mit.edu/addy/research/brdf/

BRDF, BTDF, BSDF: What's up with all these abbreviations?

- **BTDF**
  - Bidirectional **transmittance** distribution function
  - Described light transmission
- **BSDF** = BRDF+BTDF
  - Bidirectional **scattering** distribution function

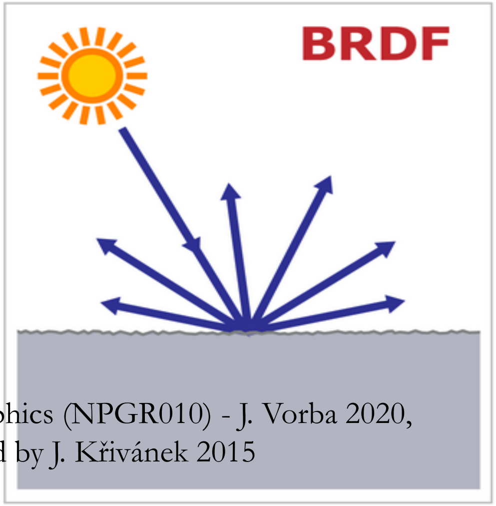


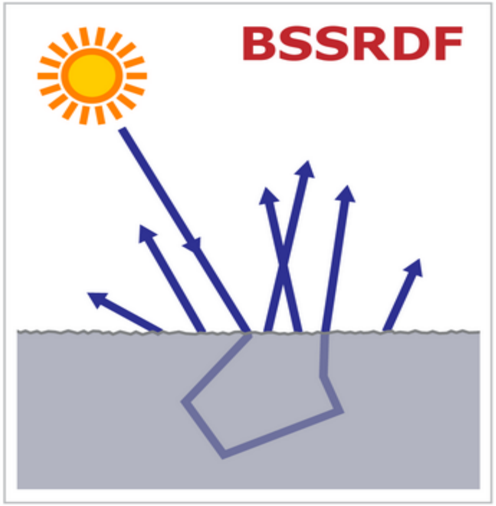
SBRDF, BTF

- **SV-BRDF … Spatially Varying BRDF**
  - BRDF parameters are spatially varying (can be given by a surface texture)
- **BTF … Bidirectional Texture Function**
  - Used for materials with complex structure
  - As opposed to the BRDF, models even the meso-scale

BSSRDF

- BRDF
  - Light arriving at a point is reflected/transmitted at the same point
  - No subsurface scattering considered
- **BSSRDF**
  - Bi-directional **surface scattering** reflectance distribution function
  - Takes into account scattering of light under the surface





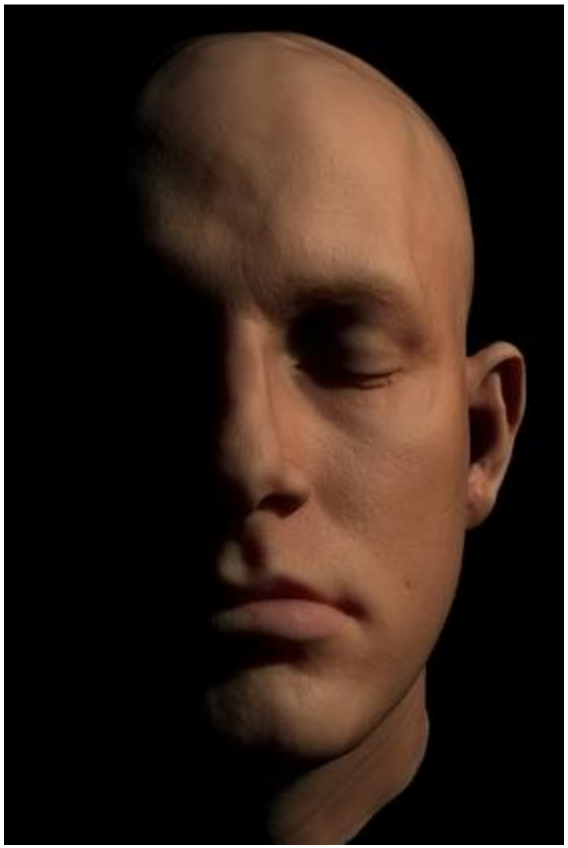
BSSRDF

- Sub-surface scattering makes surfaces looks “softer”

BRDF

BSSRDF

BSSRDF



BRDF BSSRDF

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