Radiometry and radiosity

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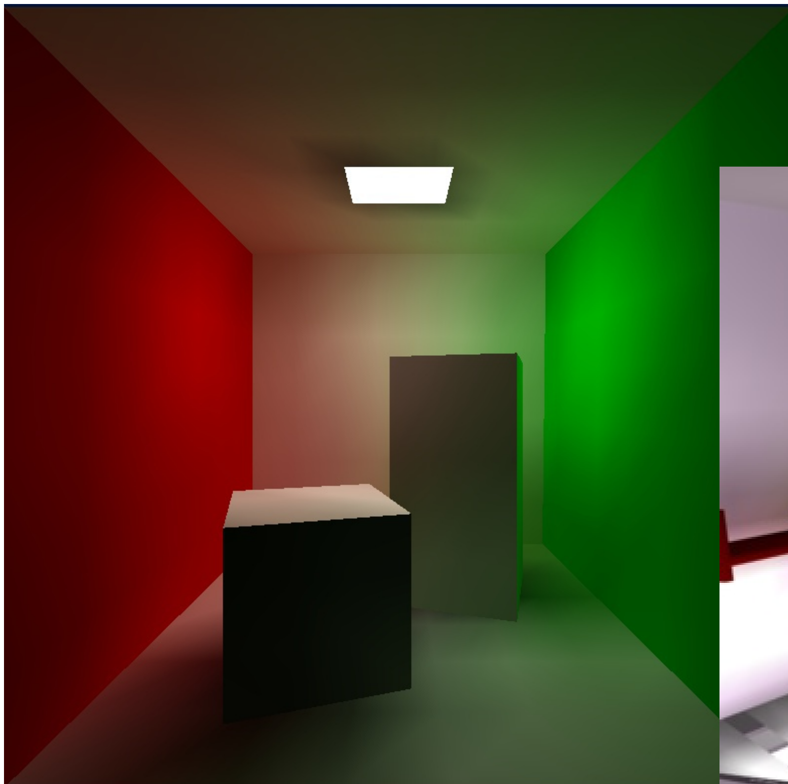
Global illumination, radiosity

- based on **physics**
  - energy transport (light transport) in simulated environment
  - first usage of radiosity in image synthesis: Cindy Goral (SIGGRAPH 1984)
- radiosity is able to compute **diffuse light**, secondary lighting, ..
- basic **radiosity** cannot do sharp reflections, mirrors, ..
- time consuming computation
  - Radiosity: light propagation only, RT: rendering

Radiosity – examples



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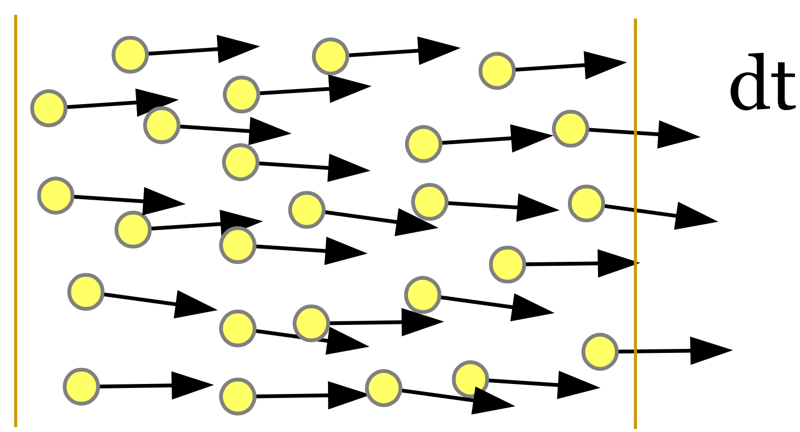
Basic radiometry I

Radiant flux, Radiant power

$$\Phi = \frac{d\,Q}{d\,t} \qquad [\,\mathrm{W}\,]$$

Number of photons (converted to energy) per time unit
(100W bulb: ~10^{19} photons/s, eye pupil from a monitor: 10^{12} p/s)

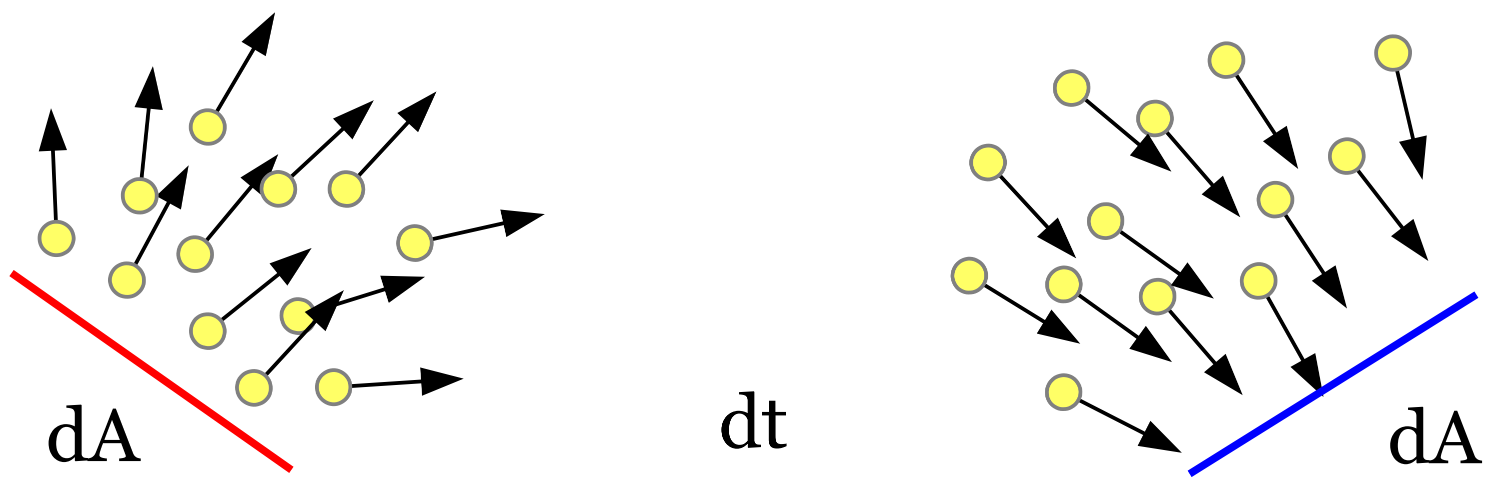


Basic radiometry II

Irradiance, Radiant exitance, Radiosity

$$E(x) = \frac{d\,\Phi(x)}{d\,A(x)} \qquad [\,\mathrm{W/m^2}\,]$$

Photon areal density (converted to energy) incident or radiated per time unit



Basic radiometry III

Radiance

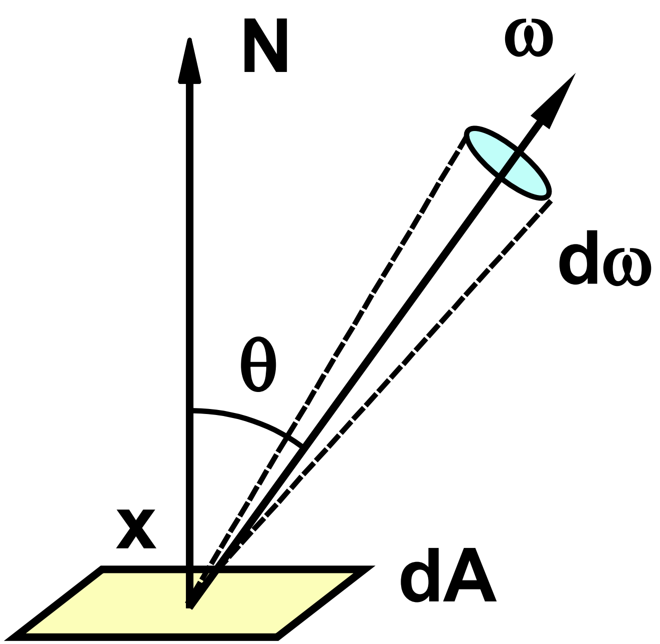
$$L(x,\omega) = \frac{d^2\Phi(x,\omega)}{d\,A_\omega^\perp(x)\,d\,\sigma(\omega)} \qquad [\,\mathrm{W/m^2/sr}\,]$$

Number of photons (converted to energy) per time unit passing through a small area perpendicular to the direction ω. Radiation is directed to a small cone around the direction ω.

Radiance is a quantity defined as a **density** with respect to $\mathbf{dA^\perp}$ and with respect to solid angle $\mathbf{d\sigma(\omega)}$.

Radiance I

- received/emitted **radiance** in direction ω:
  - $L_{in}(\omega)$ ($L_e(\omega)$, $L_{out}(\omega)$) [$W/(m^2 \cdot sr)$]

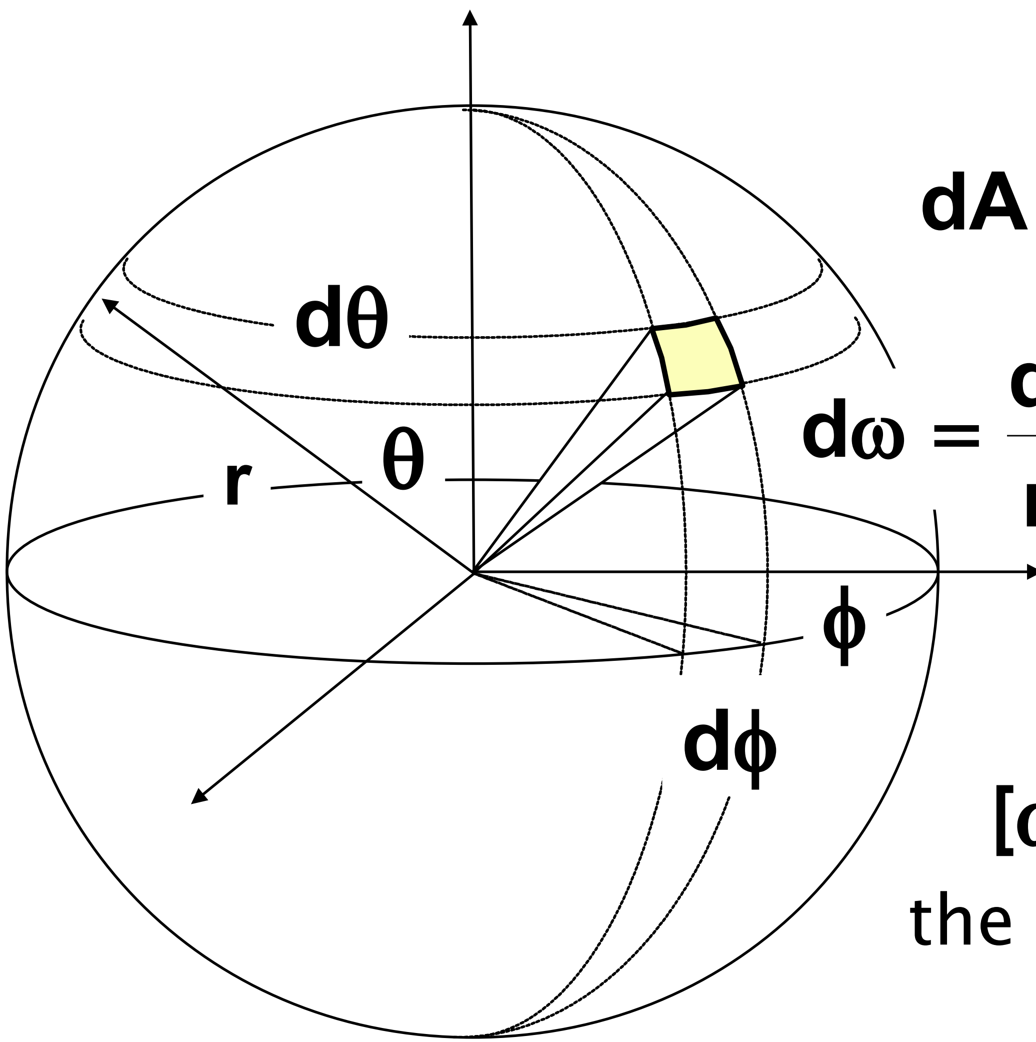


$$L_{out}(x,\omega) = \frac{d^2\Phi}{dA\, d\omega\, \cos\theta}$$

$$= \frac{dB_{out}}{d\omega\, \cos\theta}$$

$$= \frac{dI}{dA\, \cos\theta}$$

Solid angles



$$dA = r^2 \sin\theta \, d\theta \, d\phi$$

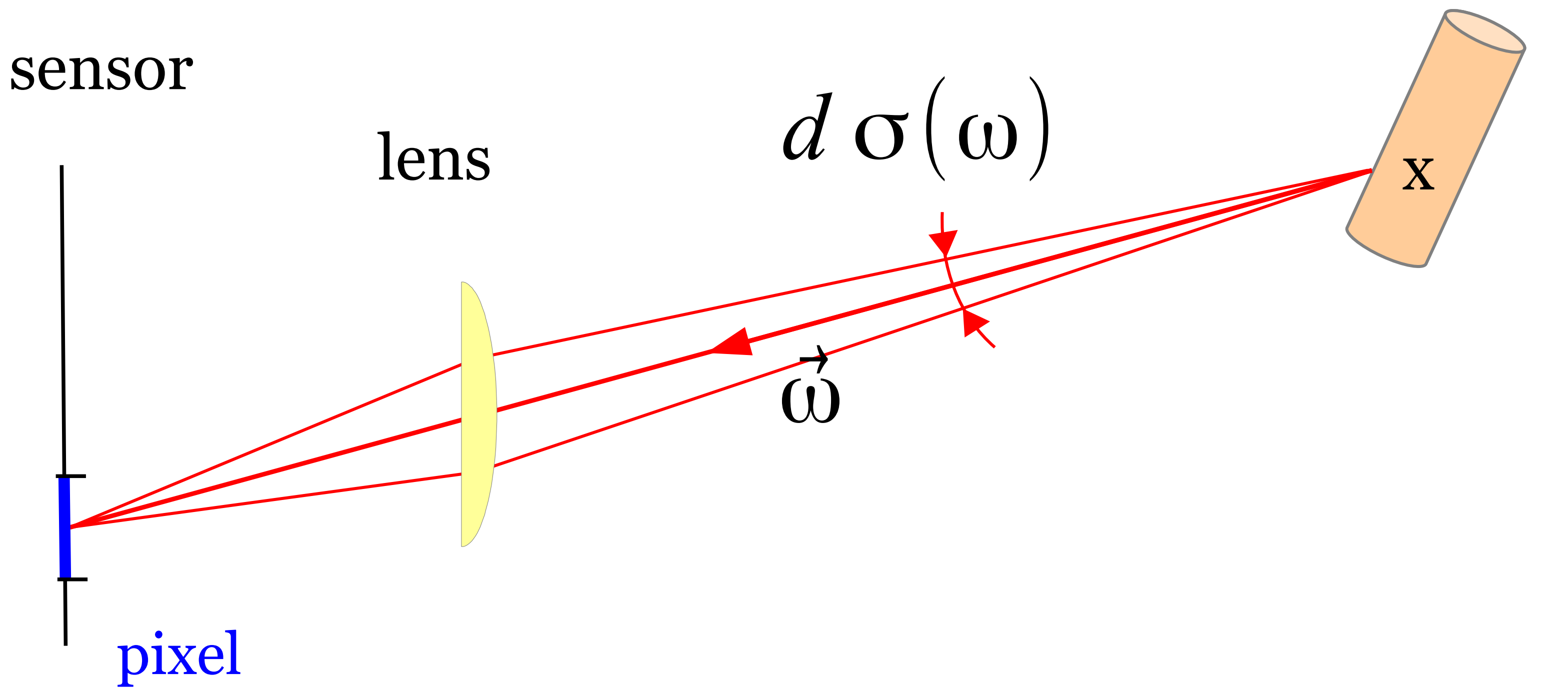
$$d\omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$$

[ω] .. steradian (sr)

the whole sphere .. **4π sr**

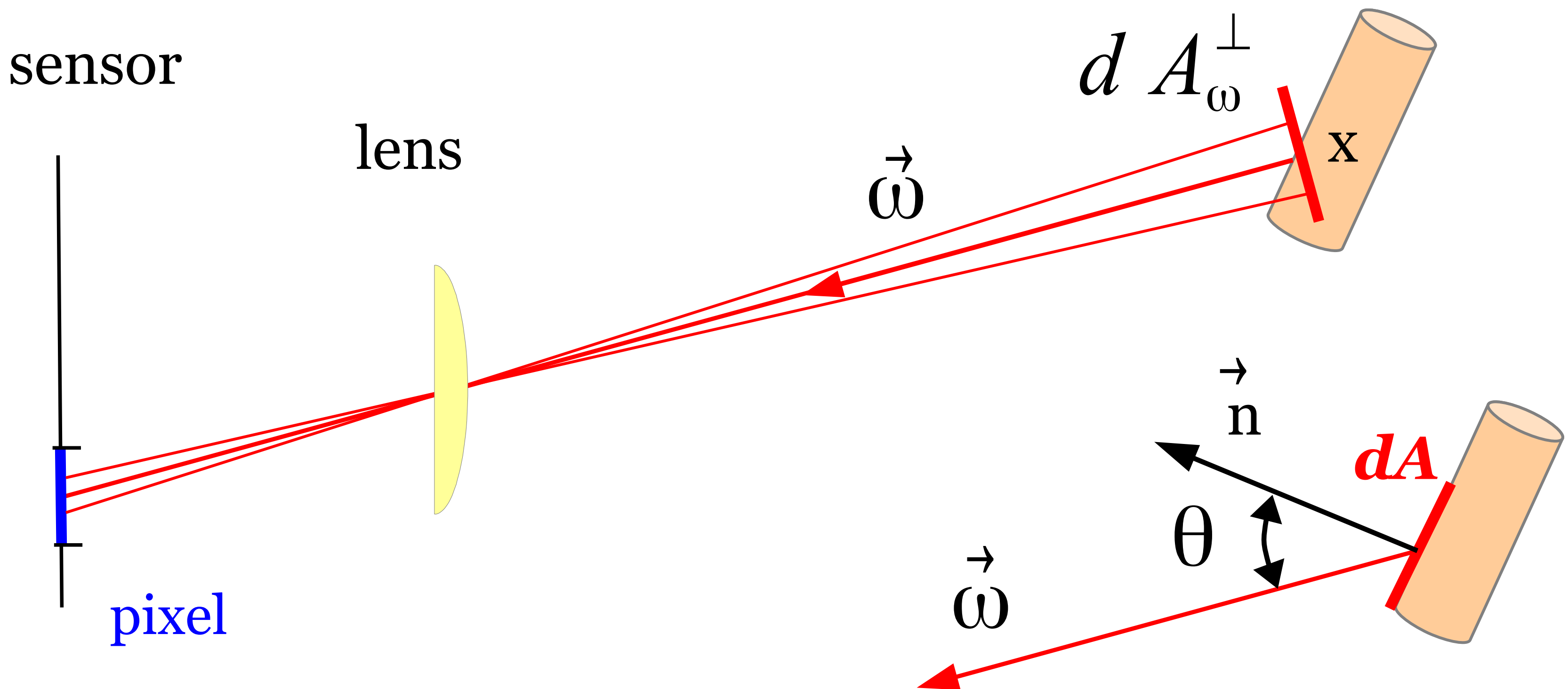
Radiance II

$$\Phi(x, \omega) \propto d\sigma(\omega)$$



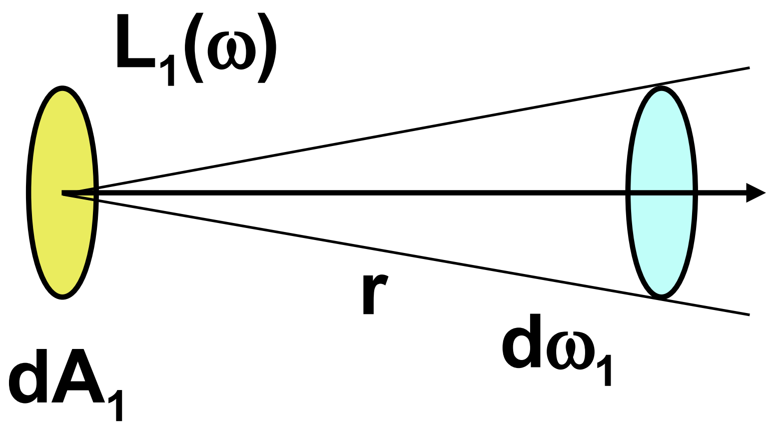
Radiance III

$$\Phi(x, \omega) \propto dA_{\omega}^{\perp}(x)$$



$$dA_{\omega}^{\perp} = dA \cdot \cos\theta$$

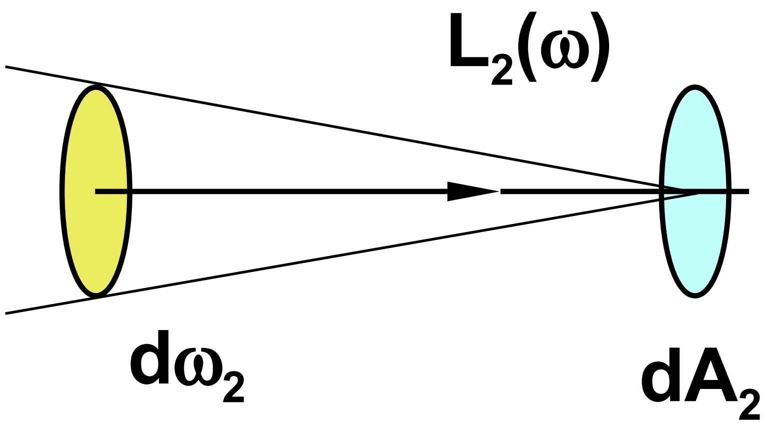
Energy preservation law (ray / fiber)



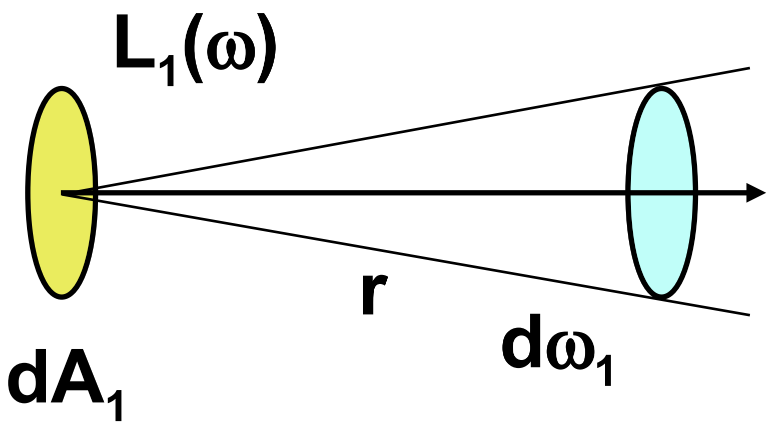
$$L_1 \, d\omega_1 \, dA_1 = L_2 \, d\omega_2 \, dA_2$$

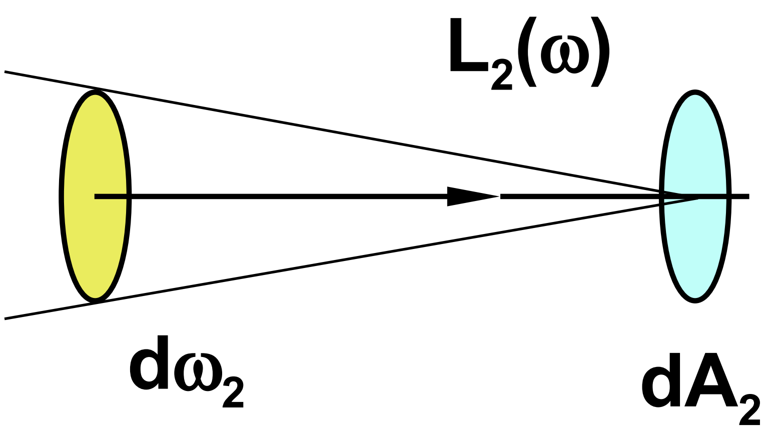
emitted power

received power



Energy preservation law (ray / fiber)





$$L_1\, d\omega_1\, dA_1 = L_2\, d\omega_2\, dA_2$$

$$\underline{T} = d\omega_1\, dA_1 = d\omega_2\, dA_2 = \frac{dA_1\, dA_2}{r^2}$$

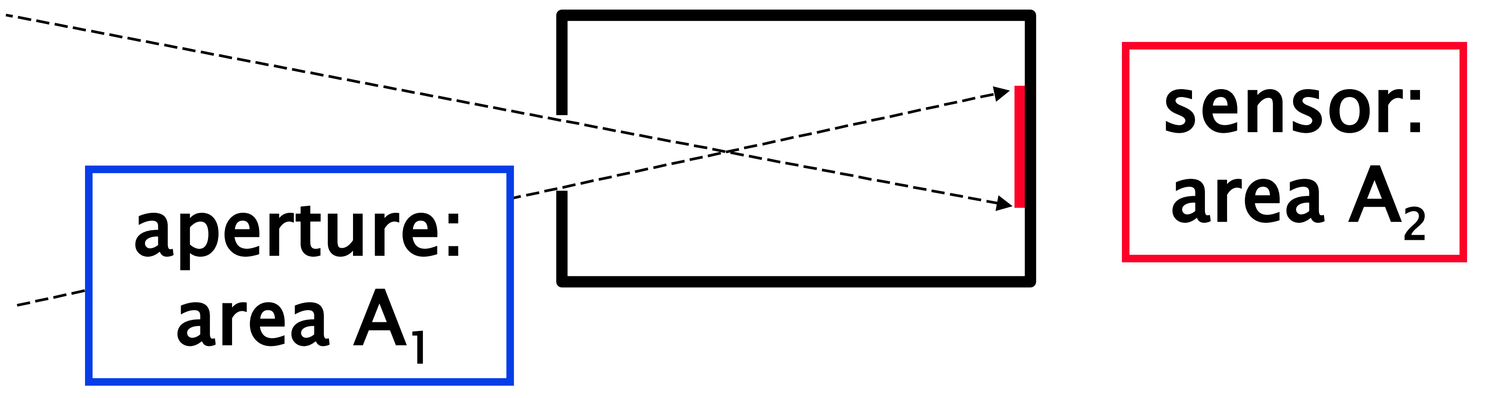
ray capacity

$$L_1 = L_2$$

ray ... radiance L

Light measurement

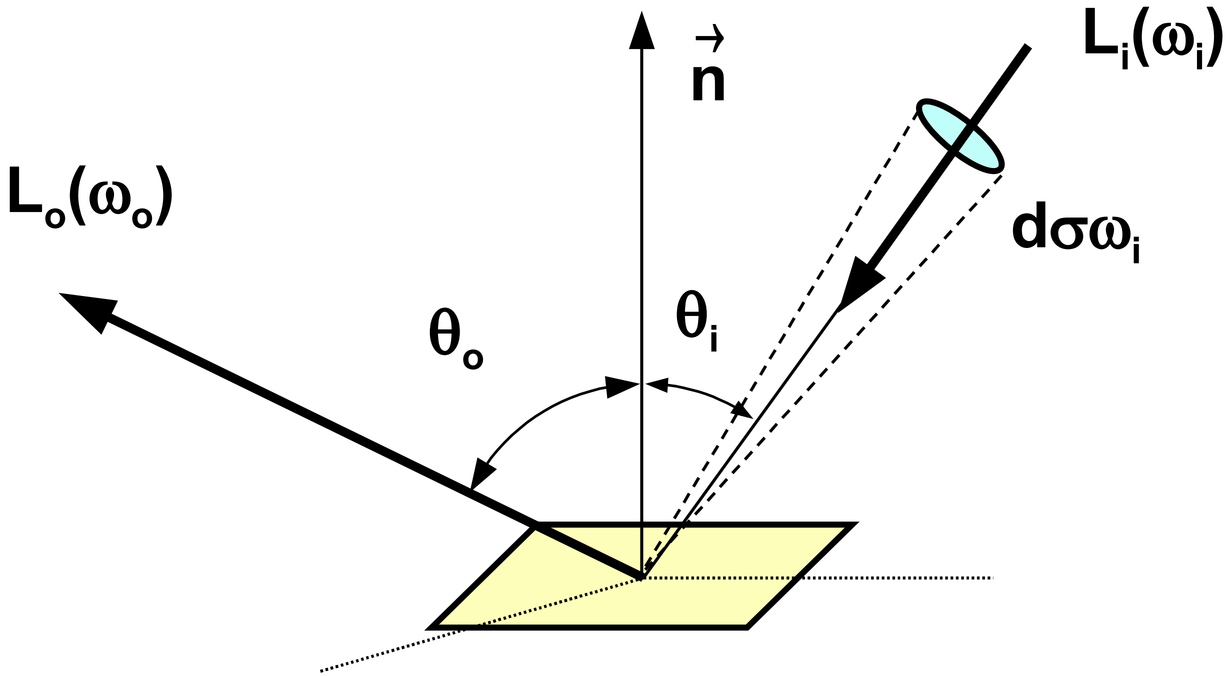
- **measured quantity** is proportional to **radiance** from visible scene



$$\underline{R} = \int_{A_2}\int_{\Omega} L_{in}(A,\omega)\cdot\cos\theta\, d\omega\, dA = \underline{L_{in}\cdot T}$$

BSDF (Local transfer function)

(„Bidirectional Scattering Distribution Function“, older term: BRDF)



$$f_s(\omega_i \to \omega_o) = \frac{d\,L_o(\omega_o)}{d\,E(\omega_i)} = \frac{d\,L_o(\omega_o)}{L_i(\omega_i)\,\cos\theta_i\;d\,\sigma^{\perp}(\omega_i)}$$

Helmholtz law (reciprocity)

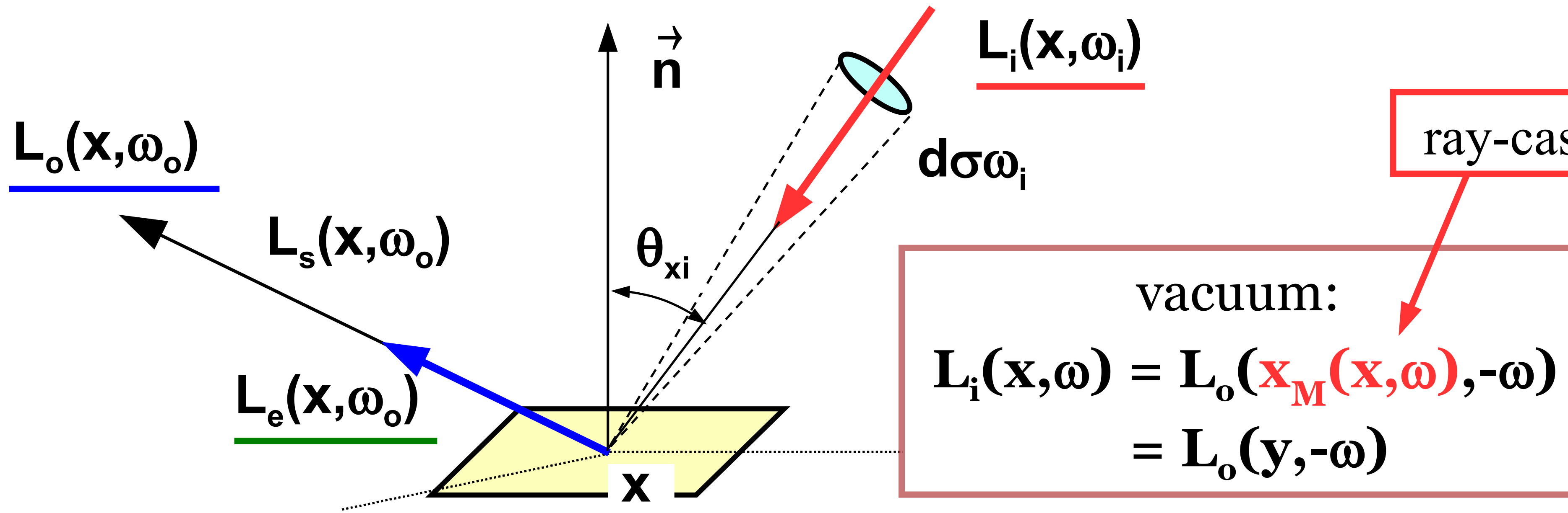
- for **real** surfaces (physically plausible):

$$f(\omega_{in} \to \omega_{out}) = f(\omega_{out} \to \omega_{in})$$

- general **BSDF** needs not be **isotropic** (invariant to rotation around surface normal)
  – metal surfaces polished in one direction, ..

$$f(\theta_{in}, \phi_{in}, \theta_{out}, \phi_{out}) \neq f(\theta_{in}, \phi_{in} + \phi, \theta_{out}, \phi_{out} + \phi)$$

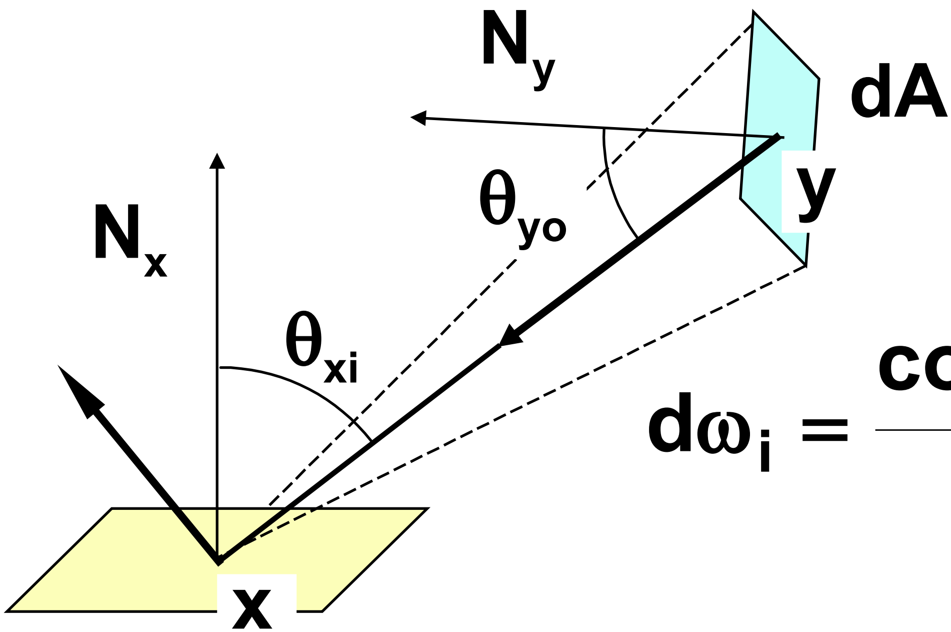
Local rendering equation



own emission at x

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \\ + \int L_o(y,-\omega_i) \cdot f_s(x,\omega_i \rightarrow \omega_o) \cdot d\sigma_x^{\perp}(\omega_i)$$

Radiance received from a surface



$$d\omega_i = \frac{\cos\theta_{yo}\, dA}{\|x - y\|^2}$$

Geometric term:

$$G(y, x) = \frac{\cos\theta_{yo} \cos\theta_{xi}}{\|x - y\|^2}$$

Radiance received from a surface

$$L_o(x,\omega_o) =$$

integral over all incoming directions

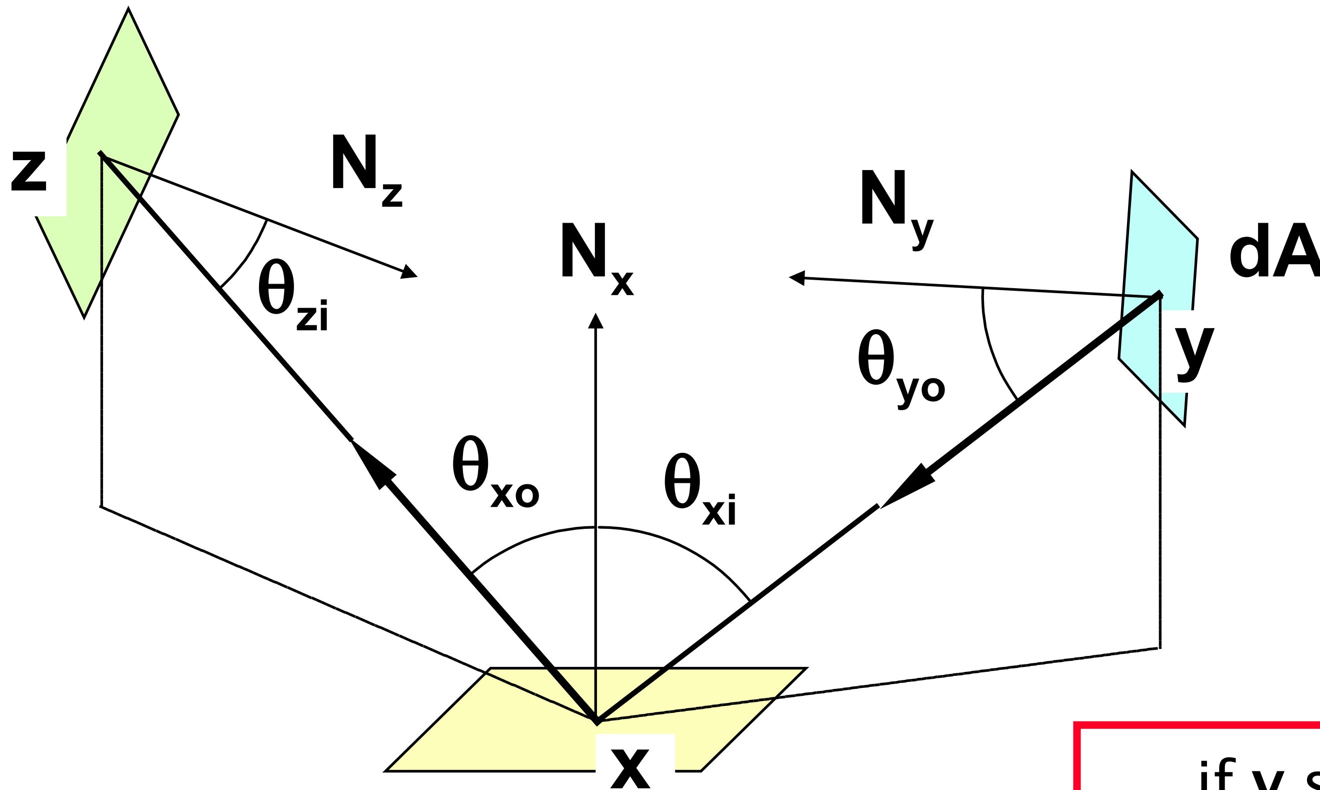
$$= L_e(x,\omega_o) + \int_{\Omega} f(x,\omega_i \to \omega_o) \cdot L_i(x,\omega_i) \cdot \cos\theta_{xi}\, d\omega_i =$$

$$= L_e(x,\omega_o) + \int_{S} f(x,\omega_i \to \omega_o) \cdot L_o(y,-\omega_i) \cdot G(y,x)\, dA$$

integral over an emitting surface

(assumption: the whole surface **S** is visible from **x**)

Reflected light



if y sees x

Terminology: $L(y, x) = L_o(y, x - y) = L_i(x, y - x)$

$f(y, x, z) = f(x, (y - x) \rightarrow (z - x))$

Indirect radiance equation

$$V(y,x) = \begin{cases} 1 & \text{if } y \text{ sees } x \\ 0 & \text{else} \end{cases}$$

$$\underline{L(x,z)} = \underline{L_e(x,z)} + \int_S \underline{f(y,x,z)} \cdot L(y,x) \cdot \underline{G(y,x) \cdot V(y,x)}\, dA$$

- $L_e(x,z)$: **own** (emitted) **radiant exitance**
- $f(y,x,z)$: **BRDF**
- $G(y,x) \cdot V(y,x)$: **geometric terms**

Radiosity equation

- assumption – **ideal diffuse (Lambertian)** surface:
  - **BRDF** is not dependent on incoming/outgoing angles
  - outgoing radiance $\mathbf{L(y,\omega)}$ independent on direction $\boldsymbol{\omega}$

$$L(x,z) = L_e(x,z) + f(x) \cdot \int_S L(y,x) \cdot G(y,x) \cdot V(y,x)\, dA$$

$$L(x,z) = B(x)/\pi, \quad L_e(x,z) = E(x)/\pi, \quad f(x) = \rho(x)/\pi$$

$$B(x) = E(x) + \rho(x) \cdot \int_S B(y) \cdot \frac{G(y,x) \cdot V(y,x)}{\pi}\, dA$$

Discrete solution

$$B(x) = E(x) + \rho(x) \cdot \int_S B(y) \cdot g(y,x)\ dA$$

where $$g(y,x) = \frac{G(y,x) \cdot V(y,x)}{\pi}$$

- solution **B** is infinit-dimensional
- discretization of the task:
  - **Monte-Carlo** ray-tracing (dependent on camera)
  - classical **radosity** (finite/boundary elements FEM)

General radiosity method

1. object surfaces divided into set of **elements**
2. definition of **knot points** on elements
   - **radiosity** will be computed there
3. choice of an **approximation method** and error metric
   - basis functions for convex blend from knot points
4. **coefficients** of linear equation system
   - "form-factors"

General radiosity method

5. solution of **linear equation system**
   – result: radiosity in knot points

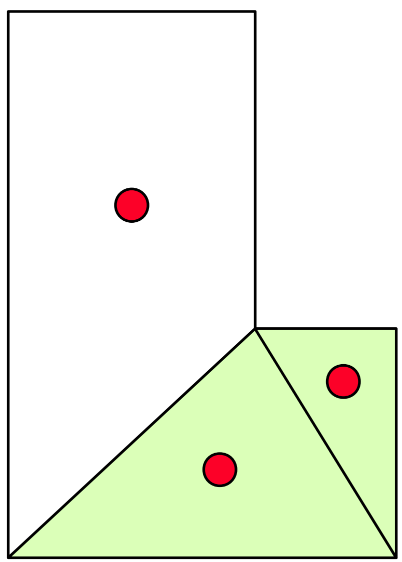
6. reconstruction of values on **whole surfaces**
   – linear blends using basis functions and knot point radiosities

7. **rendering** of results (arbitrary view)
   – light is proportional to radiosity

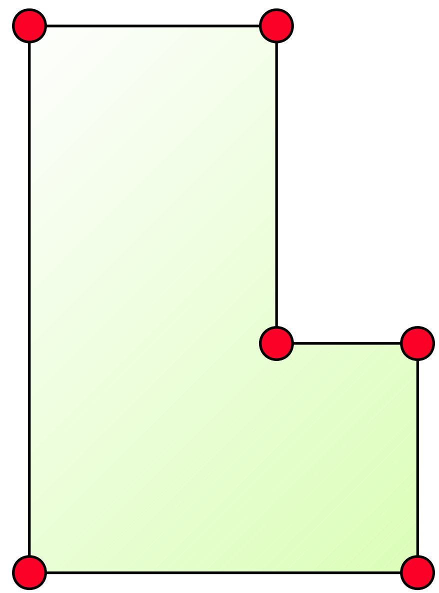
Remarks

- step ❸ is performed in **algorithm design** phase
  - does not appear in an implementation
- some **advanced methods** do not strictly follow the sequence ❶ to ❼
  - sometimes a computation flow goes back to some previous phase, some phases can be iterated,..

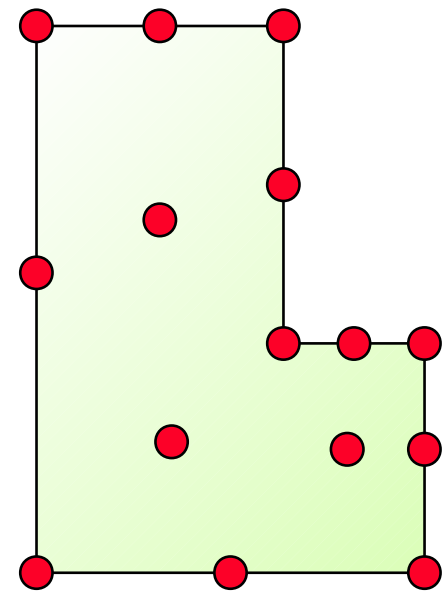
Radiosity approximation



constant
(knots in centers)



bilinear
(knots in vertices)



quadratic
(more knots in centers..)

Constant elements

- on every **element A_i constant** reflectivity is assumed ρ, radiosity **B** – average of **B(x)**:
  - terminology: ρ_i, B_i for $i = 1 .. N$

$$B(x) = E(x) + \rho(x) \cdot \int_S B(y) \cdot g(y,x)\, dA$$

average over area A_i

$$B_i = E_i + \rho_i \cdot \frac{1}{A_i} \int_{A_i} \left[\sum_{j=1}^{N} B_j \int_{A_j} g(y,x)\, dA_j \right] dA_i$$

radiosity received in point **x** (lying on A_i)

Basic radiosity equation

switching sum and integral:

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^{N} B_j \cdot \frac{1}{A_i} \int_{A_i} \int_{A_j} g(y, x) \; dA_j \; dA_i$$

geometric term – **form factor F_{ij}**
(part of energy irradiated from $\mathbf{A_i}$ received directly by $\mathbf{A_j}$)

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^{N} B_j \, F_{ij} \quad \left[\frac{W}{m^2} \right]$$

Intuitive derivation

$$B_i A_i = E_i A_i + \rho_i \cdot \sum_{j=1}^{N} B_j A_j F_{ji} \quad [\mathrm{W}]$$

emitted power = own power + reflected power

reciprocal rule: $$A_j F_{ji} = A_i F_{ij}$$

$$B_i A_i = E_i A_i + \rho_i \cdot \sum_{j=1}^{N} B_j F_{ij} A_i \quad \Big| \cdot A_i^{-1}$$

$$B_i = E_i + \rho_i \cdot \sum_{j=1}^{N} B_j F_{ij} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

System of linear equations

$$\underline{B_i} - \rho_i \cdot \sum_{j=1}^{N} \underline{B_j}\, F_{ij} = E_i \qquad i = 1\,..\,N$$

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & -\rho_1 F_{1,2} & .. & -\rho_1 F_{1,N} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & .. & -\rho_2 F_{2,N} \\ .. & .. & .. & .. \\ -\rho_N F_{N,1} & -\rho_N F_{N,2} & .. & 1-\rho_N F_{N,N} \end{bmatrix} \underline{\begin{bmatrix} B_1 \\ B_2 \\ .. \\ B_N \end{bmatrix}} = \begin{bmatrix} E_1 \\ E_2 \\ .. \\ E_N \end{bmatrix}$$

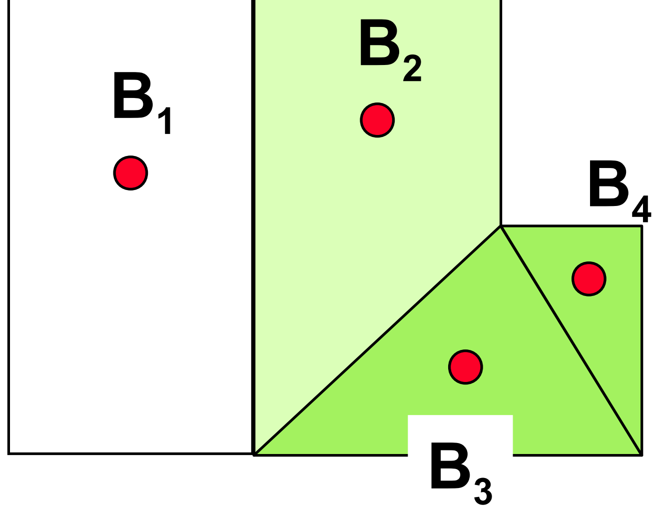
vector of unknown vars $[B_i]$

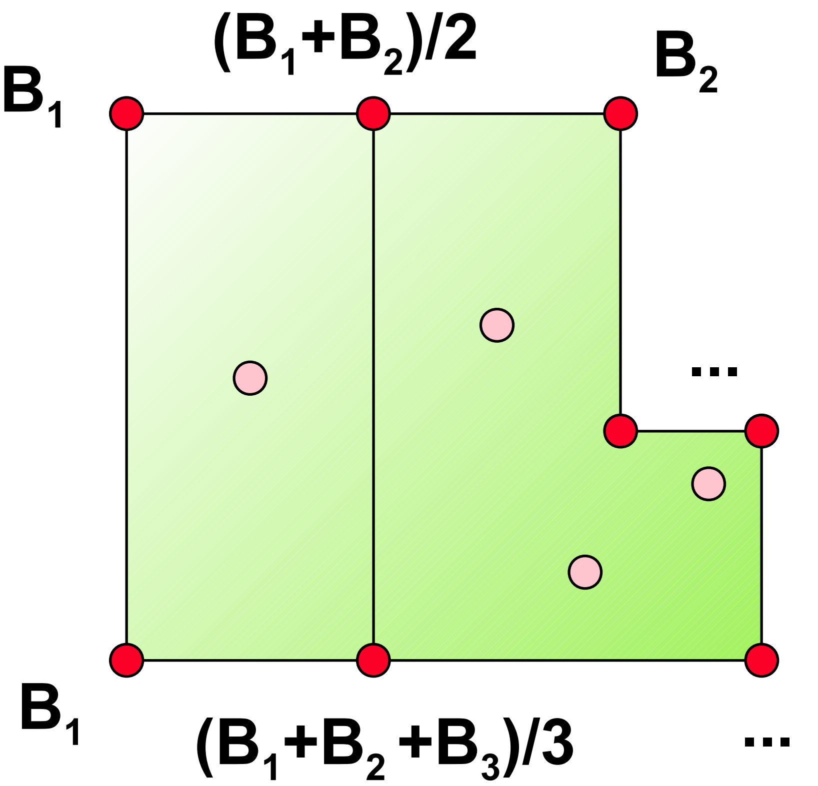
System of linear equations

- for **planar (convex) surfaces**: $F_{ii} = 0$
  - the diagonal contain only unit values
- **nondiagonal items** are usually very small (abs value)
  - matrix is “diagonally dominant”
  - ⇒ system is stable and can be solved by **iterative methods** (Jacobi, Gauss-Seidel)
- for **light change (light sources)** $[E_i]$ system needs not to be fully re-computed, only reverse phase could be done

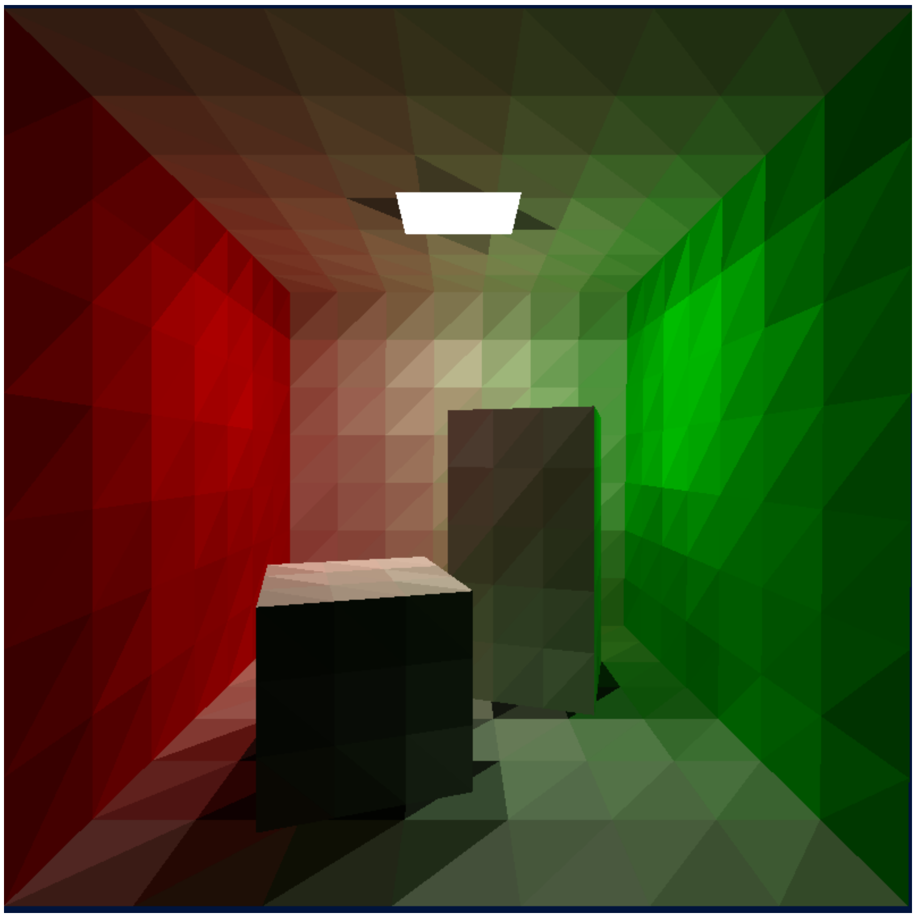
Radiosity to vertices

Even in constant element approach usage of some color interpolation method is recommended (**Gouraud**)





Linear color interpolation



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