matfyz

Computer Graphics Charles University

Image warping – deformations

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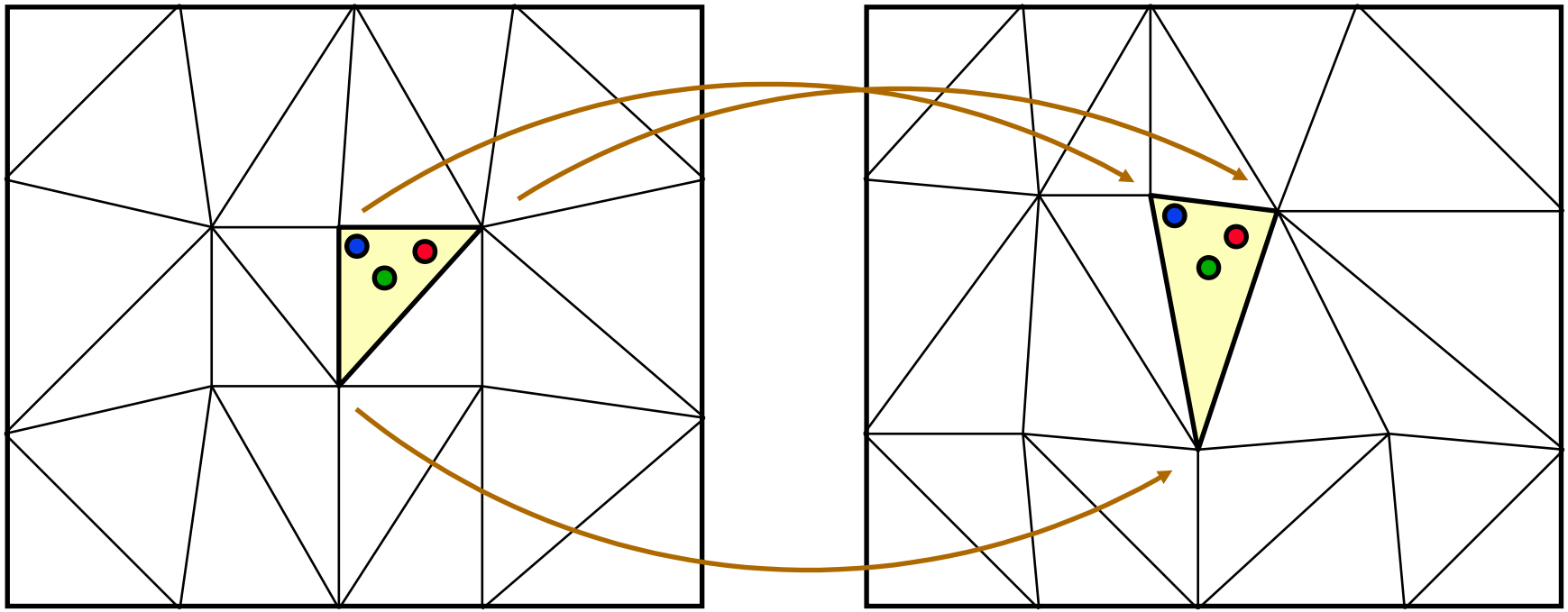
Deformation mapping

- **explicit, analytic function**
  - 3D graphics: texture mapping (surface of a solid), perspective distortion..
  - global deformation function
- **free-form functions**
  - interactive (user, GUI)
  - image space covered by function elements / handles / control points
  - change of one element has only local impact

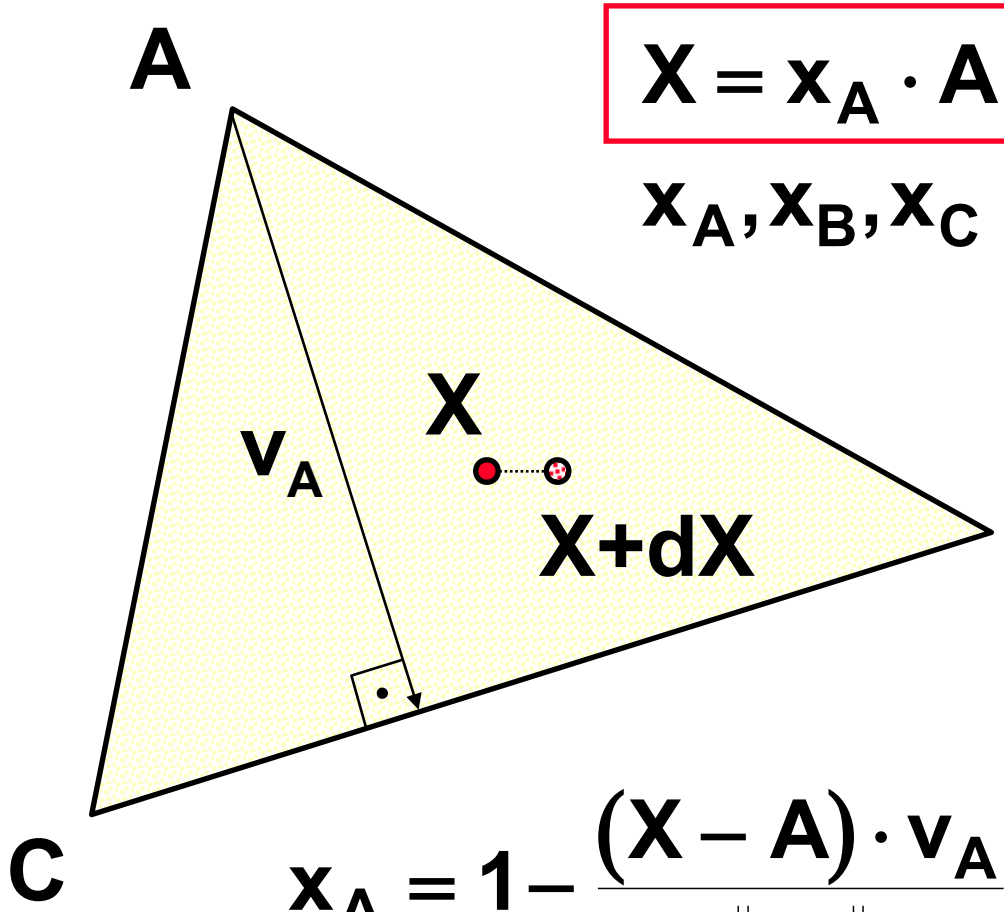
Triangle mesh



source **target**

Identical mesh topology!

Barycentric coordinates



$$X = x_A \cdot A + x_B \cdot B + x_C \cdot C$$

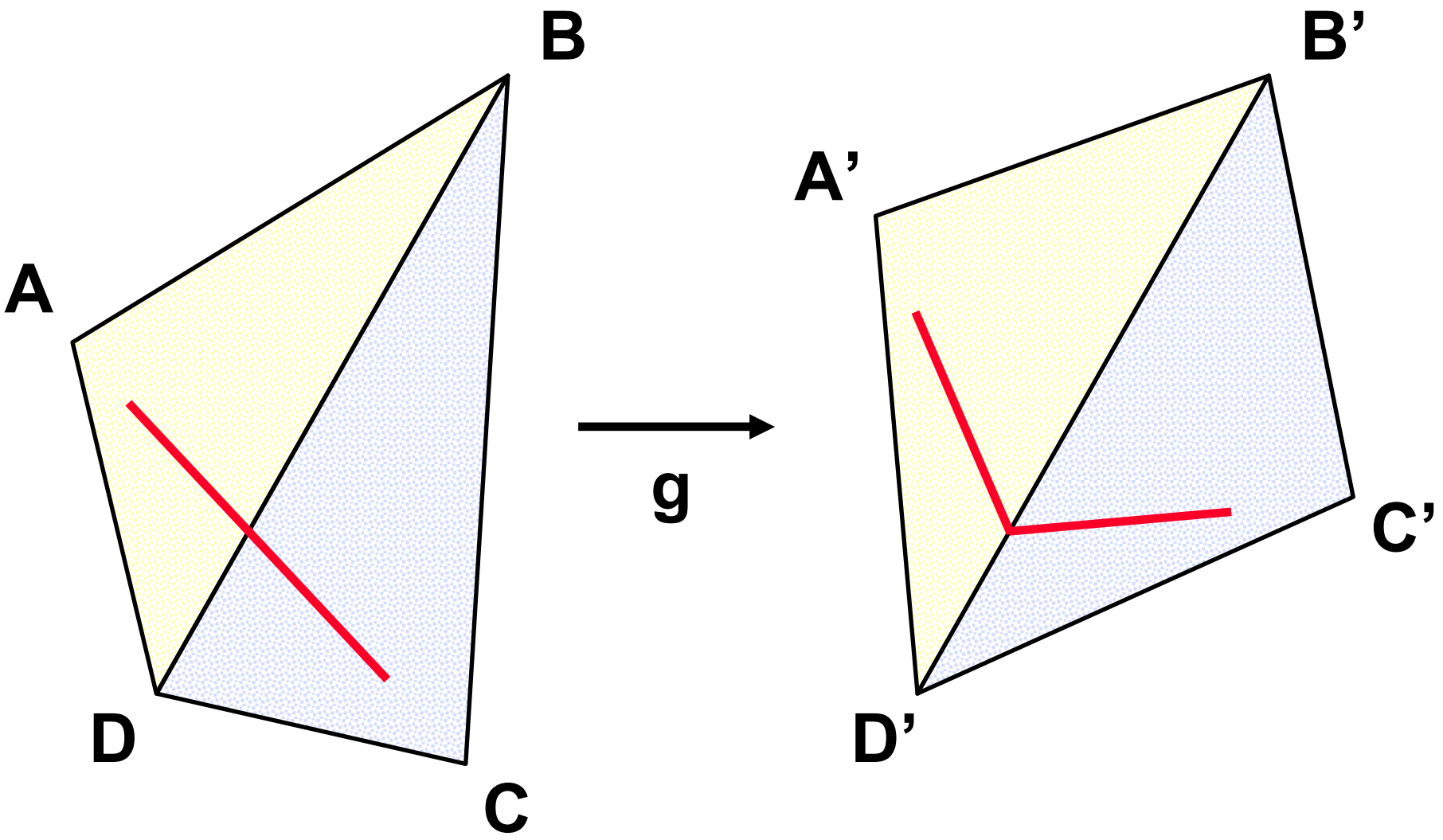
$$x_A, x_B, x_C > 0 \qquad x_A + x_B + x_C = 1$$

$$x_A = 1 - \frac{(X - A) \cdot v_A}{\|v_A\|} = \left(1 + \frac{A \cdot v_A}{\|v_A\|}\right) - \frac{X \cdot v_A}{\|v_A\|}$$

Algorithm

- **scanline pass** through **target image**
  - barycentric coordinates are linear inside a triangle (finite difference method)
- list of **sorted triangle edges**
  - crossing the edge .. changing triangles
- **next scanline**
  - delete ending edges
  - update continued edges
  - insert starting edges (consequent on ending ones)

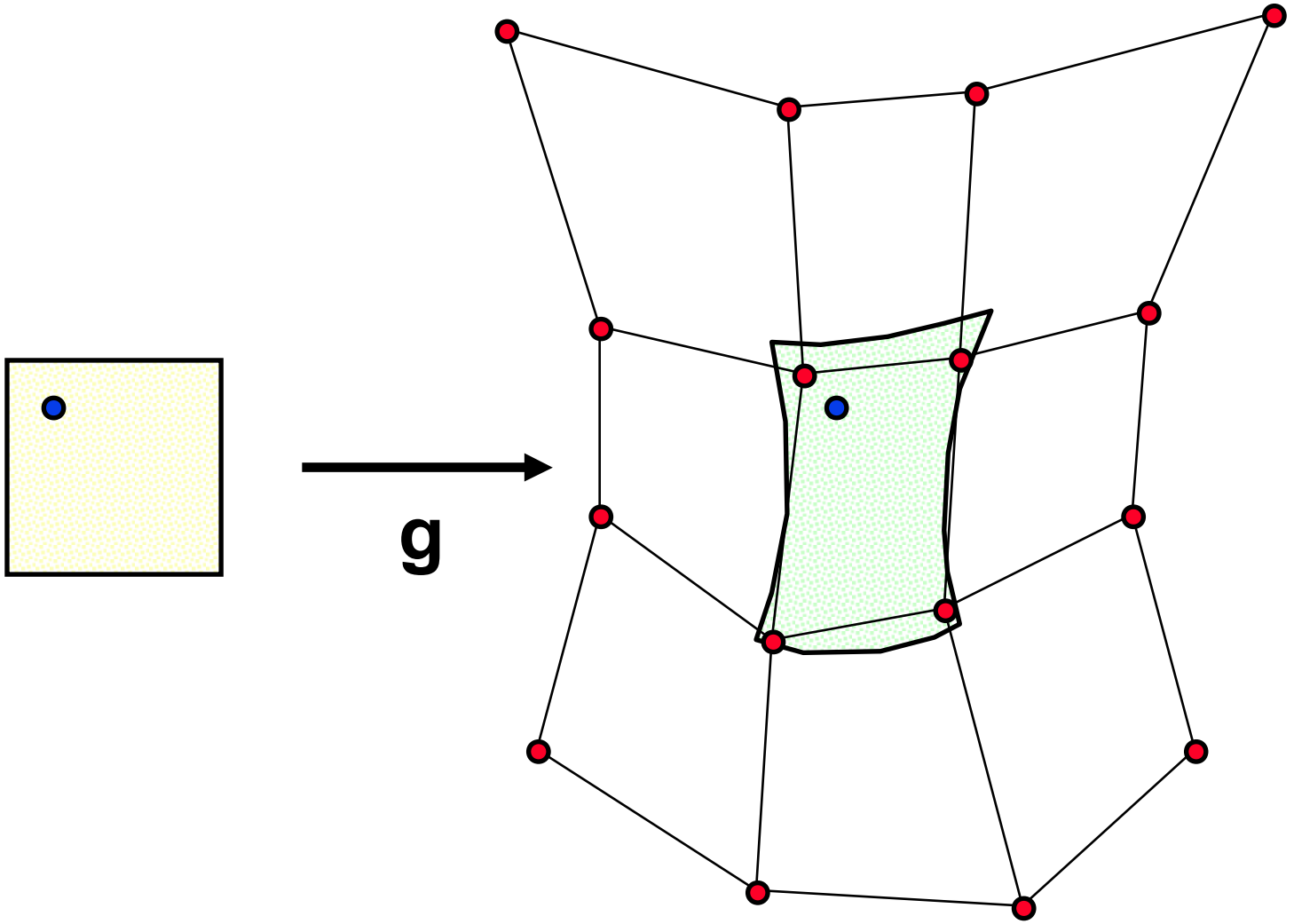
Discontinuity (1st order)



Higher-order approximations

- **quadratic** or **cubic** interpolation
  - continuity: 1st and 2nd derivatives (C^1, C^2)
- **triangle** or **quad** topology
  - need to know adjacent cells
- **inverse mapping** can be tricky
  - alternative approach ($\mathbf{g^{-1} = g_1 \circ g_2^{-1}}$)

Bicubic B-spline approximation



Bicubic B-spline approximation

$$g(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} C_i(u)\,C_j(v)\,P_{ij}$$

B-spline blending functions:

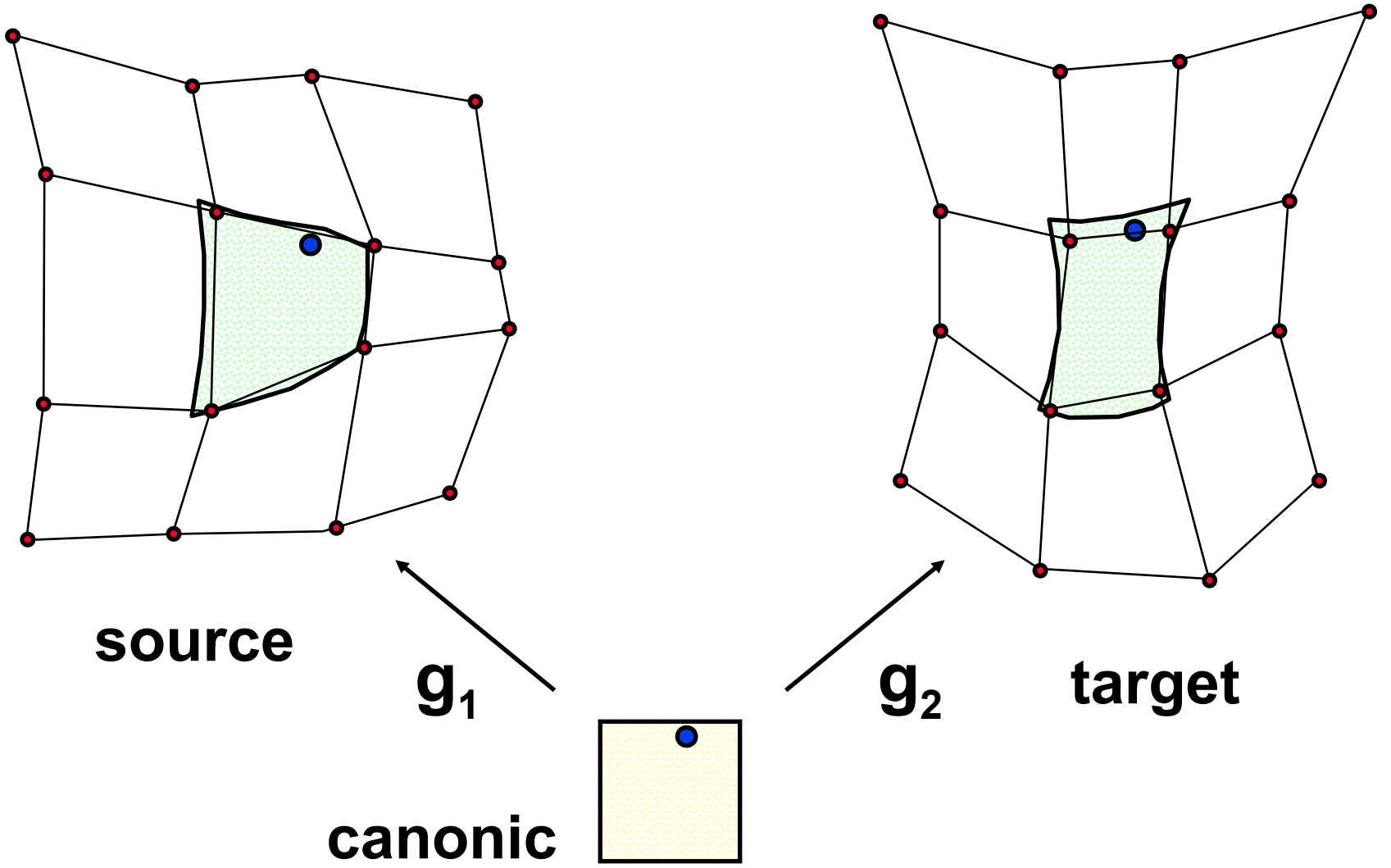
$$C_0(t) = \frac{1}{6}(1-t)^3$$

$$C_1(t) = \frac{1}{6}\left(3t^3 - 6t^2 + 4\right)$$

$$C_2(t) = \frac{1}{6}\left(-3t^3 + 3t^2 + 3t + 1\right)$$

$$C_3(t) = \frac{1}{6}t^3$$

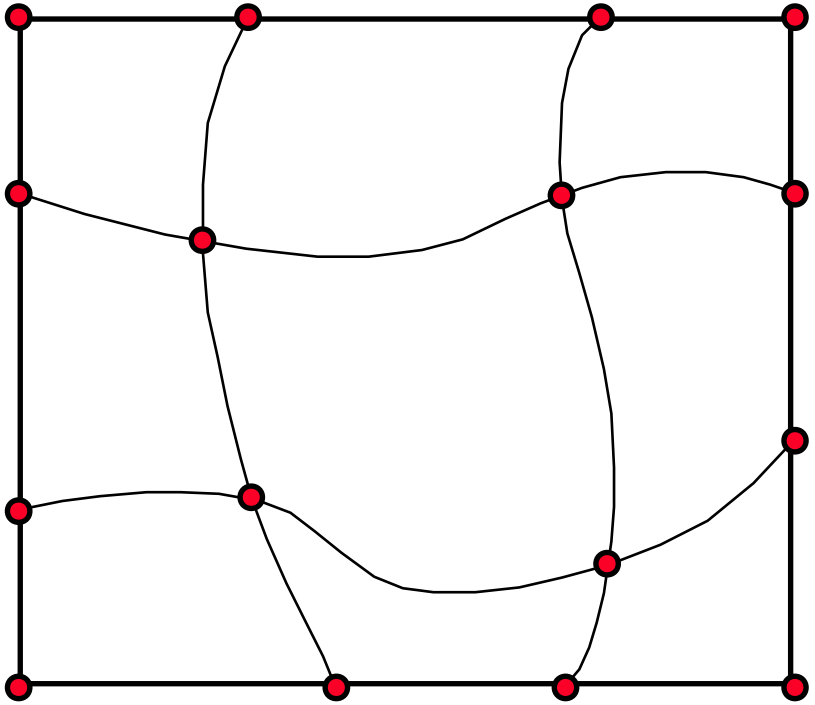
Both nets are editable



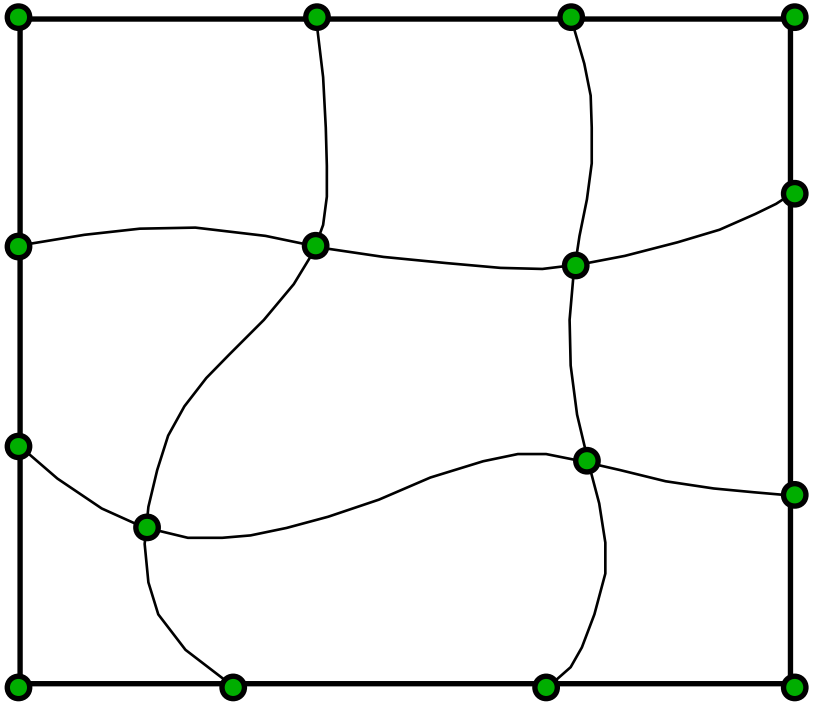
Dual-net algorithm

- only **forward mappings g_i** can be implemented efficiently (finite differences)
  - integer algorithm uses only addition and bit shifts in its inner loop
- concurrent **synchronous comput. of g_1 and g_2**
  - coverage-condition (every pixel of the target image has to be covered)
- **source pixels** are transferred using $g_2 \circ g_1^{-1}$
  - or opposite direction in case of filtering

Spline net



source

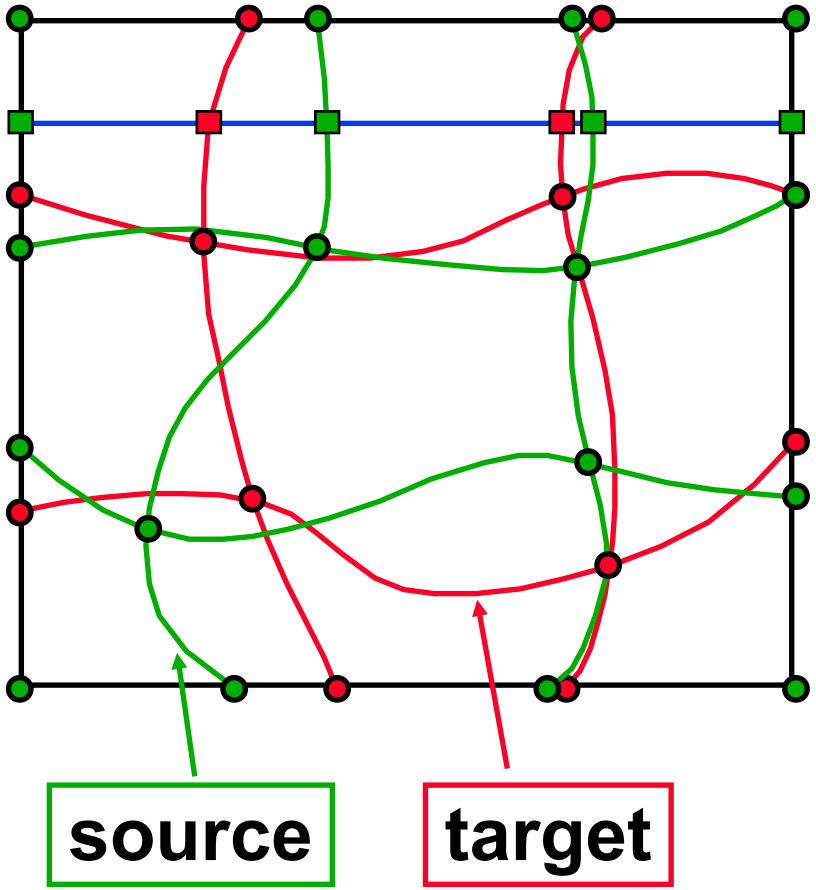


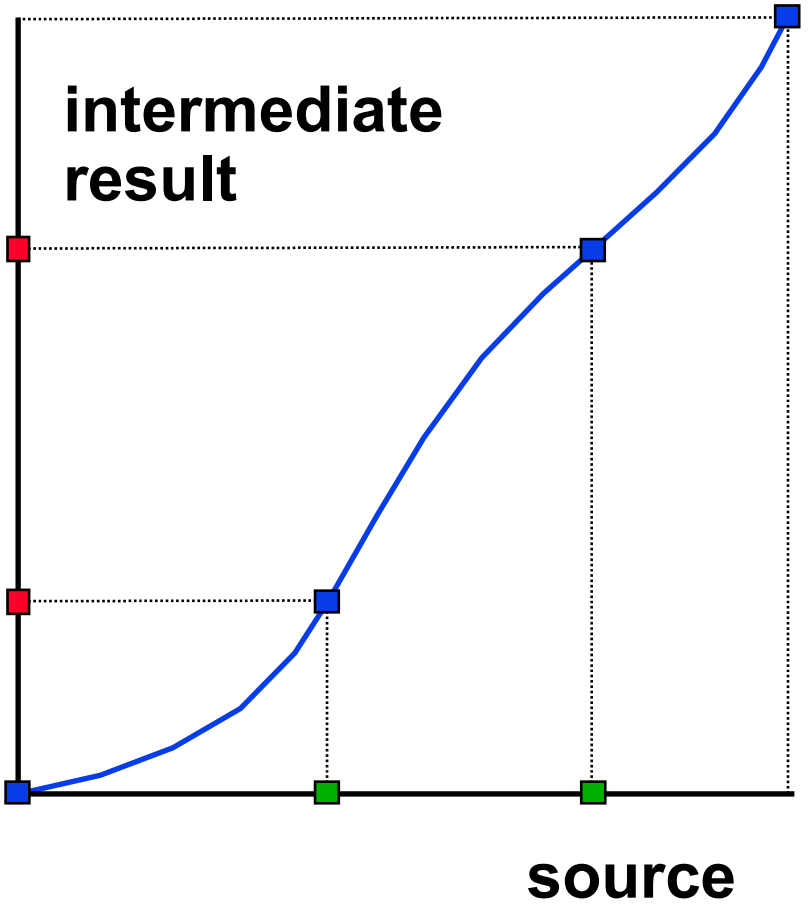
target

Two-pass algorithm

- **horizontal deformation** (each pixel row)
  - vertical spline curves intersect scanline ... control points for spline interpolation
  - horizontal splines has to be deformed as well
- **vertical deformation** (each pixel column)
  - defined by intersections of modified horizontal spline curves with vertical line

Horizontal phase – example

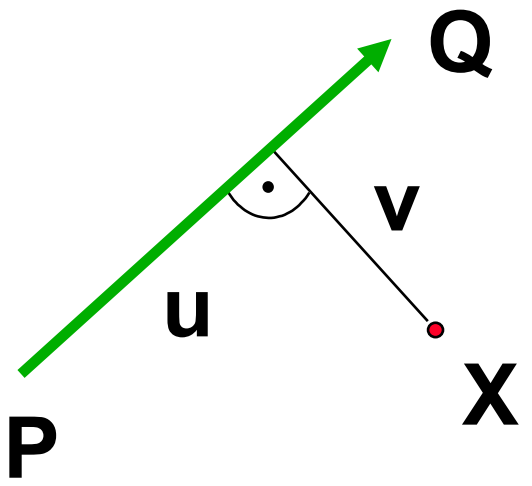




„Feature-based” warping

- **local** deformation function
  - user friendly, intuitive
  - no need to cover constant areas
- matching **pairs of oriented segments** (arrows, „features”)
  - each feature has only limited influence
- additional **numeric parameters**
  - global or feature-specific
  - feature „strength“, influence radius, etc.

One feature … affine transform



$$u = \frac{(X - P) \cdot (Q - P)}{\|Q - P\|^2} \qquad \textbf{(rel)}$$

$$v = \frac{(X - P) \cdot (Q - P)^{\perp}}{\|Q - P\|} \qquad \textbf{(abs)}$$

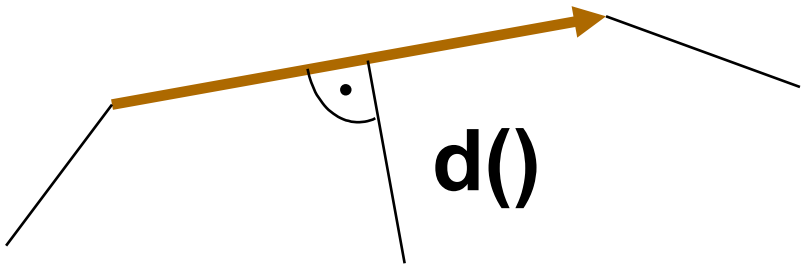
Backward mapping:

$$X' \rightarrow [u', v'] = [u, v] \rightarrow X$$

More features – radial interpolation

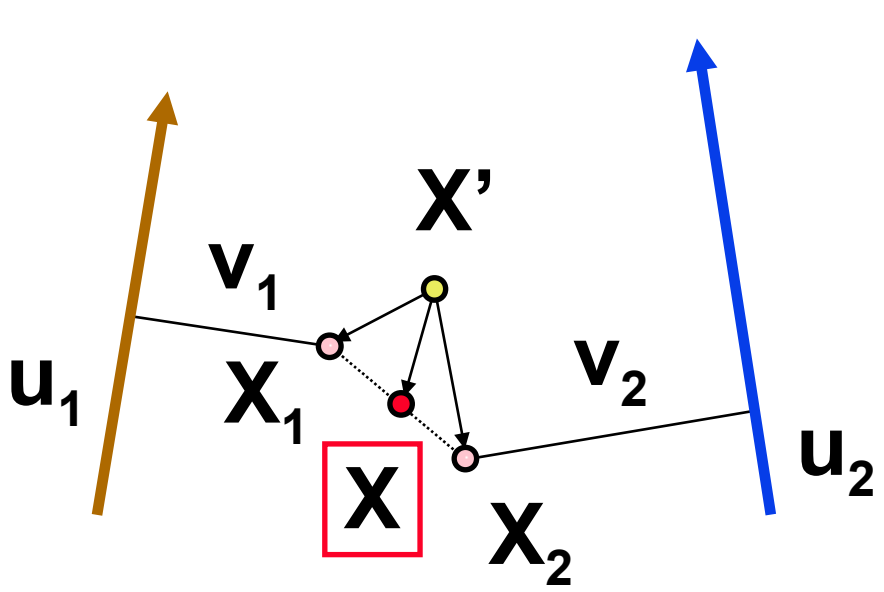
Weighting coefficient: $$w = \left[\frac{(Q' - P')^p}{a + d(P', Q', X')} \right]^b$$

- **a** accuracy on the features
- **b** descend coefficient
- **p** influence of feature-length

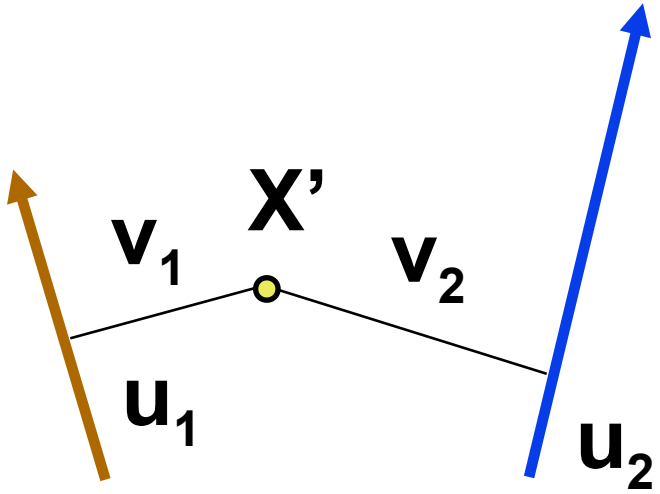


$$X = X' + \frac{\sum w_i \cdot (X_i - X')}{\sum w_i}$$

More features – example



source



target

Feature layout

- source and target features should have **„equal topology“**
  - „image folding” (non-continuous mapping)
- features **should not cross**
- if features must **touch**, it should happen in equivalent points
  - otherwise „ghosts” could appear..

(„Don't cross the streams.“ - „Why?“ - „It would be bad“)

The End

More info:

- **J. Gomes et al.**: ***Warping and Morphing of Graphical Objects***, C.N., SIGGRAPH'95
- **T. Beier, S. Neely**: ***Feature-Based Image Metamorphosis***, SIGGRAPH'92