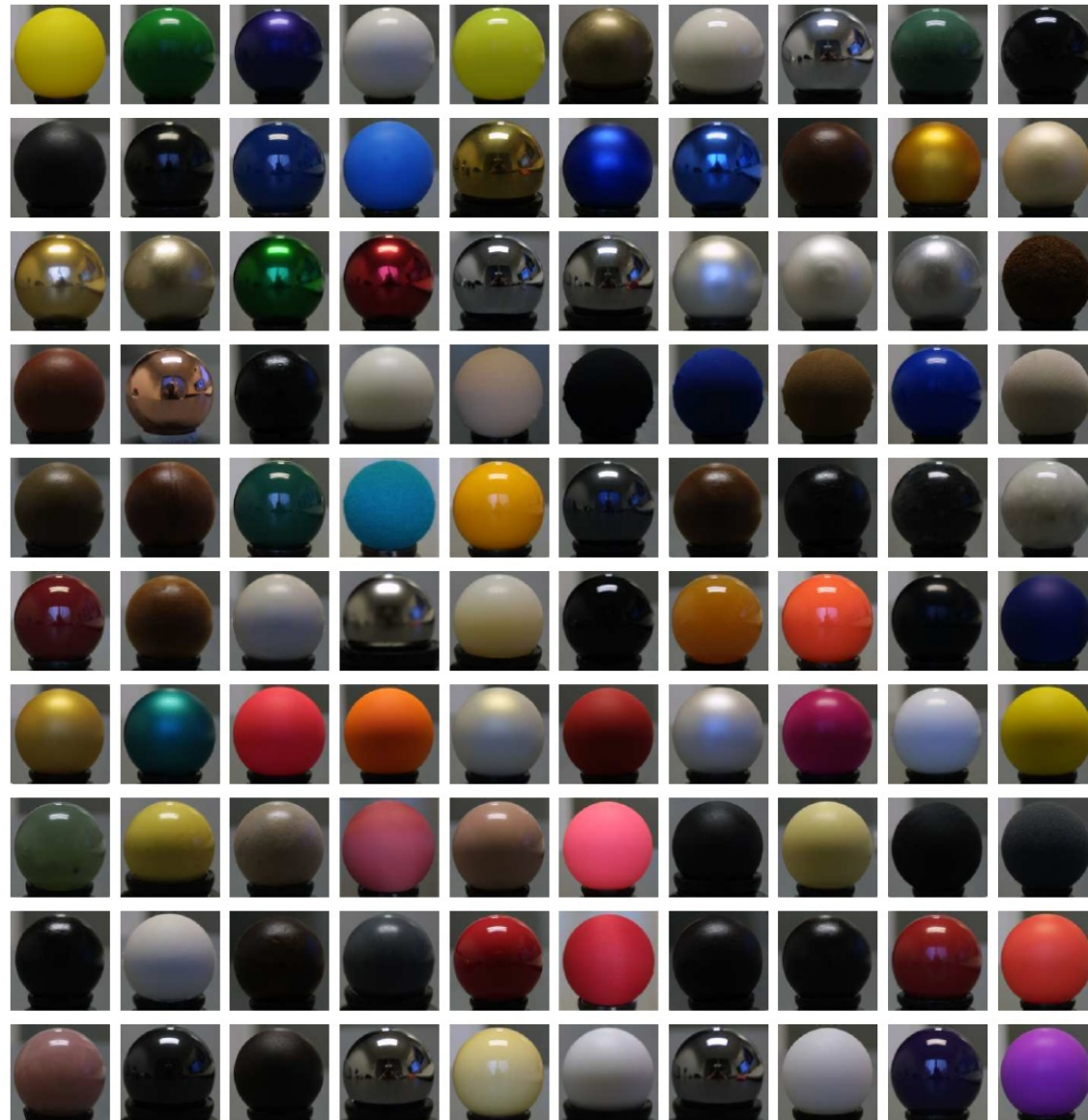


# Reflectance Models (BRDF)

© 1996-2024 Josef Pelikán  
CGG MFF UK Praha

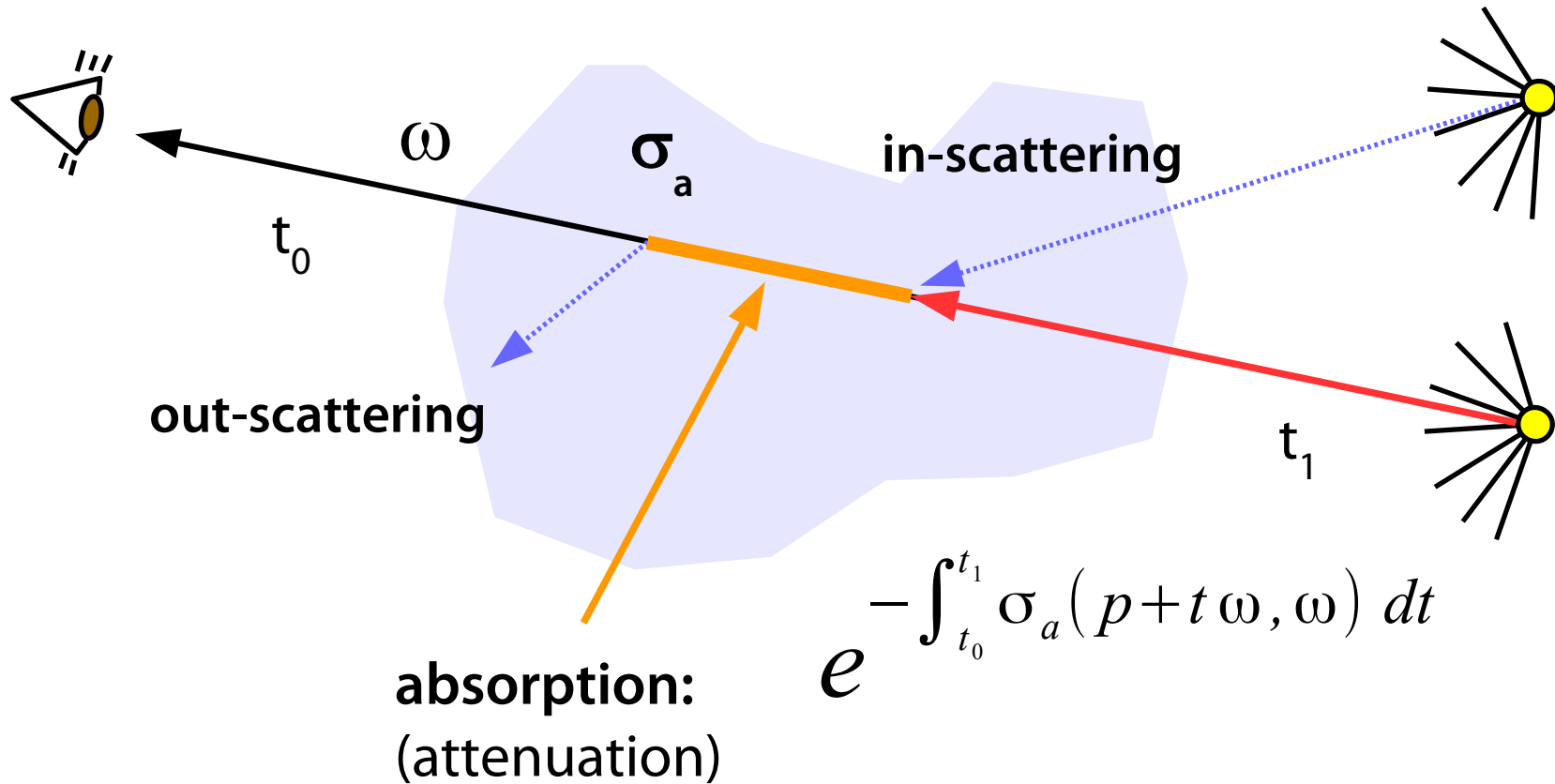
[pepca@cgg.mff.cuni.cz](mailto:pepca@cgg.mff.cuni.cz)  
<https://cgg.mff.cuni.cz/~pepca/>

# Reflectance models (“materials”)





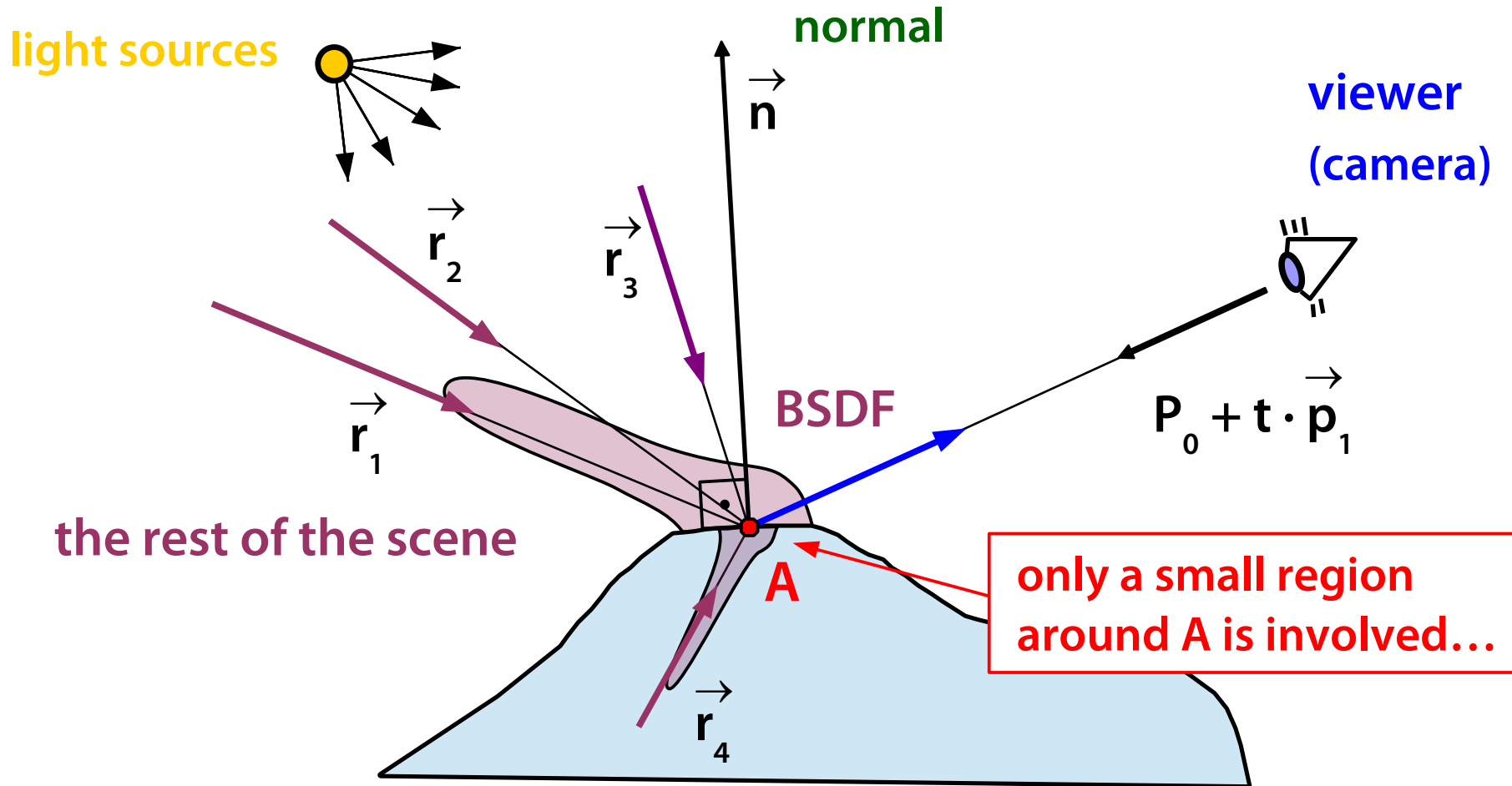
# Light travels through media



Absorption is simple, ~~scattering~~ is very complicated

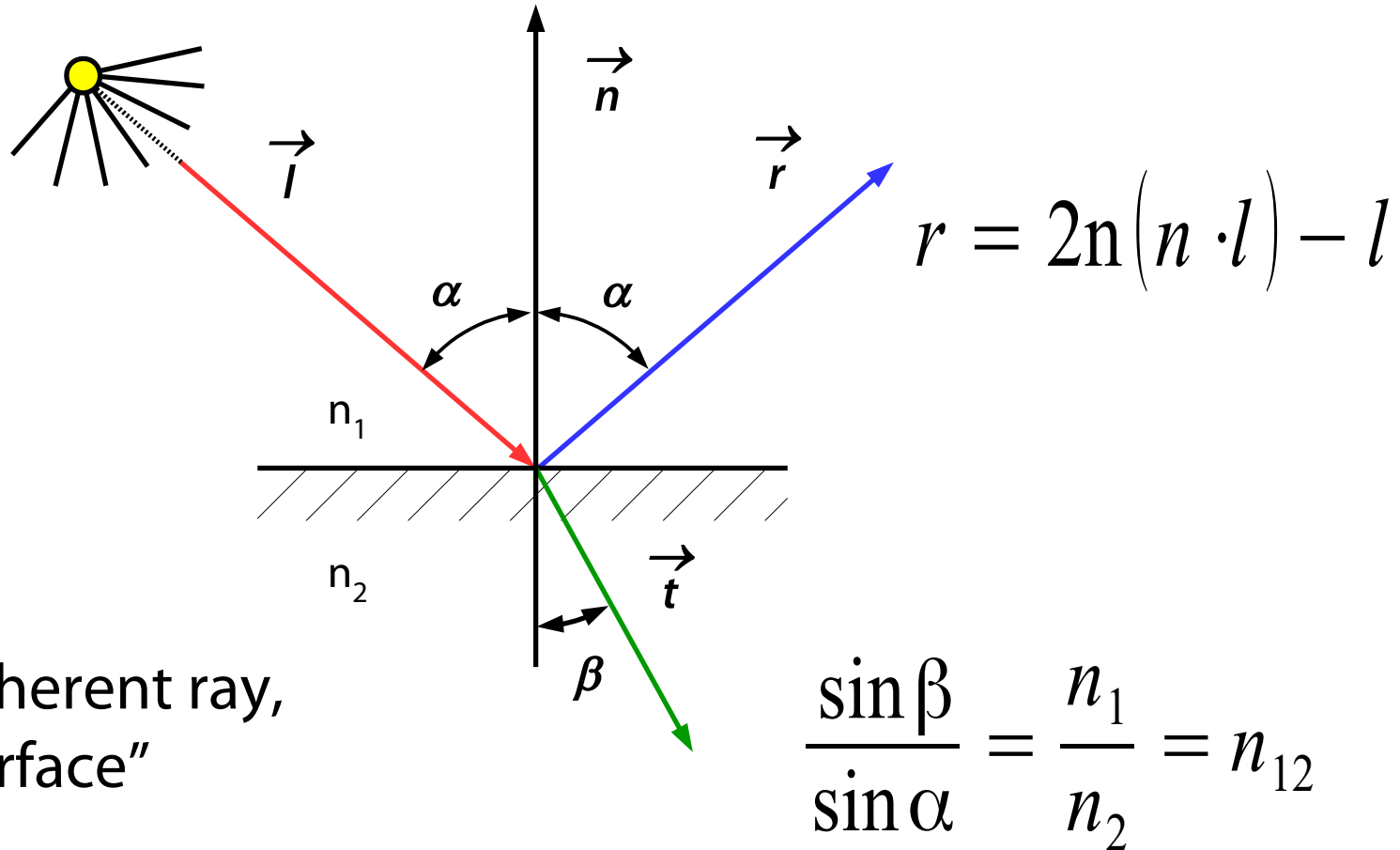


# Local reflection and refraction of light





# Light hits object surface (ideal)

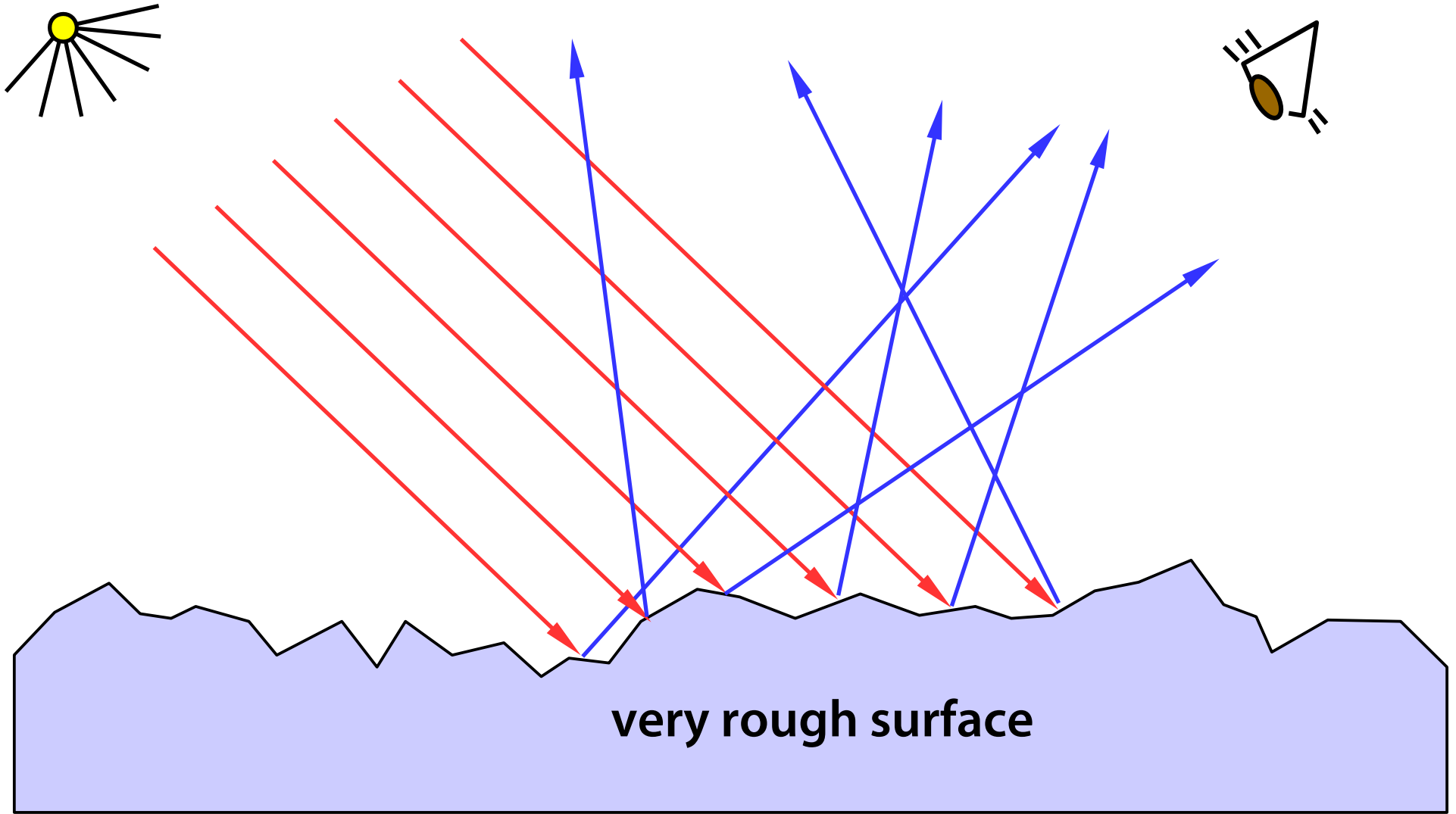


“Ideal coherent ray,  
ideal surface”

## Fresnel equations

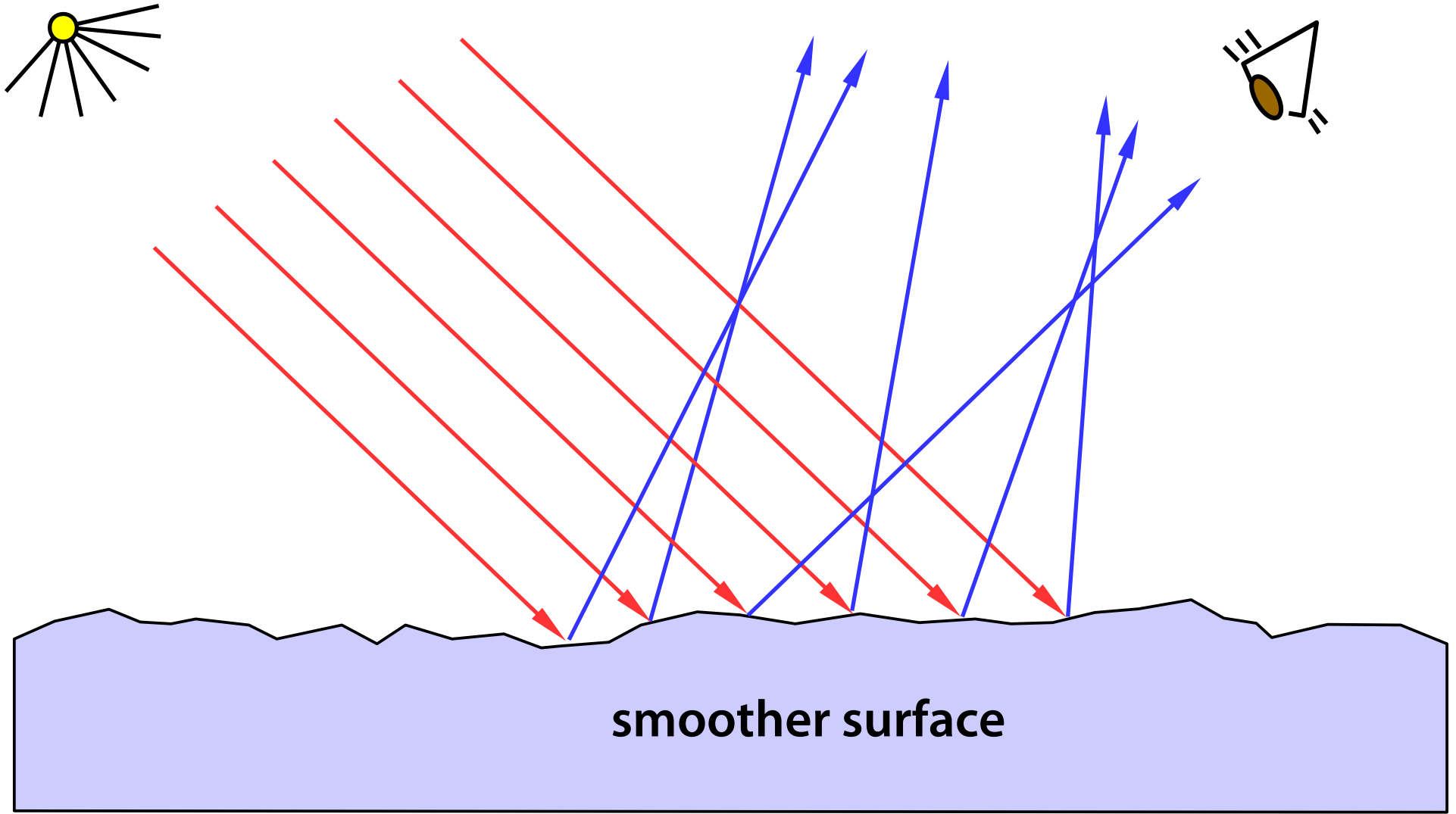


# Real surface (microscopic view)



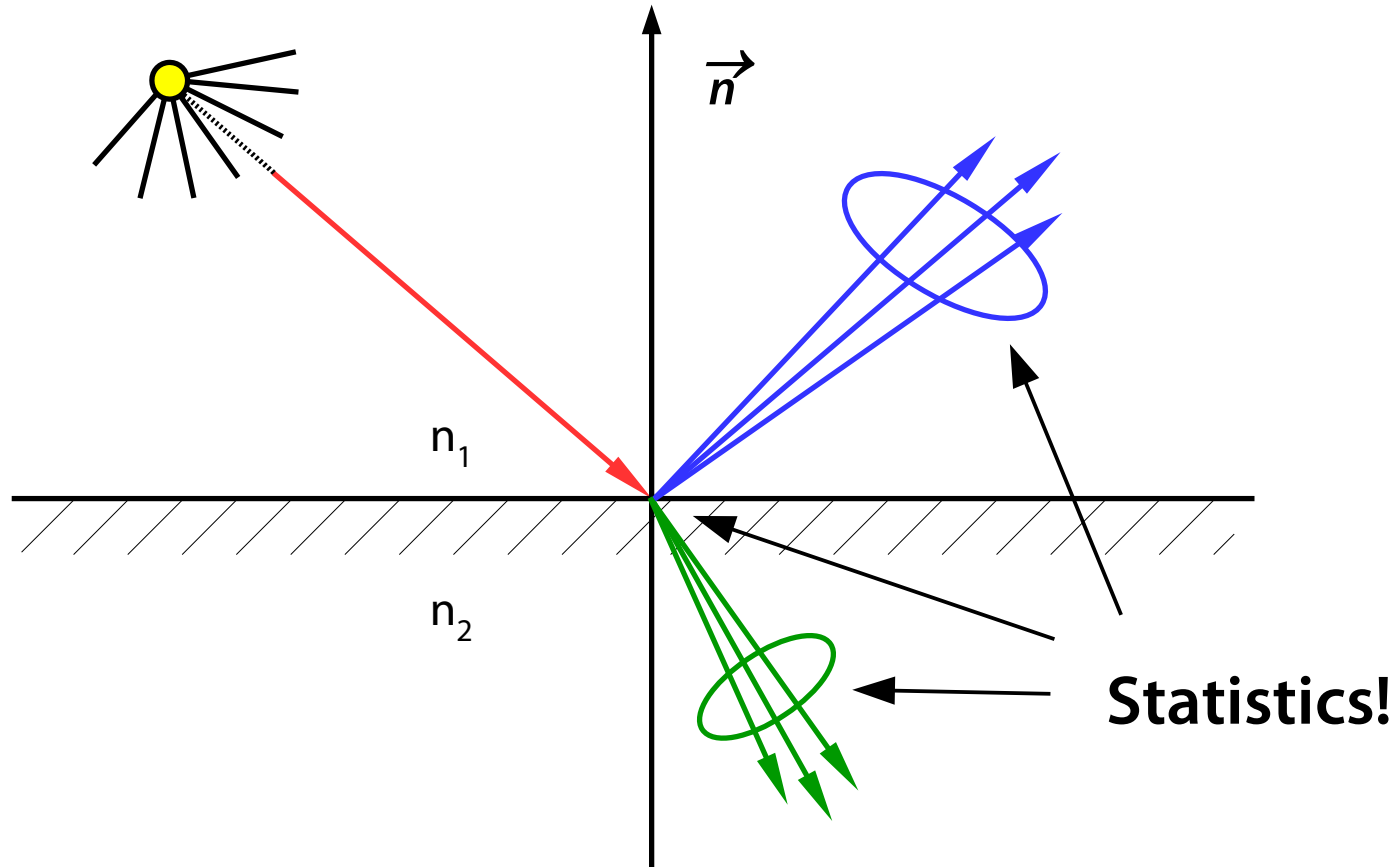


# Real surface (microscopic view)





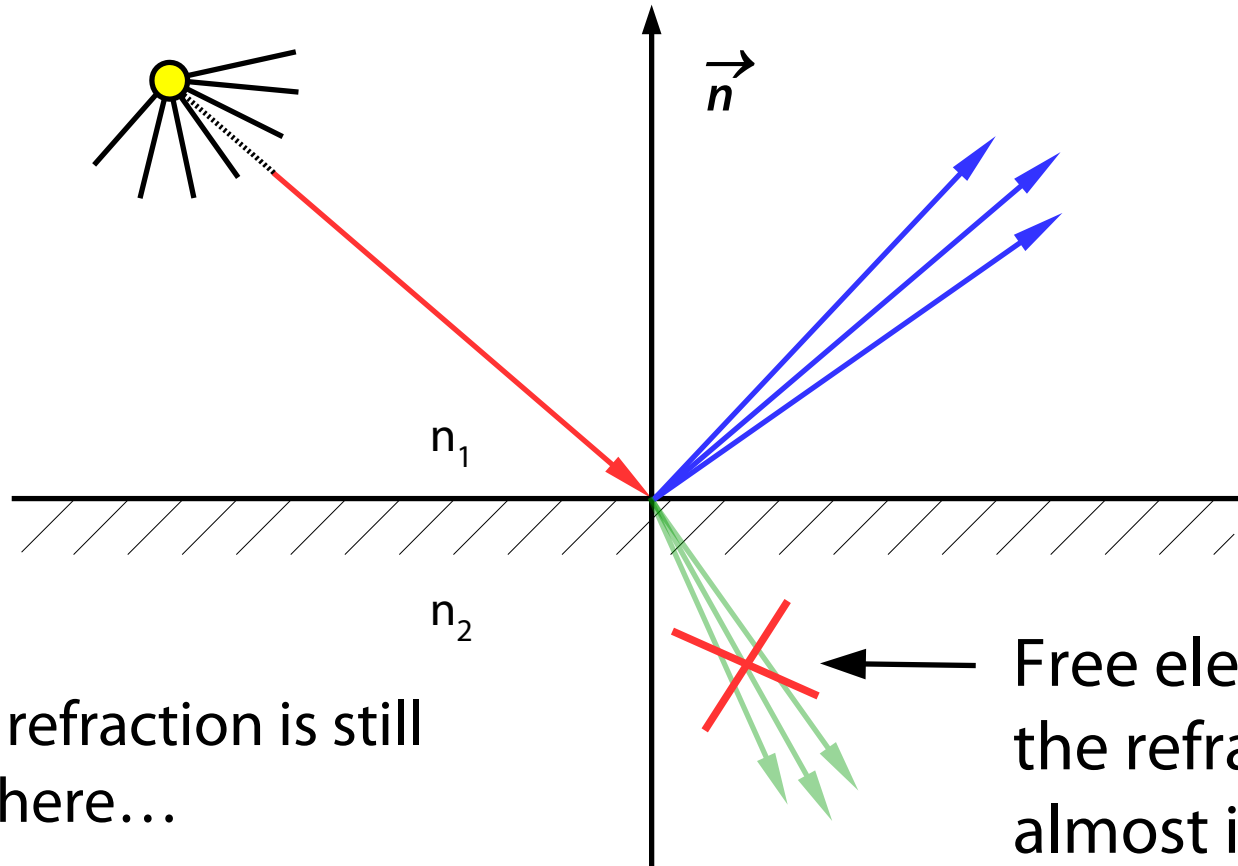
# What we see from a distance...





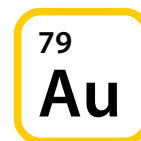


# Metals (conductors)



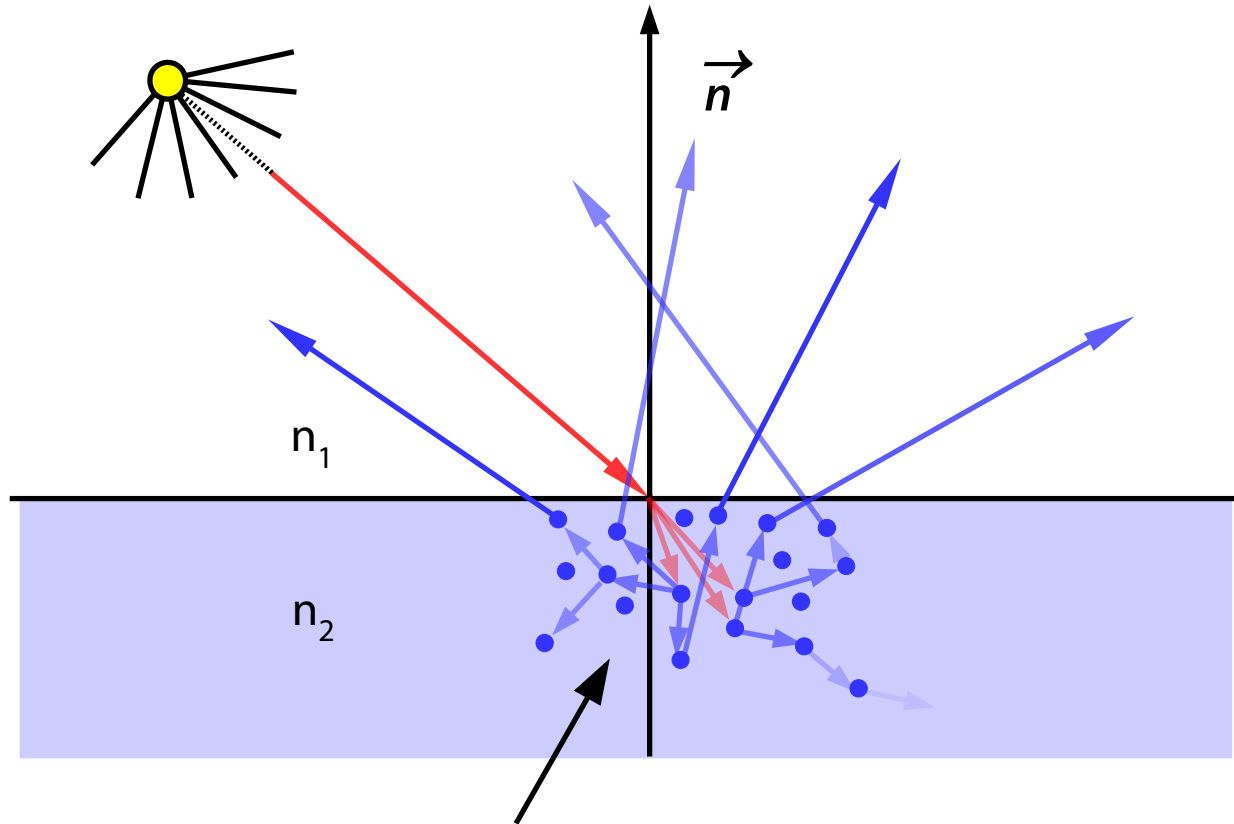
Index of refraction is still defined here...

Free electrons **absorb** the refracted light almost immediately





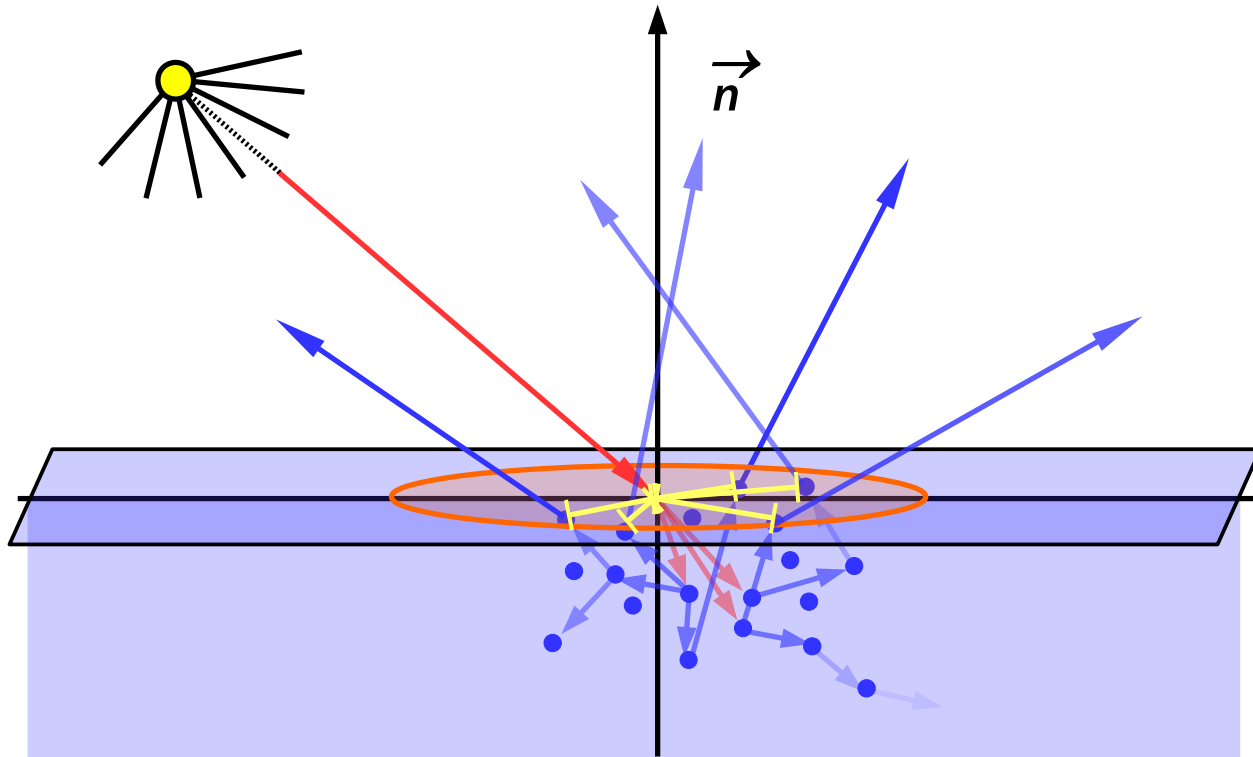
# Dielectrics (insulators)



Scattering inside the material



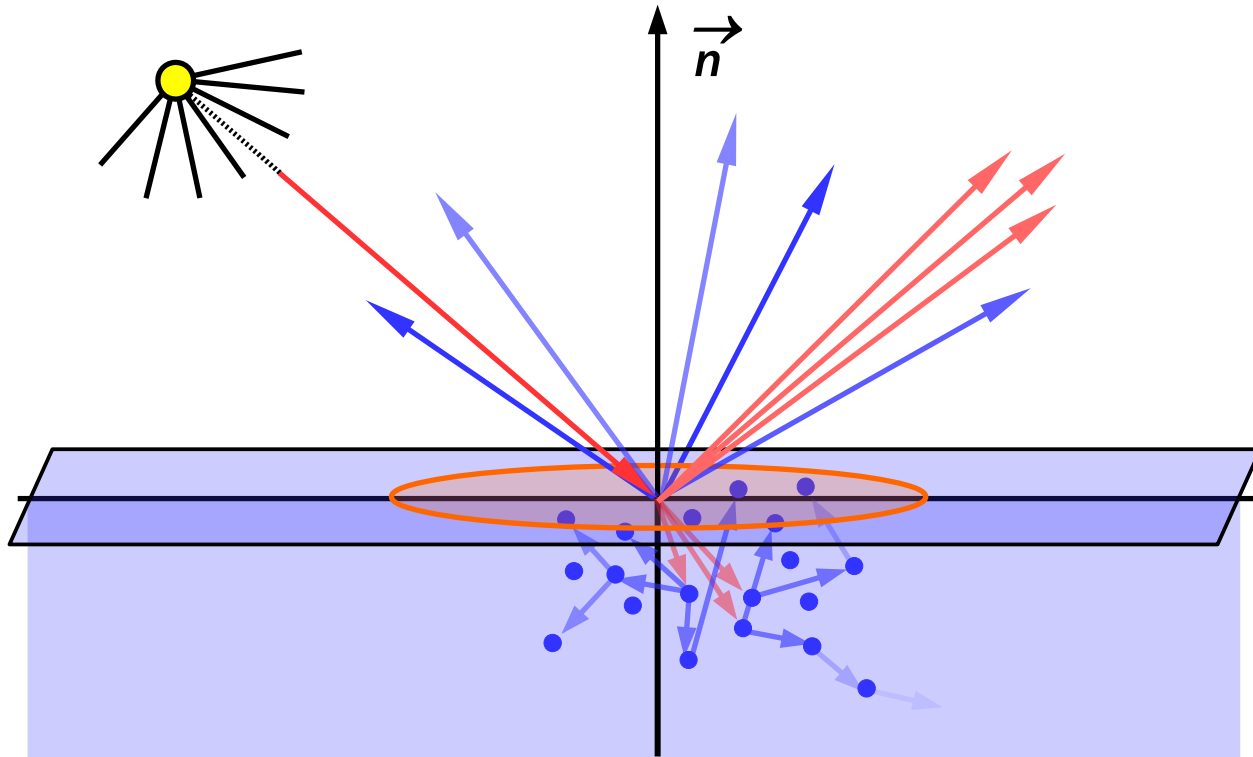
# BSSRDF idea



“Bi-directional Scattering-Surface Reflectance Distribution Function”



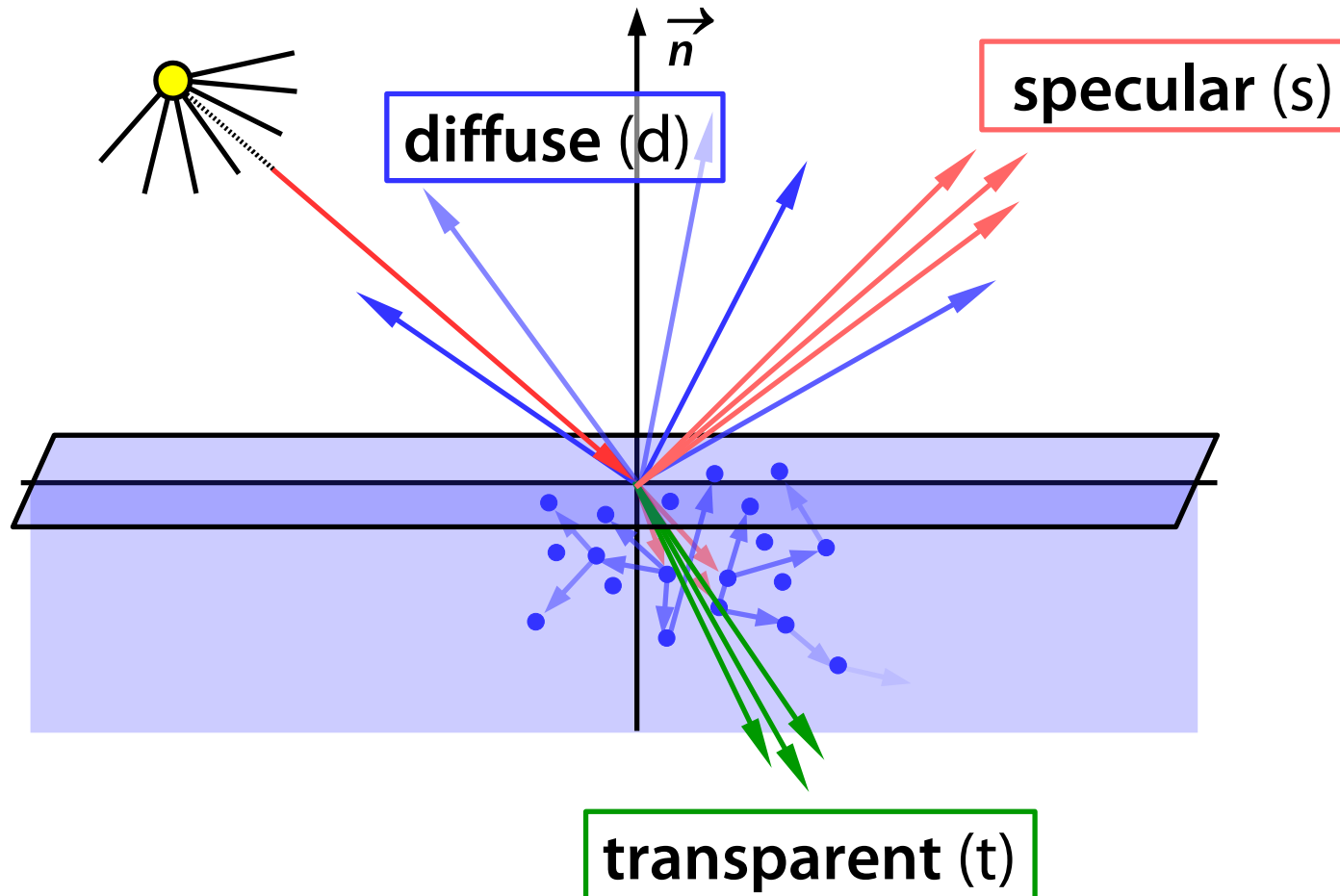
# Ignoring exit-to-entry distance



**BRDF** = “Bi-directional Reflectance Distribution Function”



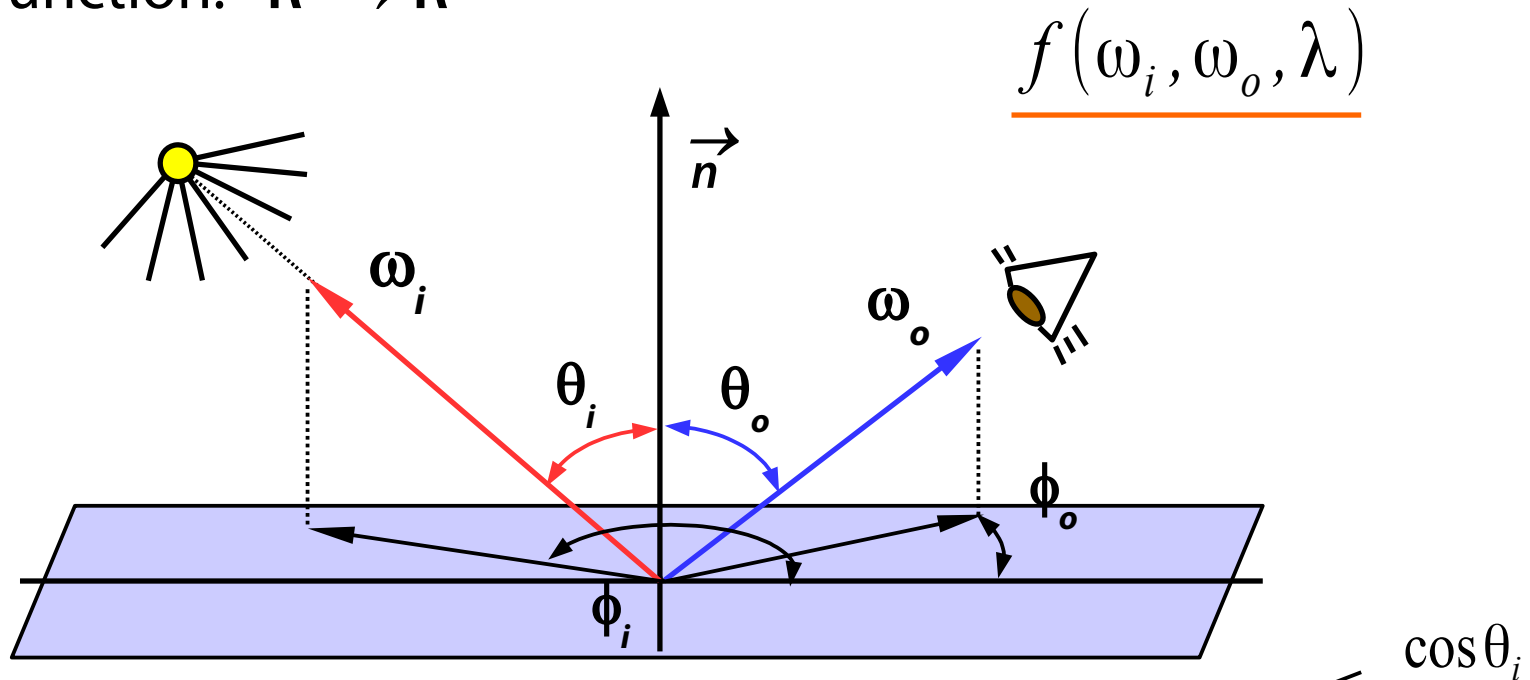
# Shading terms (components)





# BRDF formulation

BRDF function:  $\mathbb{R}^5 \rightarrow \mathbb{R}$



$$\underline{L_o(\omega_o)} = \int_{\Omega} \underline{f(\omega_i, \omega_o)} \cdot \underline{L_i(\omega_i)} (\underline{n \cdot \omega_i}) \underline{d\omega_i}$$

$\cos \theta_i$



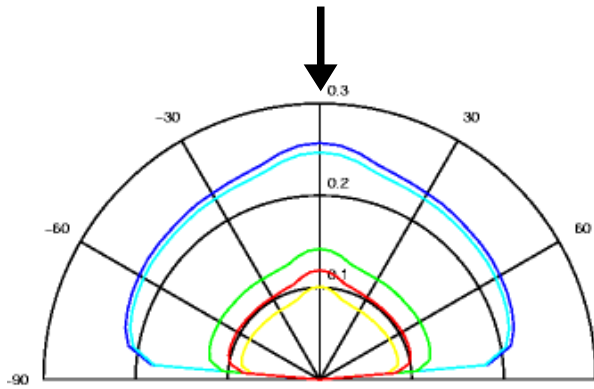
# BRDF plausibility

Non-negative  $f(\omega_i, \omega_o, \lambda) \geq 0$

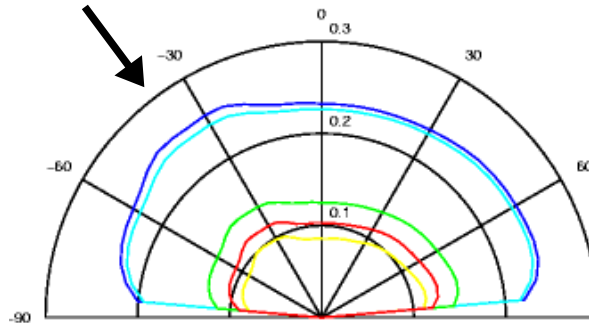
Reciprocal  $f(\omega_i, \omega_o, \lambda) = f(\omega_o, \omega_i, \lambda)$

Energy-conserving  $\int_{\Omega} f(\omega_i, \omega_o) (n \cdot \omega_i) d\omega_i \leq 1$

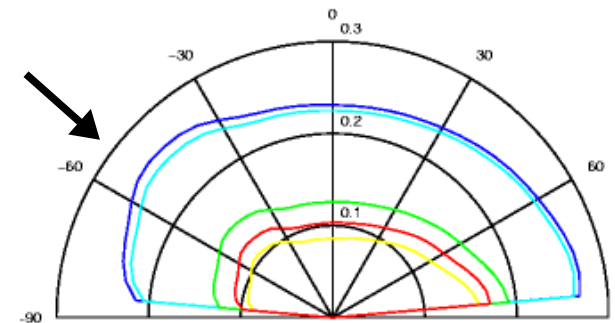
# Actual reflections (from laboratory)



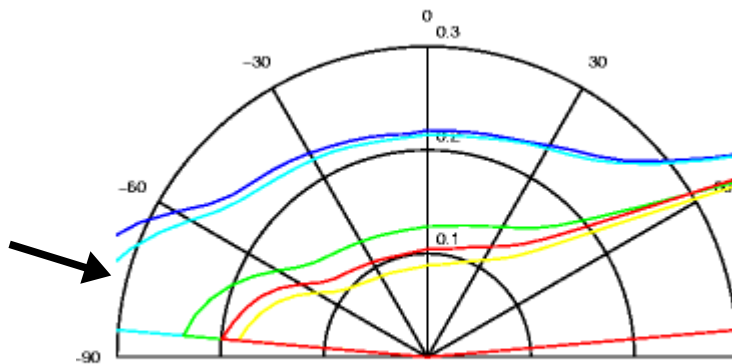
0° (perpendicular)



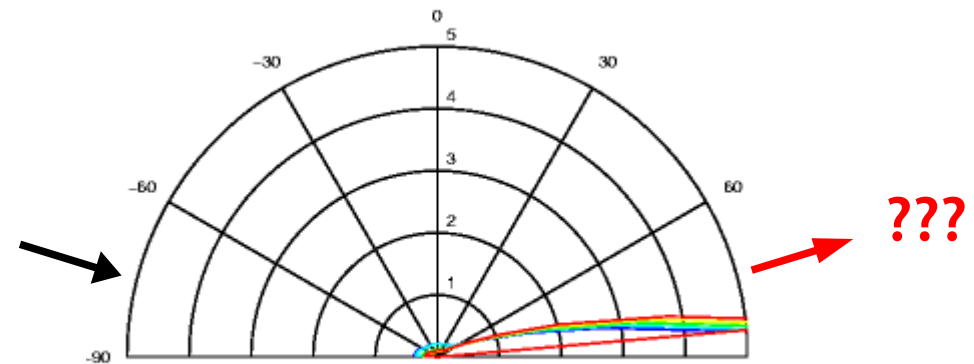
35°



55°



75°



75° (zoomed out)





# History (**physical**/**empirical**)

**Beckmann, Spizzichino** (1963): electromagnetic wave reflection on rough surfaces (optics)

**Torrance, Sparrow** (1967): off-specular reflections on rough surfaces (optics)

**Phong** (1975): famous empirical model, used many decades

**Blinn** (1977): first light reflection presentation at SIGGRAPH

**Cook, Torrance** (1981): generalization, implementation, first physically based BRDF model in computer graphics

**He** (1991): more complex wave optics, polarization, diffraction, interference...

**Ward** (1992): anisotropic material, microfacets



# History (**physical**/**empirical**)

**Schlick** (1994): fast Fresnel formula approximation, two-layer reflectance model

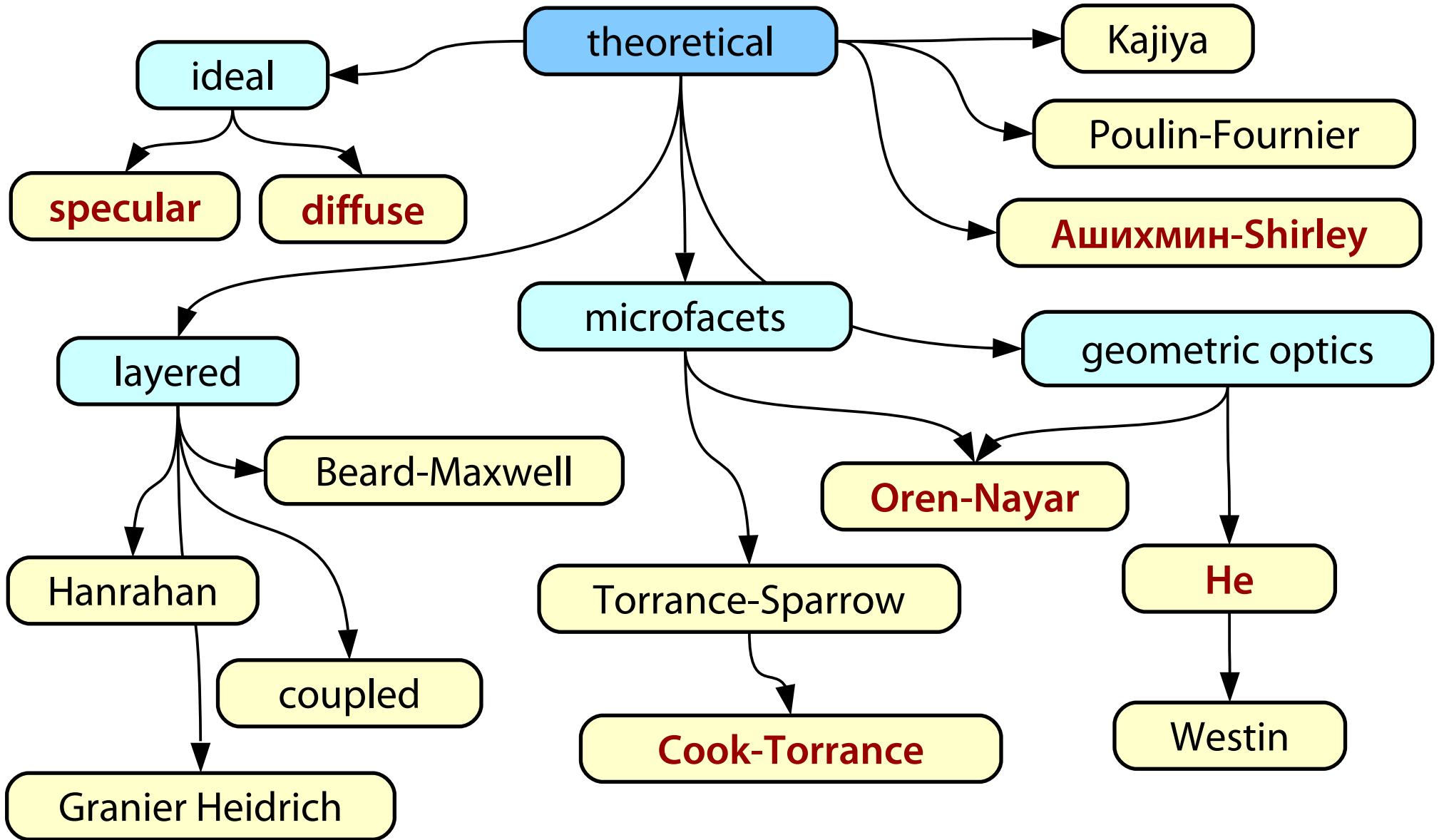
**Lafortune** (1997): multiple lobes, fitted to lab data

**Ашихмин, Shirley** (2000): anisotropic Phong

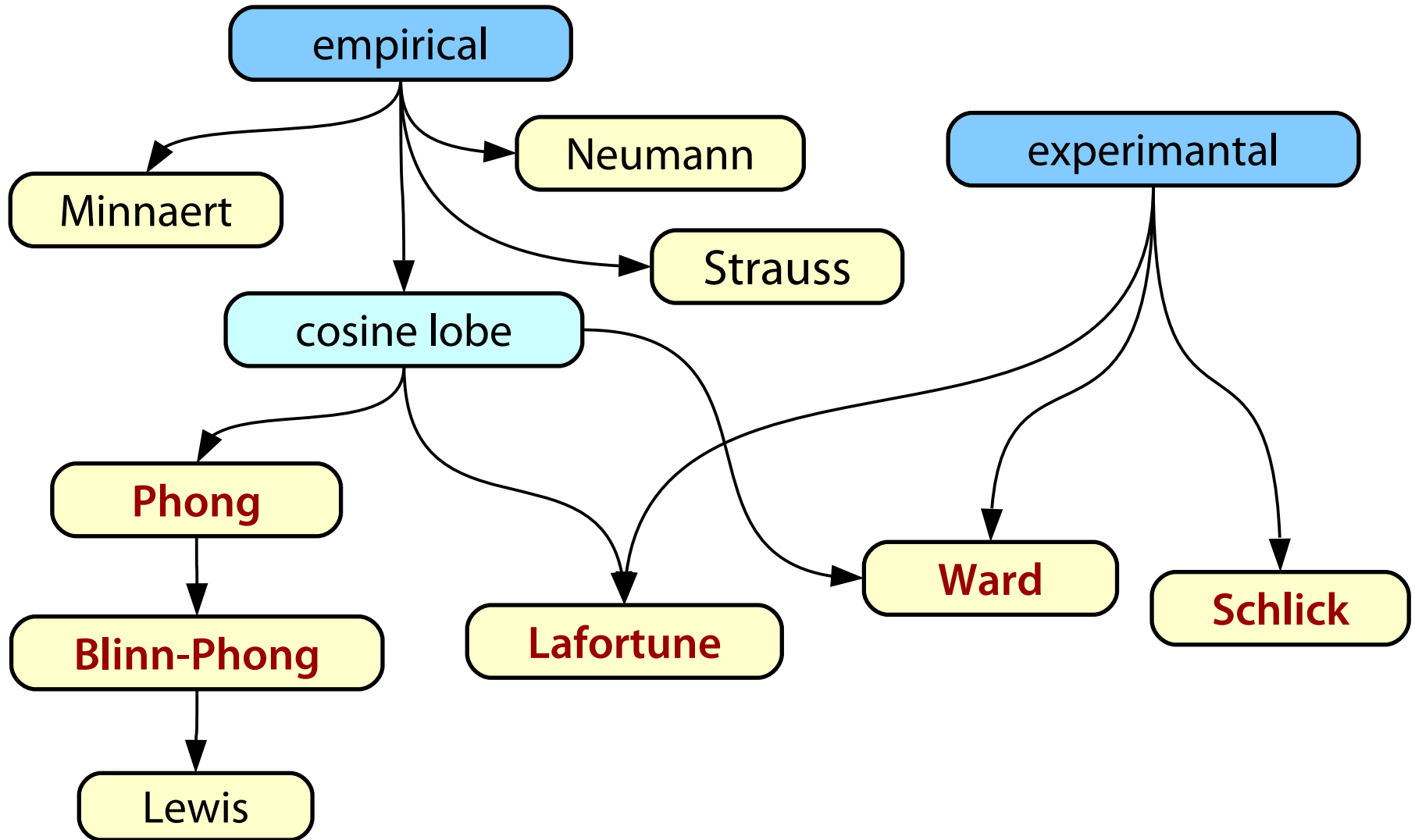
**Walter** (2007): microfacet refraction model (BSDF = Bidirectional Scattering Distribution Function)

**Ашихмин, Bagher** (2007, 2012): models based on arbitrary microfacet distribution (measured...)

# BRDF ZOO I

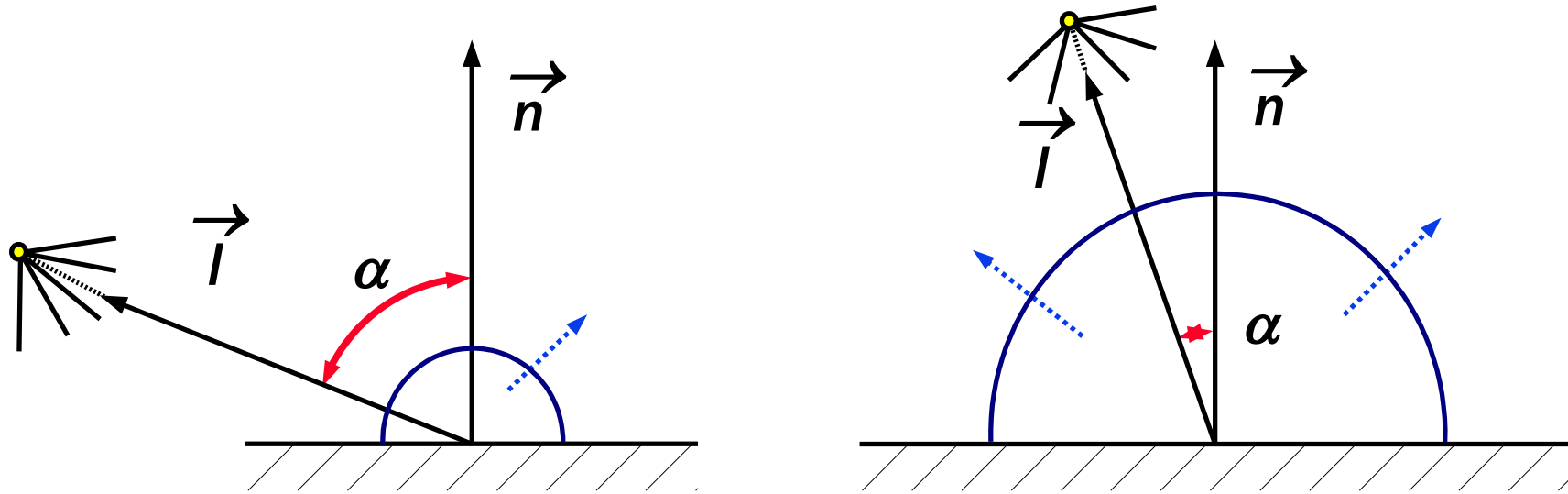


# BRDF ZOO II





# Ideal diffusion



## Ideal diffuse material (Lambertian surface)

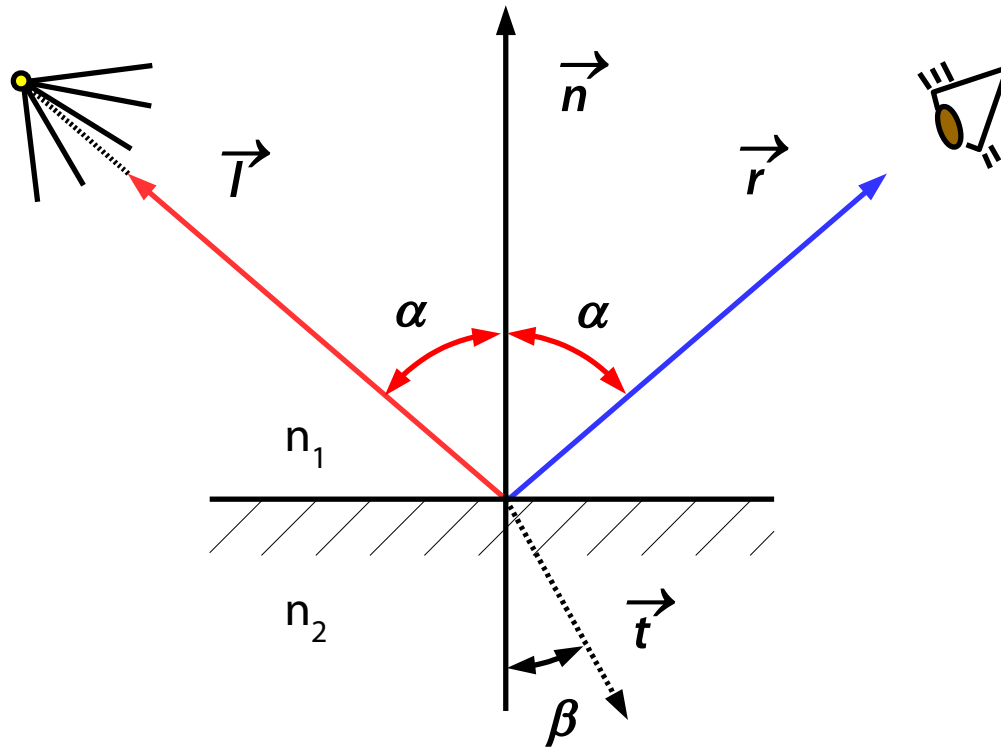
- reflection probability is constant
- examples: furry surface, noisy microstructure w/o any pattern

## Lambert law (1760)

- reflected intensity depends solely on  $\cos \alpha$



# Ideal (mirror) reflection



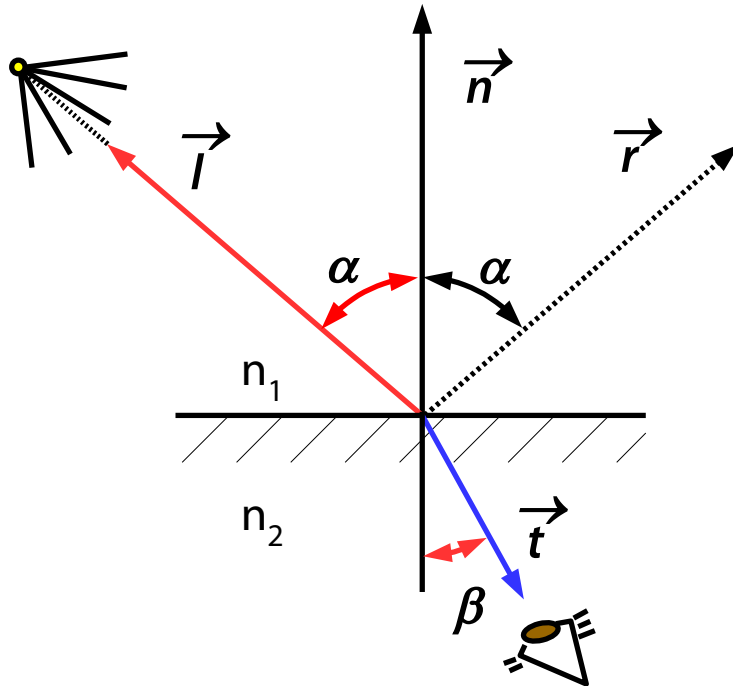
$$r = 2n(n \cdot l) - l$$

$$\frac{\sin \beta}{\sin \alpha} = \frac{n_1}{n_2} = n_{12}$$

Ratio of reflected and refracted light is determined by **Fresnel equations** (19. century)



# Refraction (Snell's law, Ibn Sahl, 984)



$$\frac{\sin \beta}{\sin \alpha} = \frac{n_1}{n_2} = n_{12}$$

$$\cos \beta = \sqrt{1 - n_{12}^2 \sin^2 \alpha} = \sqrt{1 - n_{12}^2 \cdot (1 - (n \cdot l)^2)}$$

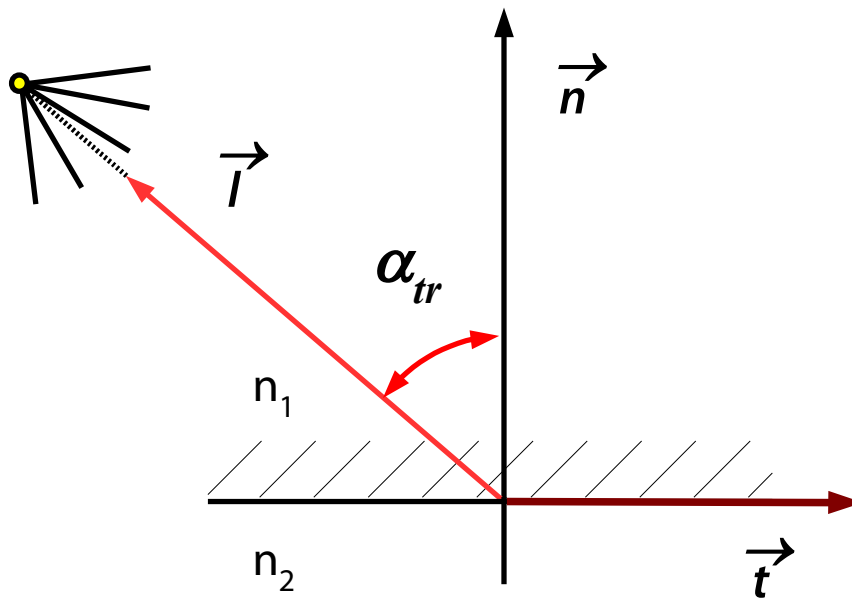
$$t = \left[ n_{12}(n \cdot l) - \sqrt{1 - n_{12}^2 \cdot (1 - (n \cdot l)^2)} \right] \cdot n - n_{12} \cdot l$$



# Total internal reflection

Going from more dense environment to less dense one ( $n_1 > n_2$ )

For incident angles greater than **critical angle**  $\alpha_{tr}$  there is no refraction at all!



$$\sin \alpha_{tr} = \frac{n_2}{n_1}$$





# Fresnel equations (polarization)

Two **polarizations** (electric field perpendicular „s“ /senkrecht/ or parallel „p“ to the incident plane)

Reflectance “R” and transmittance “T” (power ratios):

$$R_s = \left[ \frac{\sin(\beta - \alpha)}{\sin(\beta + \alpha)} \right]^2 \quad T_s = 1 - R_s$$

$$R_p = \left[ \frac{\tan(\beta - \alpha)}{\tan(\beta + \alpha)} \right]^2 \quad T_p = 1 - R_p$$



# Fresnel equations (alternative)

No need to compute angles (cosines are easy)

$$R_s = \left[ \frac{n_1 \cos \alpha - n_2 \cos \beta}{n_1 \cos \alpha + n_2 \cos \beta} \right]^2$$

$$R_p = \left[ \frac{n_1 \cos \beta - n_2 \cos \alpha}{n_1 \cos \beta + n_2 \cos \alpha} \right]^2$$



# Unpolarized light

Averaging values  $R_s$  a  $R_p$

$$R = \frac{1}{2} \frac{(a-u)^2 + b^2}{(a+u)^2 + b^2} \left[ \frac{(a+u-1/u)^2 + b^2}{(a-u+1/u)^2 + b^2} + 1 \right]$$

$$a^2 = \frac{1}{2} \left( \sqrt{(n_\lambda^2 - k_\lambda^2 + u^2 - 1)^2 + 4n_\lambda^2 k_\lambda^2} + n_\lambda^2 - k_\lambda^2 + u^2 - 1 \right)$$

$$b^2 = \frac{1}{2} \left( \sqrt{(n_\lambda^2 - k_\lambda^2 + u^2 - 1)^2 + 4n_\lambda^2 k_\lambda^2} - n_\lambda^2 + k_\lambda^2 - u^2 + 1 \right)$$

$$u = \cos \alpha = n \cdot l \quad n = n_\lambda - i k_\lambda \quad (\text{for dielectric } k_\lambda = 0)$$



# Dielectric (insulator) materials

$$k_\lambda = 0 \quad \Rightarrow \quad a^2 = n_\lambda^2 + u^2 - 1 \quad b = 0$$

$$R = \frac{1}{2} \frac{(a-u)^2}{(a+u)^2} \left( \frac{[u(a+u)-1]^2}{[u(a-u)+1]^2} + 1 \right)$$



# Remarks (Fresnel)

If  $\alpha = \pi/2$  (i.e.  $\mathbf{u} = \mathbf{0}$ ), then reflectance  $R_\lambda(\mathbf{90}) = 1$  regardless of the wavelength  $\lambda$

For perpendicular ray ( $\alpha = \mathbf{0}$ ):  $R_0 = R_s = R_p = \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2$

$$T_0 = T_s = T_p = 1 - R_0 = \frac{4n_1n_2}{(n_2 + n_1)^2}$$



# Wavelength $\lambda$

For  $\mathbf{l}$  and  $\mathbf{v}$  perpendicular to the surface (i.e.  $\mathbf{a} = \mathbf{0}$ ):

$$F(\lambda, \mathbf{0}) = \left( \frac{n_\lambda - 1}{n_\lambda + 1} \right)^2 \quad \text{a} \quad n_\lambda = \frac{1 + \sqrt{F(\lambda, \mathbf{0})}}{1 - \sqrt{F(\lambda, \mathbf{0})}}$$

Quantities  $F_\lambda(\mathbf{0})$  were measured in labs for many real materials  
(both conductors and insulators)

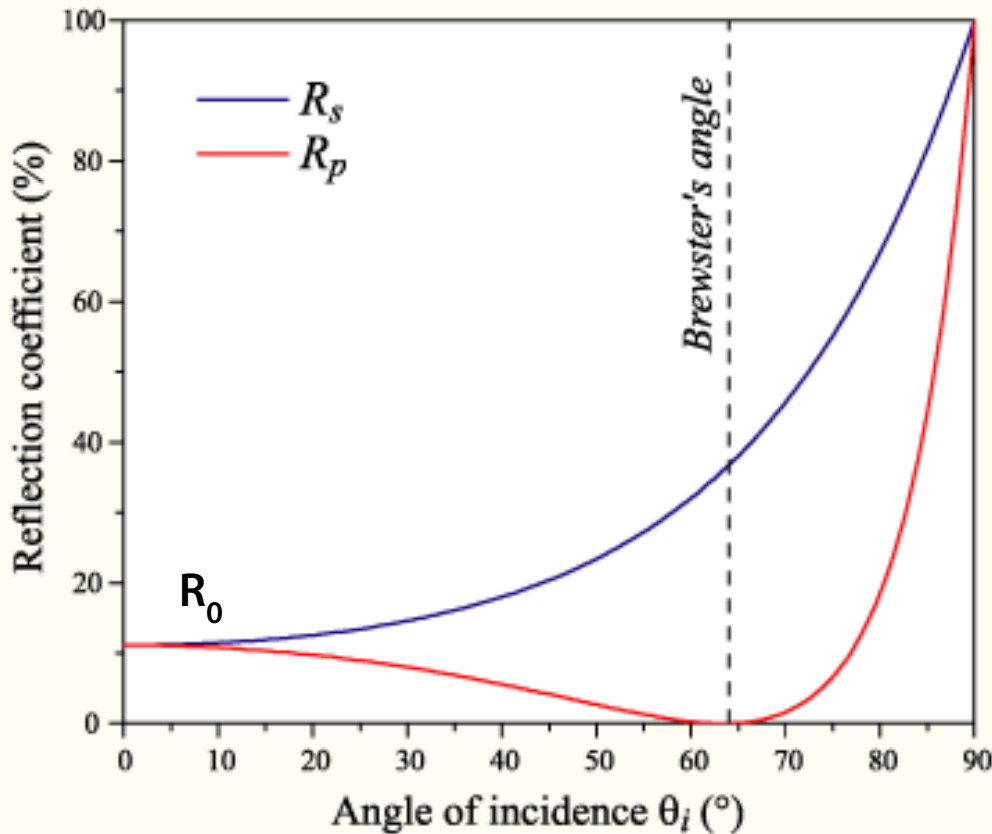
– so we know the  $n_\lambda$  indices

Specular reflection **depends on  $\lambda$**  (except for  $\alpha = \pi/2$ )

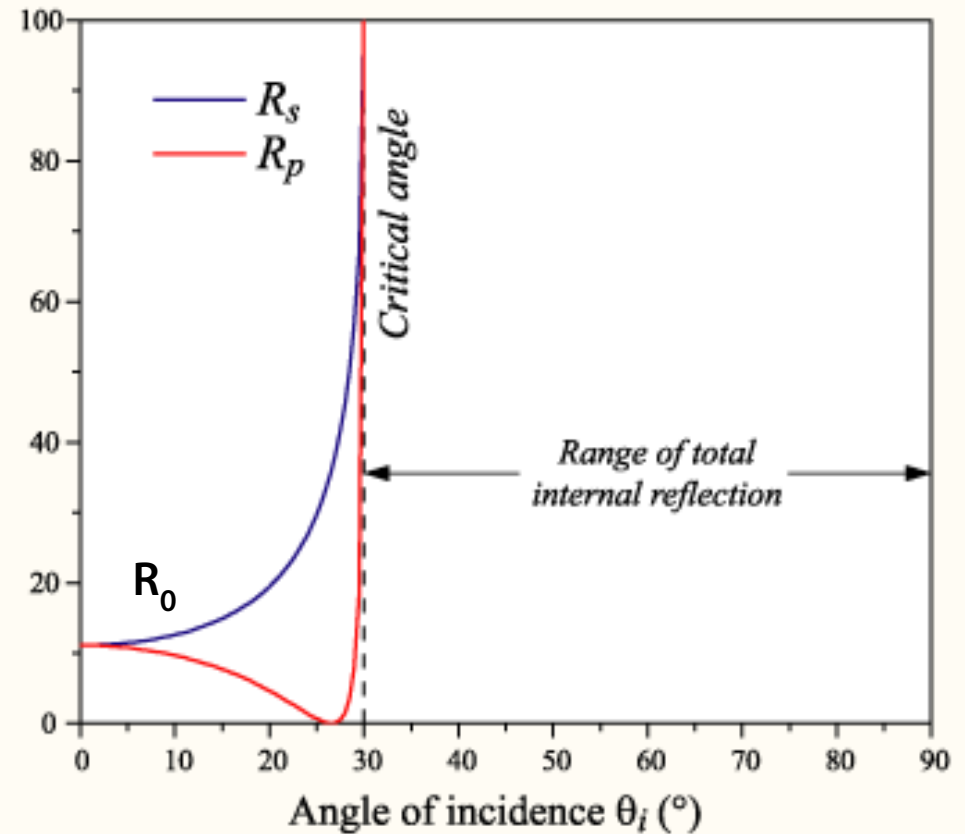


# Reflectance – dielectric material

$n_1 = 1.0, n_2 = 2.0$



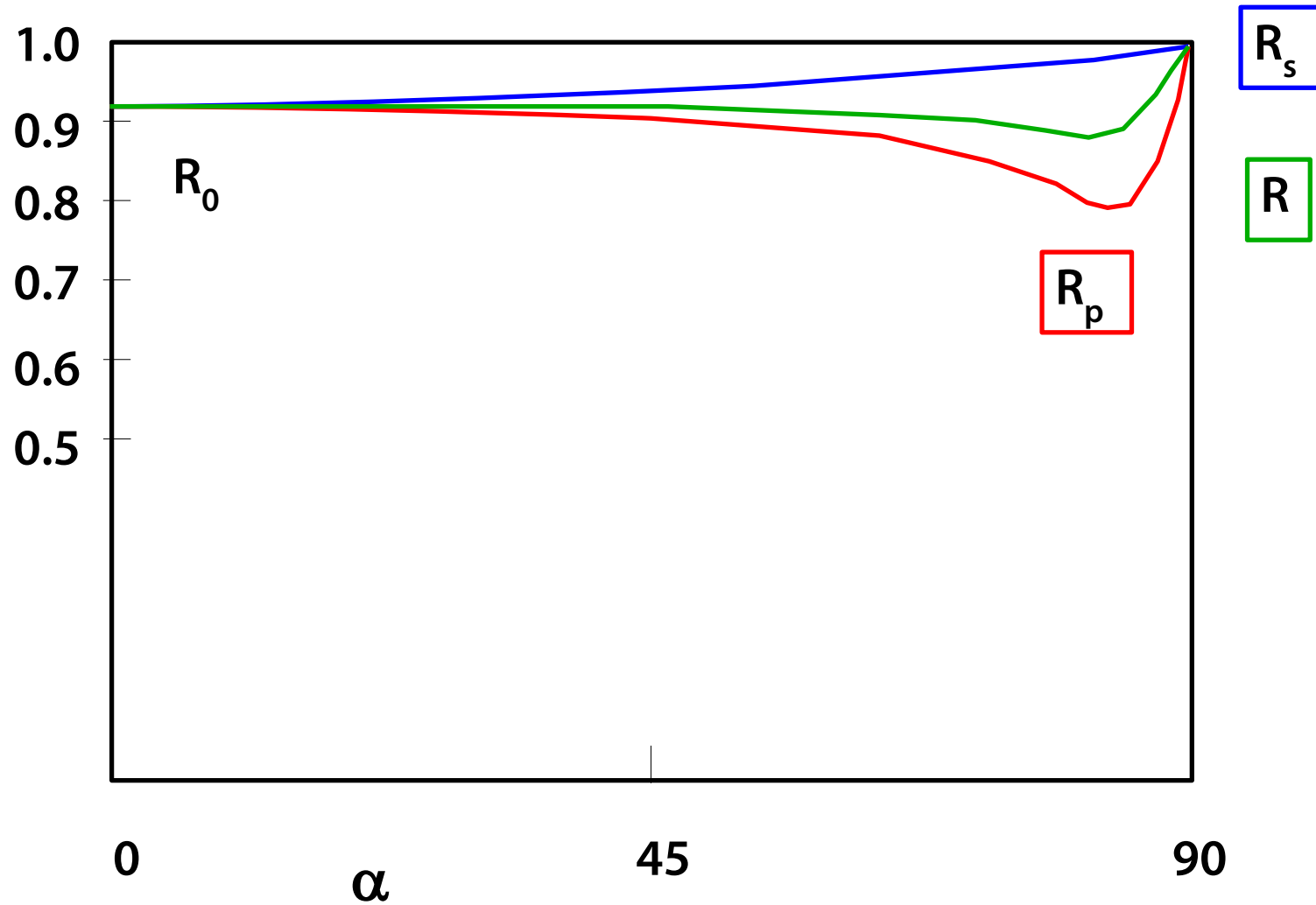
$n_1 = 2.0, n_2 = 1.0$



© Ulflund, Wiki



# Reflectance - metal

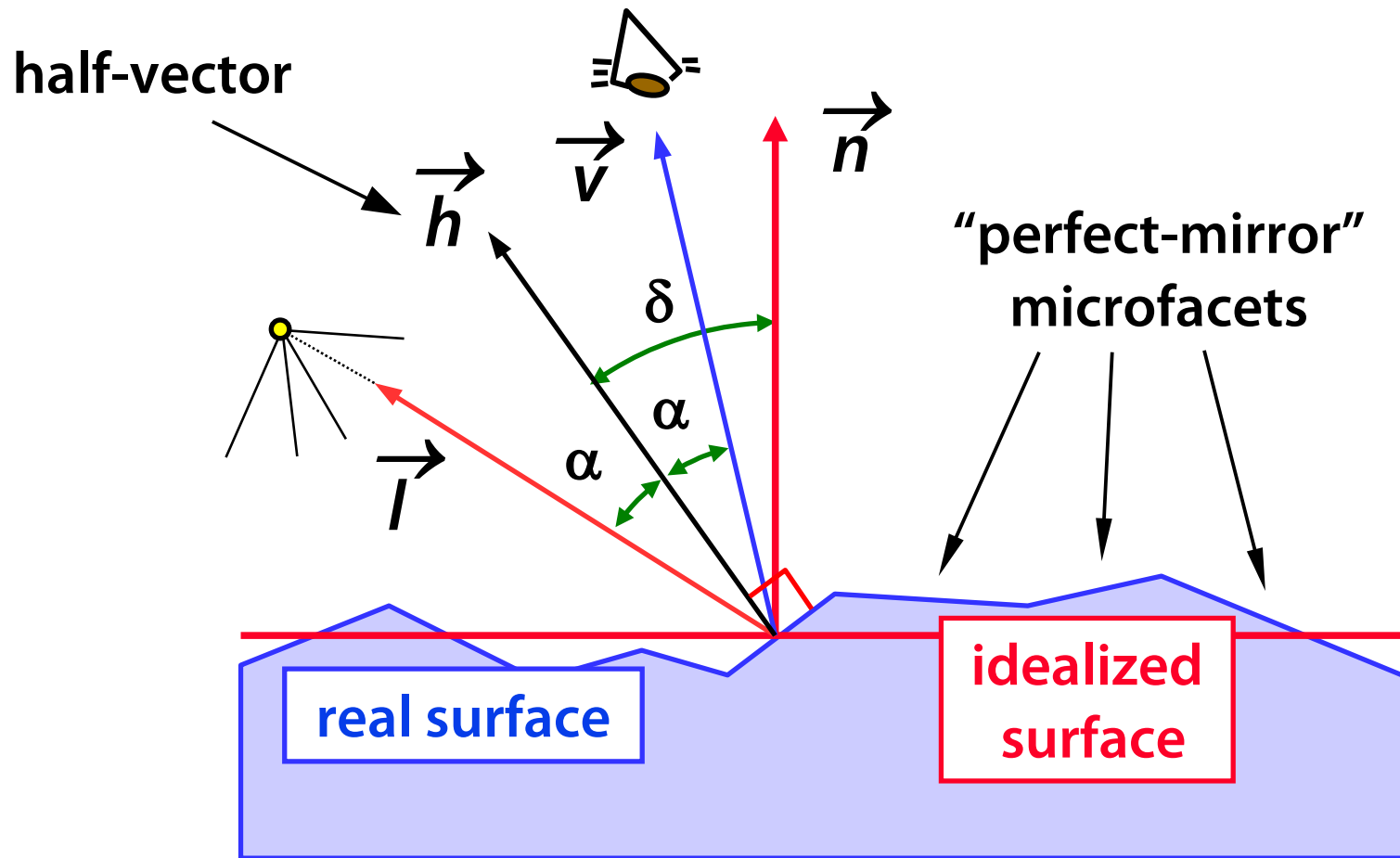






# Microfacet theory

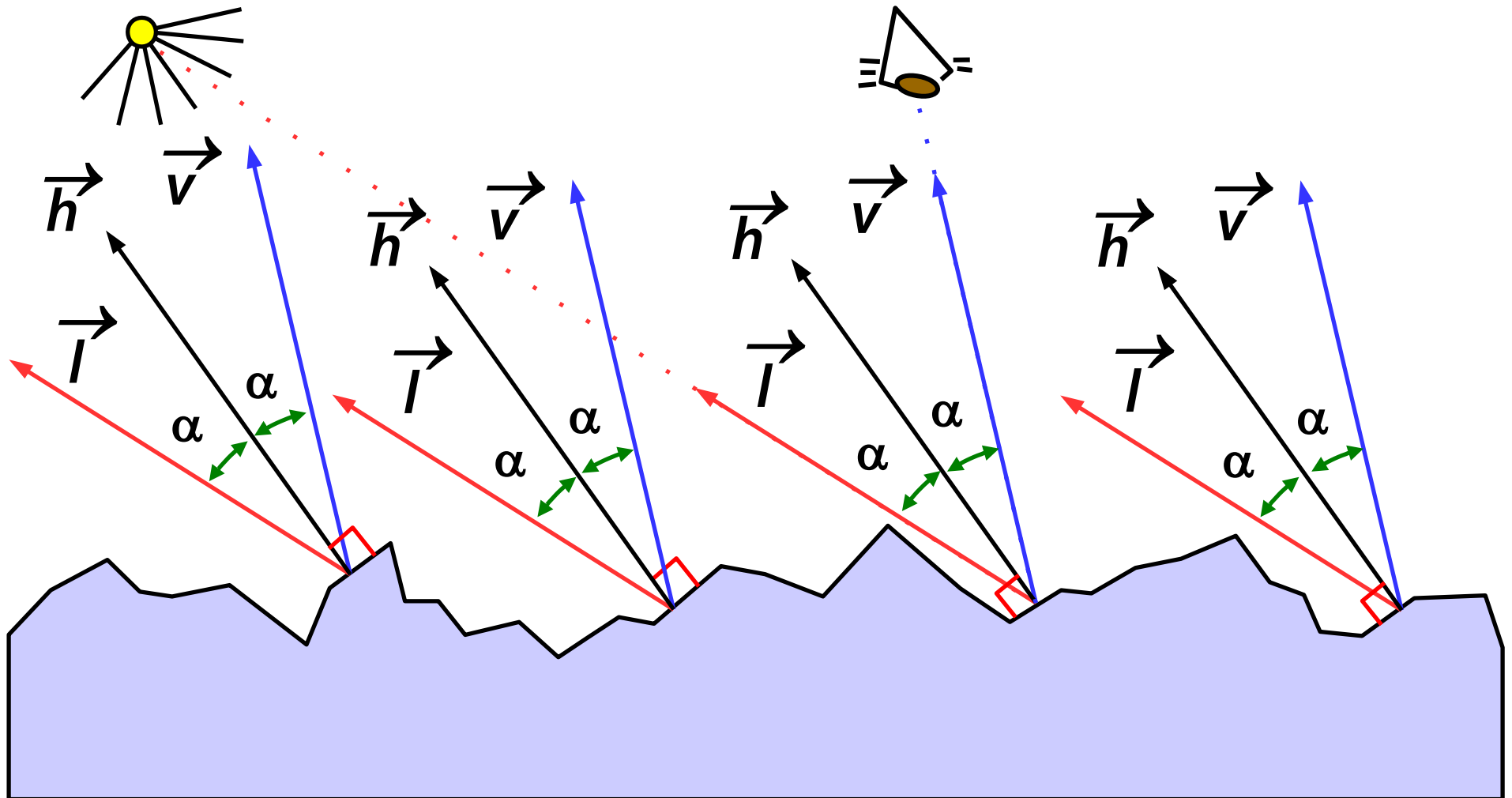
Beckmann, Spizzichino (63), Torrance, Sparrow (67)



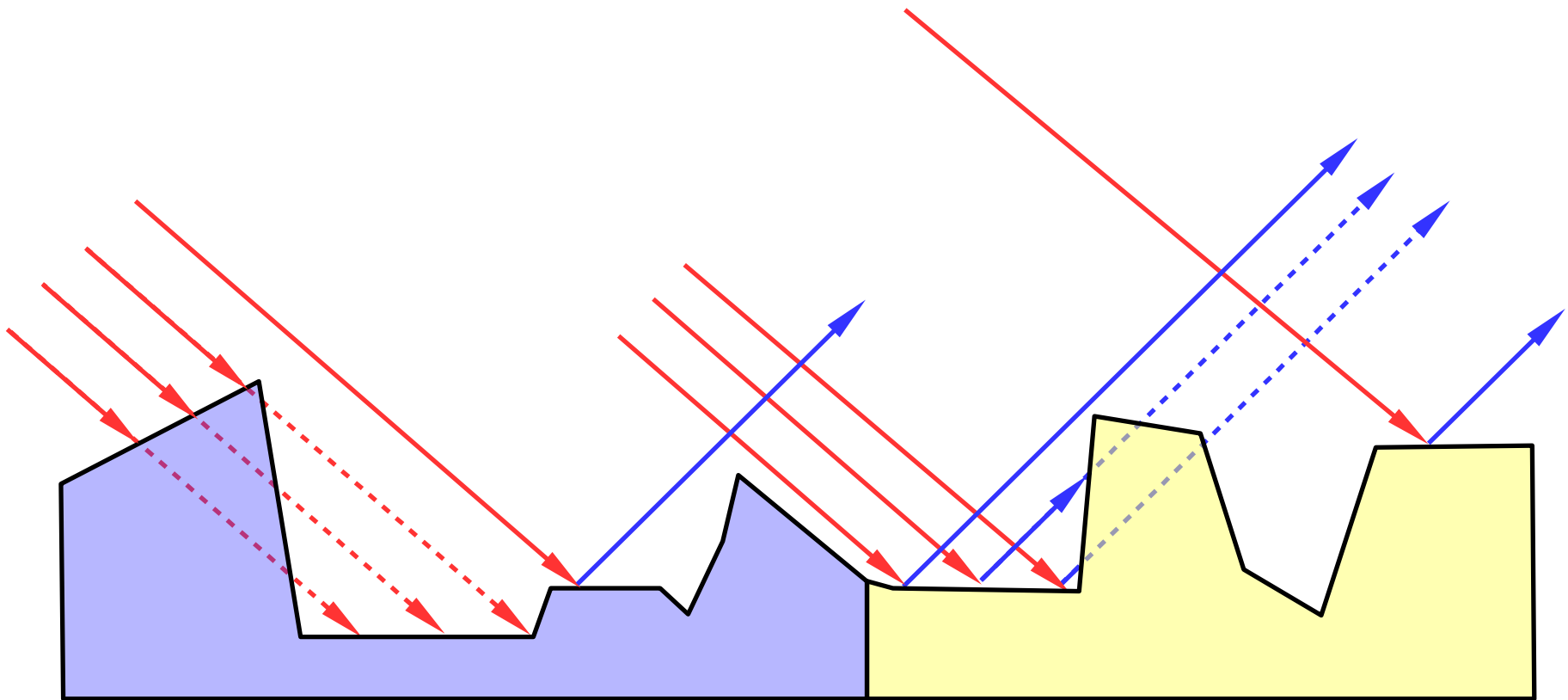
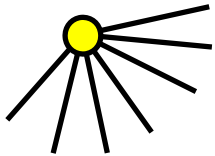


# Perfect reflection for half-angle

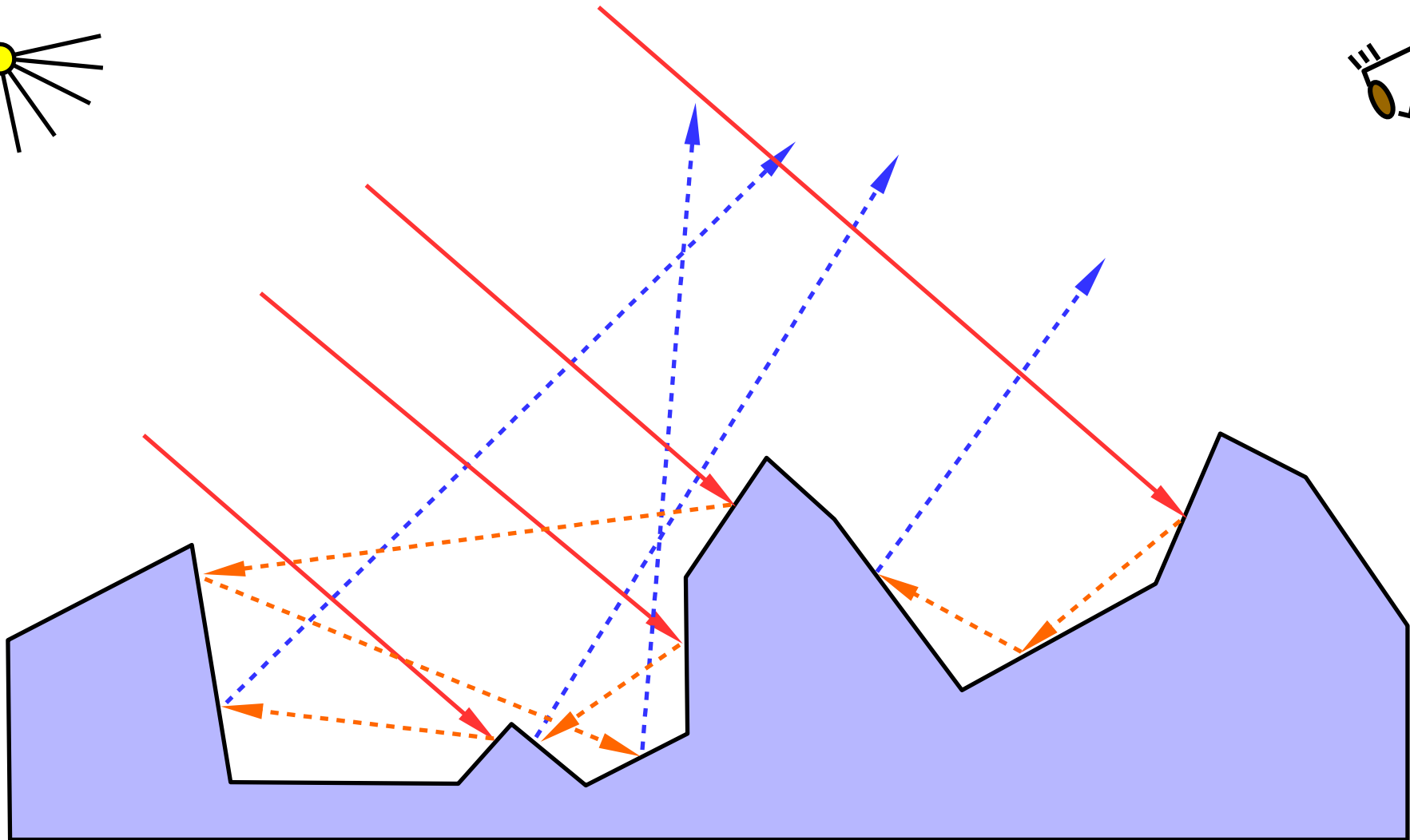
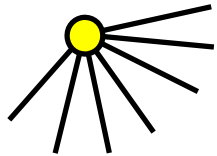
Only ideal half-angle microfacets can contribute !



# Shadowing and masking



# Multiple bounces are lost





# Microfacet specular BRDF

$$R_{\lambda}(h) = \frac{F_{\lambda}(\alpha)}{4} \cdot \frac{D(h) \cdot G(l, v, h)}{(n \cdot l)(n \cdot v)}$$

$R_{\lambda}(h)$  ... specular reflectance for wavelength  $\lambda$

$F_{\lambda}(\alpha)$  ... Fresnel ideal reflectance for wavelength  $\lambda$  and incident angle  $\alpha$

$D(h)$  ... microfacet PDF ("how many microfacets" have normal vector  $h$ )

$G(l, v, h)$  ... geometric factor (shadowing & masking)



# Fresnel term F

Fresnel term for unpolarized light

$$F(\lambda, \beta) = \frac{1}{2} \cdot \frac{(\mathbf{g} - \mathbf{c})^2}{(\mathbf{g} + \mathbf{c})^2} \left\{ 1 + \frac{[\mathbf{c}(\mathbf{g} + \mathbf{c}) - \mathbf{1}]^2}{[\mathbf{c}(\mathbf{g} - \mathbf{c}) - \mathbf{1}]^2} \right\}$$

for  $\mathbf{c} = \cos \beta = (\mathbf{V} \cdot \mathbf{H})$ ,

$$\mathbf{g}^2 = n_\lambda^2 + \mathbf{c}^2 - 1$$

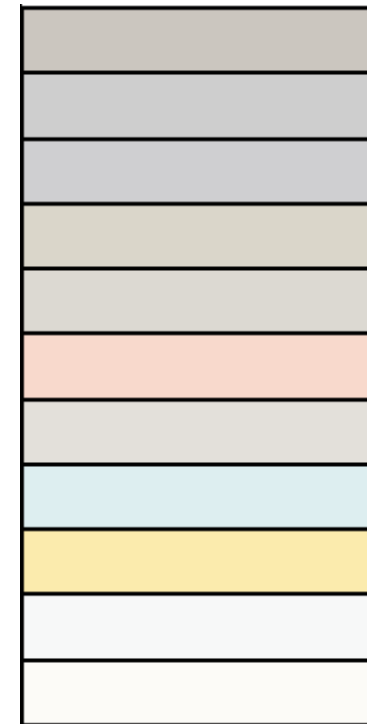
$n_\lambda$  ... index of refraction for wavelength  $\lambda$

For conductors  $\mathbf{n}'_\lambda = \mathbf{n}_\lambda - i \kappa_\lambda$  ( $\kappa_\lambda$  ... absorption coefficient)



# Fresnel – base specular color

Metal	F(0) [linear]	F(0) [sRGB]
Titanium	0.542, 0.497, 0.449	194, 187, 179
Chromium	0.549, 0.556, 0.554	196, 197, 196
Iron	0.562, 0.565, 0.578	198, 198, 200
Nickel	0.660, 0.609, 0.526	212, 205, 192
Platinum	0.673, 0.637, 0.585	214, 209, 201
Copper	0.955, 0.638, 0.538	250, 209, 194
Palladium	0.733, 0.697, 0.652	222, 217, 211
Zinc	0.664, 0.824, 0.850	213, 234, 237
Gold	1.022, 0.782, 0.344	255, 229, 158
Aluminum	0.913, 0.922, 0.924	245, 246, 246
Silver	0.972, 0.960, 0.915	252, 250, 245





# Schlick's approximation

Fresnel term for other angles, based on  $F_{\lambda}(\mathbf{0}) = c$

$$F_{\text{schlick}}(c, \vec{l}, \vec{h}) = c + (1 - c) (1 - (\vec{l} \cdot \vec{h}))^5$$





# Any angle, any $\lambda$ (R. Hall)

Let's assume we have a **base material color**  $F_\lambda(\mathbf{0})$  and an **angle-function** for some (standard)  $\lambda_0$

- set of wavelengths can be limited (3÷6 components)

$$F_\lambda(\alpha) \approx F_\lambda(\mathbf{0}) + (1 - F_\lambda(\mathbf{0})) \frac{\max(0, F_{\lambda_0}(\alpha) - F_{\lambda_0}(\mathbf{0}))}{1 - F_{\lambda_0}(\mathbf{0})}$$



# Normal distribution function

Fast and simple formula – Gaussian distribution:

$$\underline{D(\vec{h}, m)} = \chi(\vec{n} \cdot \vec{h}) (\vec{n} \cdot \vec{h}) \cdot e^{-\left(\frac{\delta}{m}\right)^2}$$

$$\chi(a) = a > 0 ? 1 : 0$$

$$\cos \delta = \vec{n} \cdot \vec{h}$$

$m$  ... “surface roughness” (standard deviation of the surface slope)

< 0.1 ... very smooth

> 0.8 ... rough (almost diffuse)



# Beckmann's distribution (~normalised)

$$\begin{aligned} \underline{D_{be}(\vec{h}, m)} &= \frac{\chi(\vec{n} \cdot \vec{h})}{\pi m^2 (\vec{n} \cdot \vec{h})^4} e^{-\left(\frac{\tan \delta}{m}\right)^2} \\ &= \frac{\chi(\vec{n} \cdot \vec{h})}{\pi m^2 (\vec{n} \cdot \vec{h})^4} e^{\frac{(n \cdot h)^2 - 1}{m^2 (n \cdot h)^2}} \end{aligned}$$



# Blinn-Phong (normalised)

$$\underline{D_{bp}}(\vec{h}, m) = \chi(\vec{n} \cdot \vec{h}) \frac{m+2}{2\pi} (\vec{n} \cdot \vec{h})^m$$



$$\underline{D_{tr}(\vec{h}, m)} = \frac{\chi(\vec{n} \cdot \vec{h}) m^2}{\pi \left( (\vec{n} \cdot \vec{h})^2 (m^2 - 1) + 1 \right)^2}$$

$m$  can be greater than 1



# GGX (Walter et al. 2007)

$$\underline{D_{GGX}(\vec{h}, m)} = \frac{\chi(\vec{n} \cdot \vec{h}) m^2}{\pi (\vec{n} \cdot \vec{h})^4 (m^2 + \tan^2 \delta)^2}$$



# Isotropic Ward (1992)

$$\underline{D_{wiso}(\vec{h}, m)} = \frac{\chi(\vec{n} \cdot \vec{h})}{\pi m^2} e^{-\frac{\tan^2 \delta}{m^2}}$$



# Anisotropic Ward (1992)

$$\underline{D_w(\vec{h}, m_x, m_y)} = \frac{\chi(\vec{n} \cdot \vec{h})}{\pi m_x m_y} e^{-\tan^2 \delta \left( \frac{\cos^2 \phi_h}{m_x^2} + \frac{\sin^2 \phi_h}{m_y^2} \right)}$$

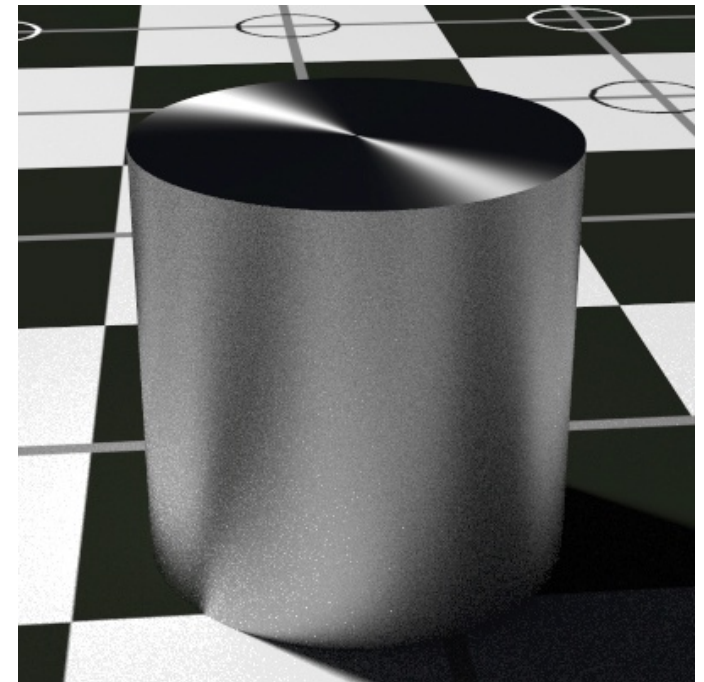
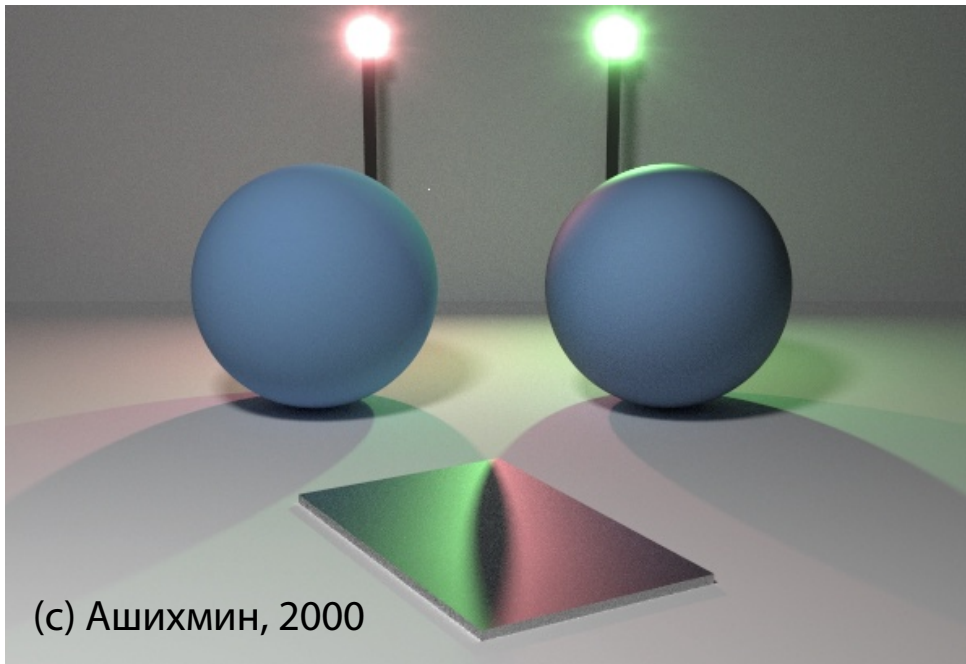
$\phi_h$  ... azimuth angle of the half-vector





# Ашихмин-Shirley anisotropic (2000)

$$\underline{D_{as}(\vec{h}, e_x, e_y)} = \sqrt{(e_x + 1)(e_y + 1)} (\vec{h} \cdot \vec{n})^{e_x \cos^2 \phi_h + e_y \sin^2 \phi_h}$$





# Material blends

Idea of **blending several materials together**  
makes sense:  $\mathbf{m}_1 \dots \mathbf{m}_k$

$$\mathbf{D}(\alpha) = \sum_{i=1}^k \mathbf{w}_i \cdot \mathbf{D}(\mathbf{m}_i, \alpha)$$

$\mathbf{w}_i \dots$  weight coefficients

$$\sum \mathbf{w}_i = 1$$



# Geometric term G (Cook-Torrance)

Compensation for masking and shadowing

$$\underline{G_{ct}}(\vec{l}, \vec{v}, \vec{h}) = \min\left(1, \frac{2(\vec{n} \cdot \vec{h})(\vec{n} \cdot \vec{v})}{(\vec{v} \cdot \vec{h})}, \frac{2(\vec{n} \cdot \vec{h})(\vec{n} \cdot \vec{l})}{(\vec{v} \cdot \vec{h})}\right)$$

Cook and Torrance assumed infinitely long “V”-shaped grooves  
(not exactly true)

- optimized: **Kelemen**, more accurate: **Smith**



# G alternative (GGX)

$$\underline{G_{GGX}}(\vec{l}, \vec{v}, \vec{h}) = \chi\left(\frac{\vec{v} \cdot \vec{h}}{\vec{v} \cdot \vec{n}}\right) \frac{2}{1 + \sqrt{1 + m^2 \tan^2 \theta_v}}$$

$m$  ... roughness from the GGX distribution

# Cheap G (Kelemen-Szirmay-Kalos)



$$\frac{G_{KSK}(\vec{l}, \vec{v}, \vec{h})}{(\vec{n} \cdot \vec{l})(\vec{n} \cdot \vec{v})} \approx \frac{1}{(\vec{l} \cdot \vec{h})^2}$$



# Blinn's contribution (1977)

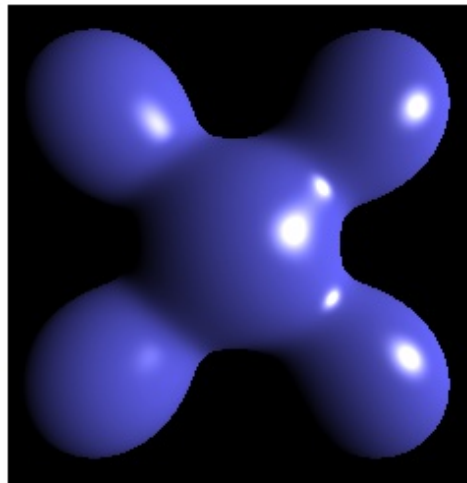
Blinn-Phong model (here  $\beta = \angle \vec{v}, \vec{r}$ )

– light source and viewer in infinity

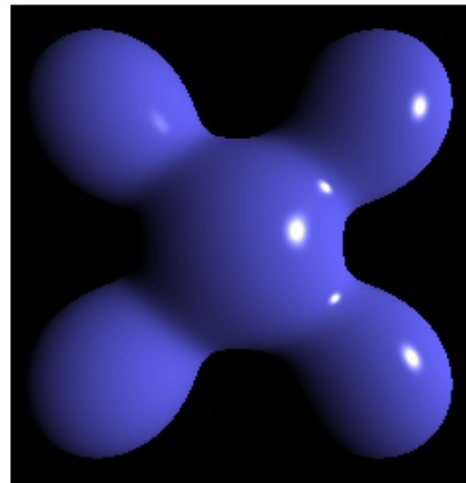
$$\cos^h \beta \approx \cos^{4h} \beta / 2$$

$$(\vec{r}_i \cdot \vec{v})^h \approx (\vec{h}_i \cdot \vec{n})^{4h}$$

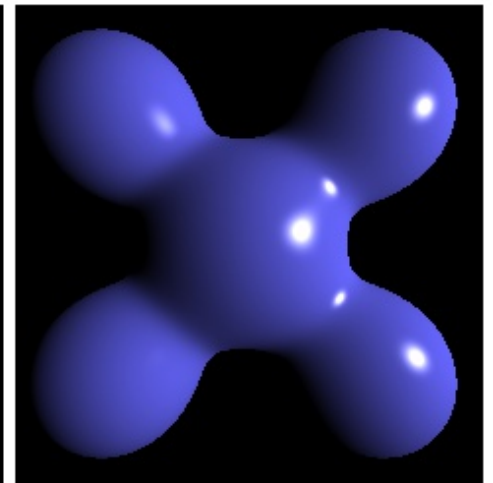
(cc) Wiki



**Blinn-Phong**



**Phong**



**Blinn-Phong**  
(higher exponent)



# Schlick's contribution

**Christophe Schlick** (1994) was experimenting with approximate substitution in Fresnel term

Fraction instead of power function

$$S^h \approx S / (h - hS + S)$$

– slightly less sharp highlight compared to Blinn-Phong

Fresnel term substitute (32× faster, error <1%)

$$R_{\text{schlick}}(\mathbf{c}, \vec{\mathbf{l}}, \vec{\mathbf{n}}) = \mathbf{c} + (1 - \mathbf{c}) (1 - (\vec{\mathbf{l}} \cdot \vec{\mathbf{n}}))^5$$



# Lafortune model (1997)

## Generalized cosine lobe model

– derived using Householder matrix ( $3 \times 3$ )

1. classical specular term (Phong) ...

$$f_r(l, v) = \rho_s C_s \cos^h \beta$$

2. ... rewritten using Householder matrix notation

$$\begin{aligned} f_r(l, v) &= \rho_s C_s (r \cdot v)^h \\ &= \rho_s C_s \left[ l^T (2nn^T - I) v \right]^h \end{aligned}$$





# General plausible cosine lobe

Householder matrix  $\mathbf{M}$  ( $3 \times 3$ )

– for reciprocity it must be **symmetrical**

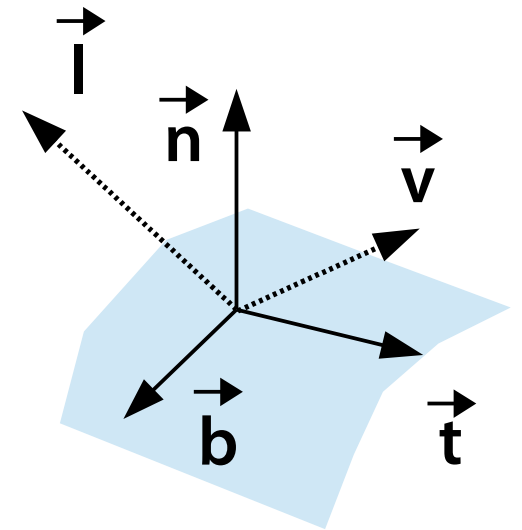
$$f_r(l, v) = \rho_s [l^T M v]^h$$

SVD decomposition of matrix  $\mathbf{M}$ :

$$f_r(l, v) = \rho_s [l^T Q^T D Q v]^h$$

$\mathbf{Q}$  ... coordinate transform,  $\mathbf{D}$  ... diagonal matrix

$$f_r(l, v) = \rho_s [C_b l_b v_b + C_t l_t v_t + C_n l_n v_n]^h$$





# Cosine lobe options

Phong lobe:  $-C_b = -C_t = C_n = \sqrt{C_s}$

General isotropic reflection:  $C_b = C_t$

Anisotropy:  $C_b \neq C_t$

Isotropic diffuse term:  $C_b = C_t = 0, C_s = (h+2)/2\pi$

Off-specular reflection:  $C_n < -C_b = -C_t$

Retro-reflections:  $C_b, C_t, C_n > 0$

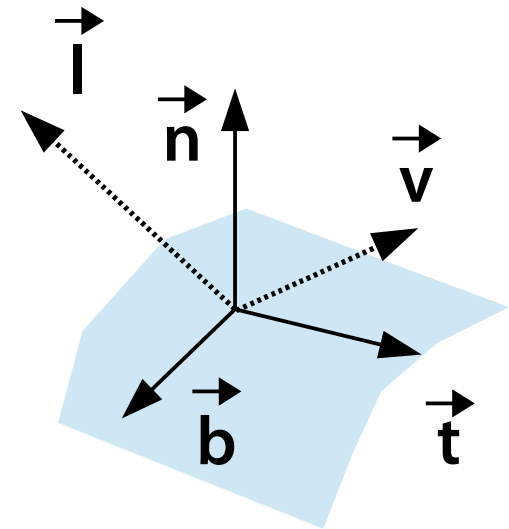


# Compound model

Superposition of several lobes

– each one is defined by:  $C_{b,i}$   $C_{t,i}$   $C_{n,i}$   $h_i$

$$f_r(\vec{l}, \vec{v}) = \sum_i \left[ C_{b,i} \vec{l}_b \vec{v}_b + C_{t,i} \vec{l}_t \vec{v}_t + C_{n,i} \vec{l}_n \vec{v}_n \right] h_i$$





# Approximation from measured data

## Roughened aluminium

- three lobes
- optimized for minimal difference from measured data

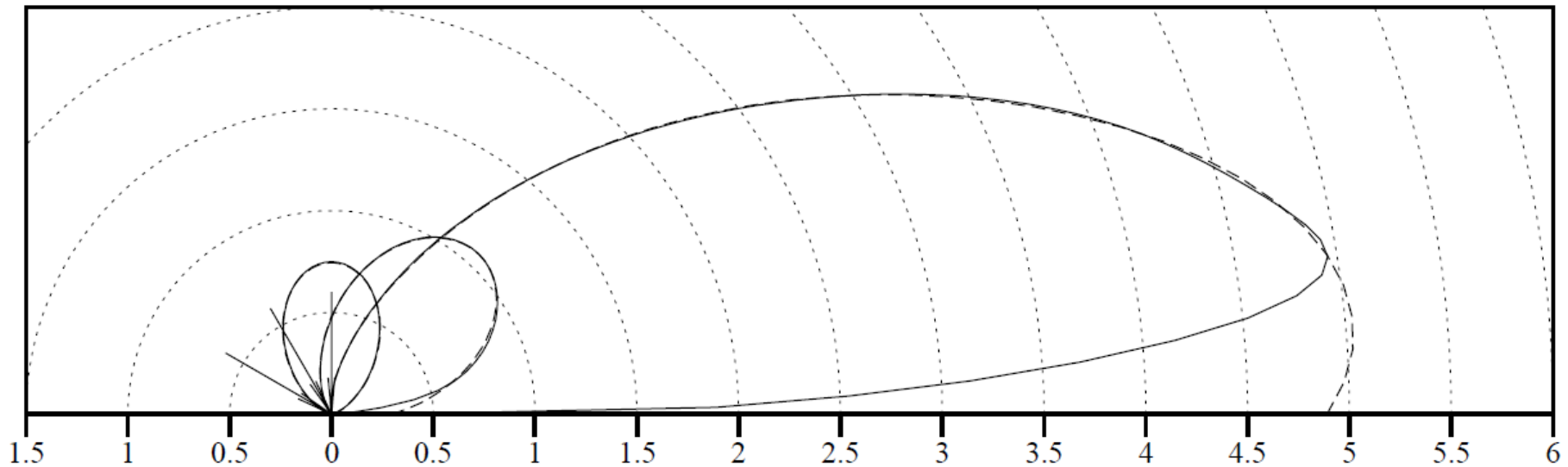


Figure 11: Polar plots of the fitted reflectance model (dashed lines) against the original physically-based model of a roughened aluminum surface (solid lines) in the plane of incidence, for  $\theta = 0^\circ, 30^\circ, 60^\circ$ , at  $500nm$ . The reflectance function becomes more off-specular and strongly increases in size towards grazing angles. The sum of generalized cosine functions captures these effects.

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# Lafortune – results

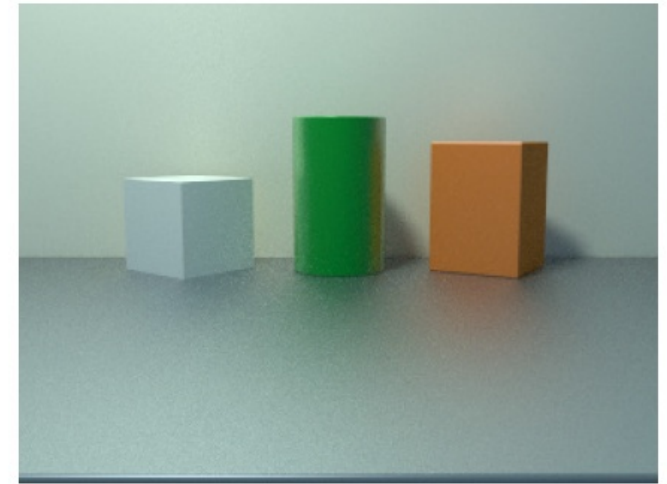
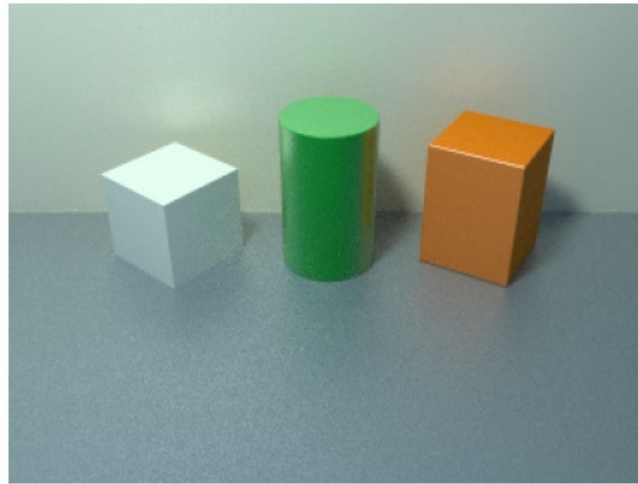
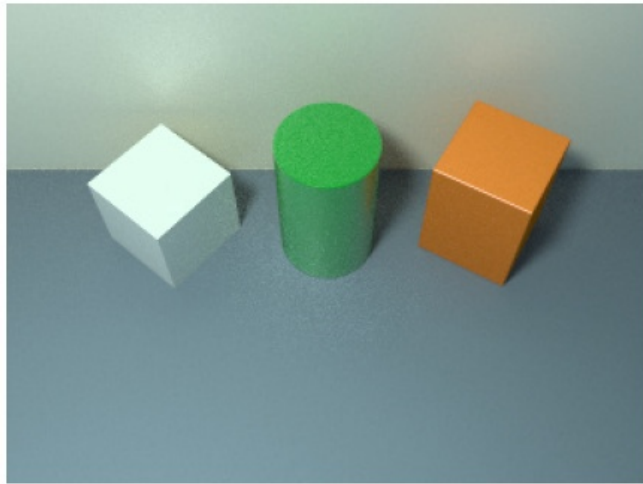
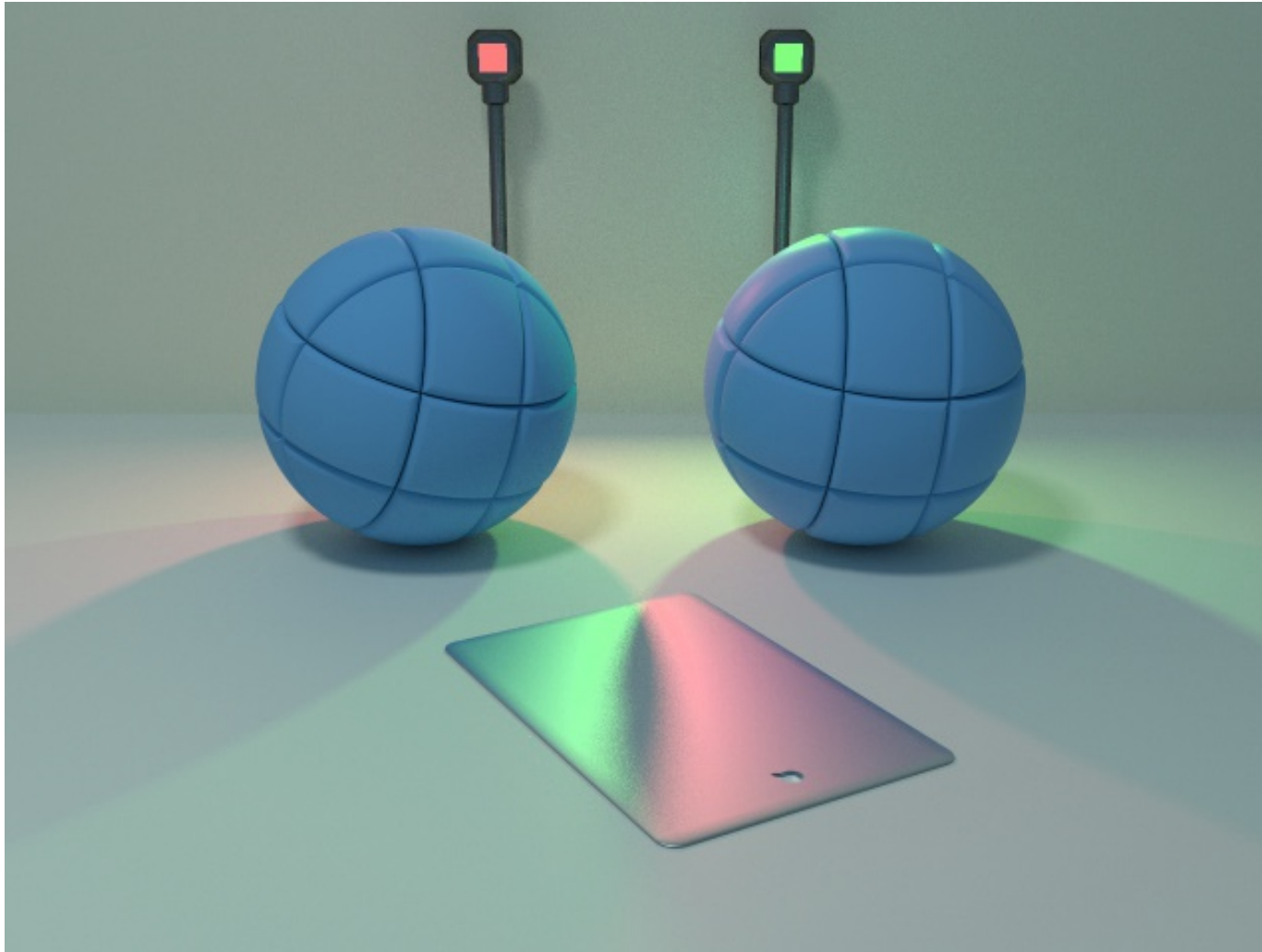


Figure 10: Rendered pictures of a scene with the generalized cosine lobe model. The off-specular directional-diffuse reflectance of the table surface gradually increases for grazing angles.

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# Lafortune – results



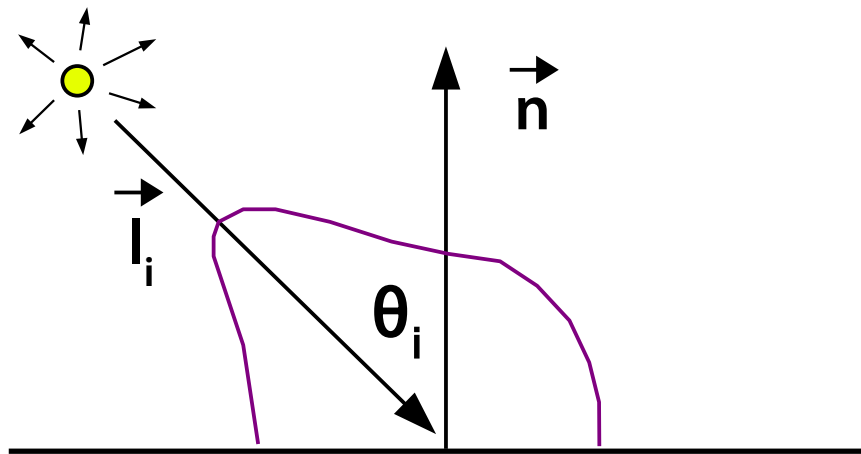
© 1997 Lafortune et al.



# Lambert law is not perfect...

Pure "cosine" surface is not as common in the nature

- rough, grainy surfaces (sandpaper, sand, etc.)
- **full moon** – contours should be darker but actually **they are not!**
- "back-scattering" effect, see reflecting (passive) taillights





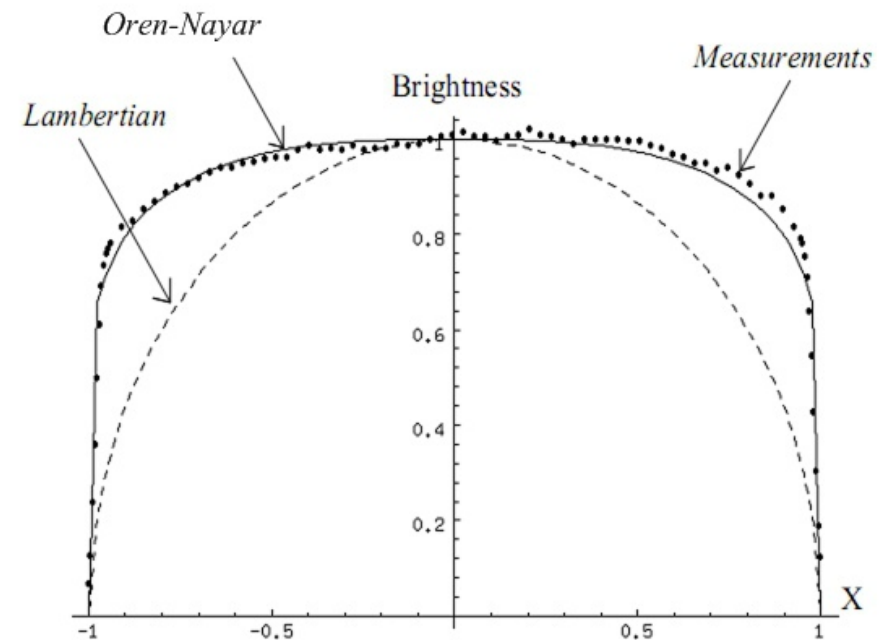
# Oren-Nayar model (1993)

Based on **microfacet idea**

- diffuse reflection on microfacets
- simplified formulas – only most important ones

$$E_d = \frac{\rho}{\pi} \cdot E_0 \cdot \cos(\theta_i) \cdot (A + B \cdot \max(0, \cos(\phi_o - \phi_i))) \cdot \sin(\alpha) \cdot \tan(\beta)$$

$\theta_i$	incoming angle ( $\angle \vec{l}_i, \vec{n}$ )
$\theta_o$	outgoing angle ( $\angle \vec{v}, \vec{n}$ )
$\Phi_i$	incoming azimuth of $\omega_i$
$\Phi_o$	outgoing azimuth of $\omega_o$
$\alpha$	$\max\{\theta_i, \theta_o\}$
$\beta$	$\min\{\theta_i, \theta_o\}$







# Oren-Nayar, final formula

$$E_d = (C_L \circ C_D) \cdot \cos(\theta_i) \cdot (A + B \cdot \max(0, \cos(\phi_o - \phi_i))) \cdot \sin(\alpha) \cdot \tan(\beta)$$

$$A = 1 - 0.5 \cdot \frac{\sigma^2}{\sigma^2 + 0.33} \quad (\text{value in denominator} - \text{up to } 0.57)$$

$$B = 0.45 \cdot \frac{\sigma^2}{\sigma^2 + 0.09}$$

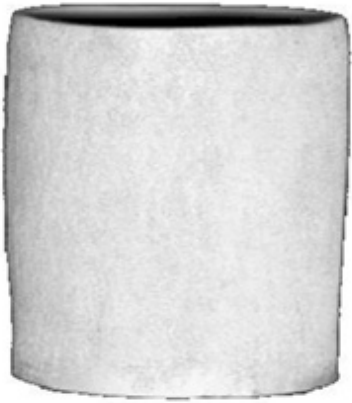
$\sigma$       **roughness:** mean value of  $h$  (see Cook-Torrance)

$C_L$       light source color

$C_D$       material color



# Oren-Nayar – examples



Real Image



Lambertian Model

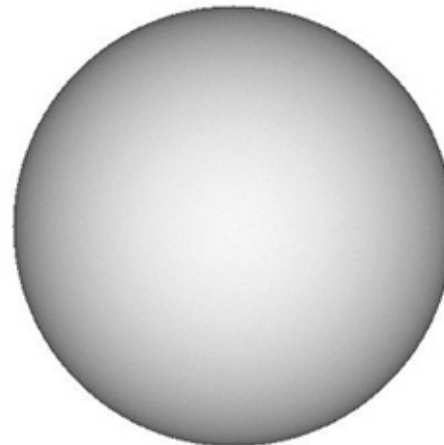


Oren-Nayar Model

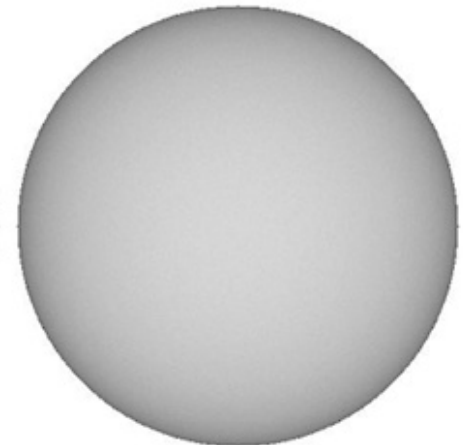
© Jwgu, Wiki,  
public domain



$\sigma = 0$



$\sigma = 0.1$



$\sigma = 0.3$



# Layered model (Weidlich-Wilkie)

a)	b)	c)	d)	e)
Glossy Paint	Tinted Glazing	Frosted Paint	Metal Foil	Metallic Paint
<b>Interfaces:</b>		Diffuse	<b>Materials:</b>	
Torrance-Sparrow		Smooth	Coloured Solid	Metal
				Colourless

© 2007 A. Weidlich, A. Wilkie



# Layered model (Weidlich-Wilkie)

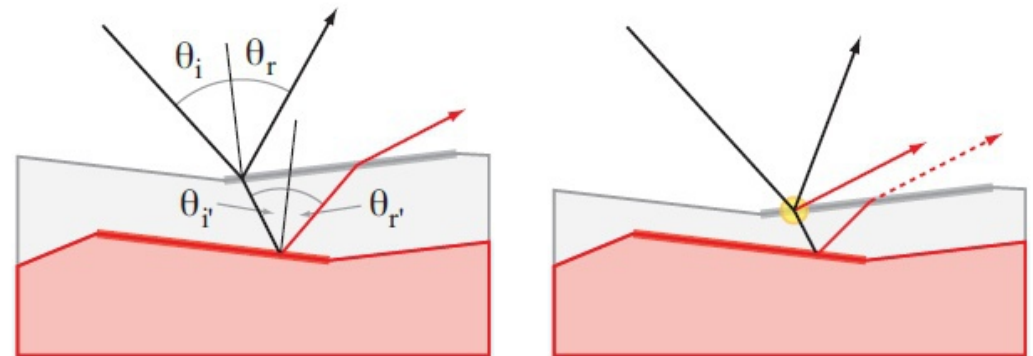
Several **very thin** transparent/tinted layers of “paint”

- clear or tinted material

**Interface** options

- smooth, diffuse or microfacets

A couple of geometric simplifications/assumptions



© 2007 A. Weidlich, A. Wilkie

# Weidlich-Wilkie – results

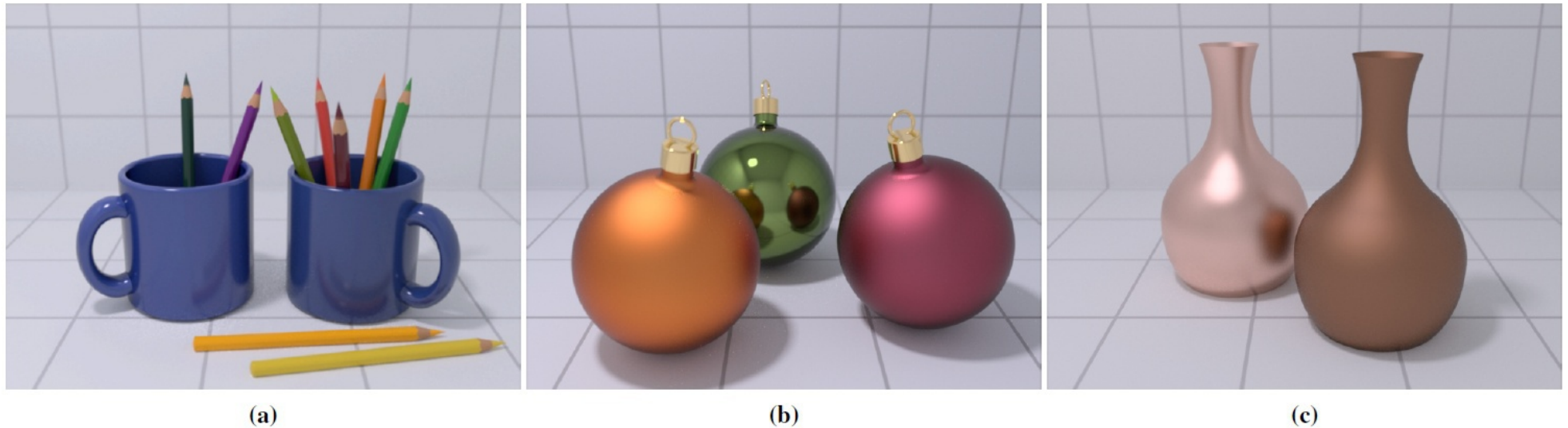


Figure 8: Left: Examples of surface type **a**) applied to crayons and two mugs. Middle: Examples of surface types **d**) (metal foil, the sphere in the background) and **f**) (frosted metal, the two spheres in the foreground). Right: Two copper vases without (left) and with patina (right). The one with patina is an example of surface type **g**).

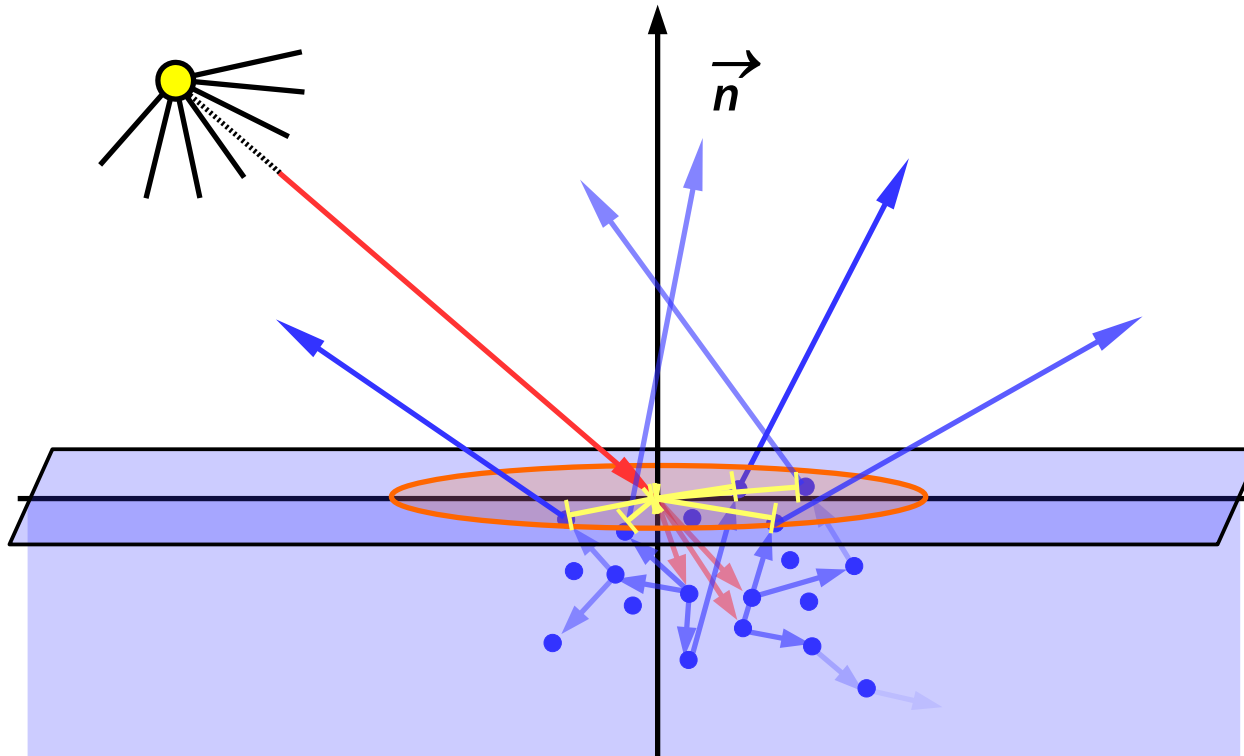


Figure 5: Diffuse white spheres with a yellow varnish layer of varying thickness. Layer thickness values are 0.0, 0.4, 0.8, 1.6, 3.0, 5.0 and 15.0. Note the progressive changes in colour, saturation and hue.

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# Sub-surface scattering of light (BSSRDF)



“Bi-directional Scattering-Surface Reflectance Distribution Function”

# BSSRDF – marble

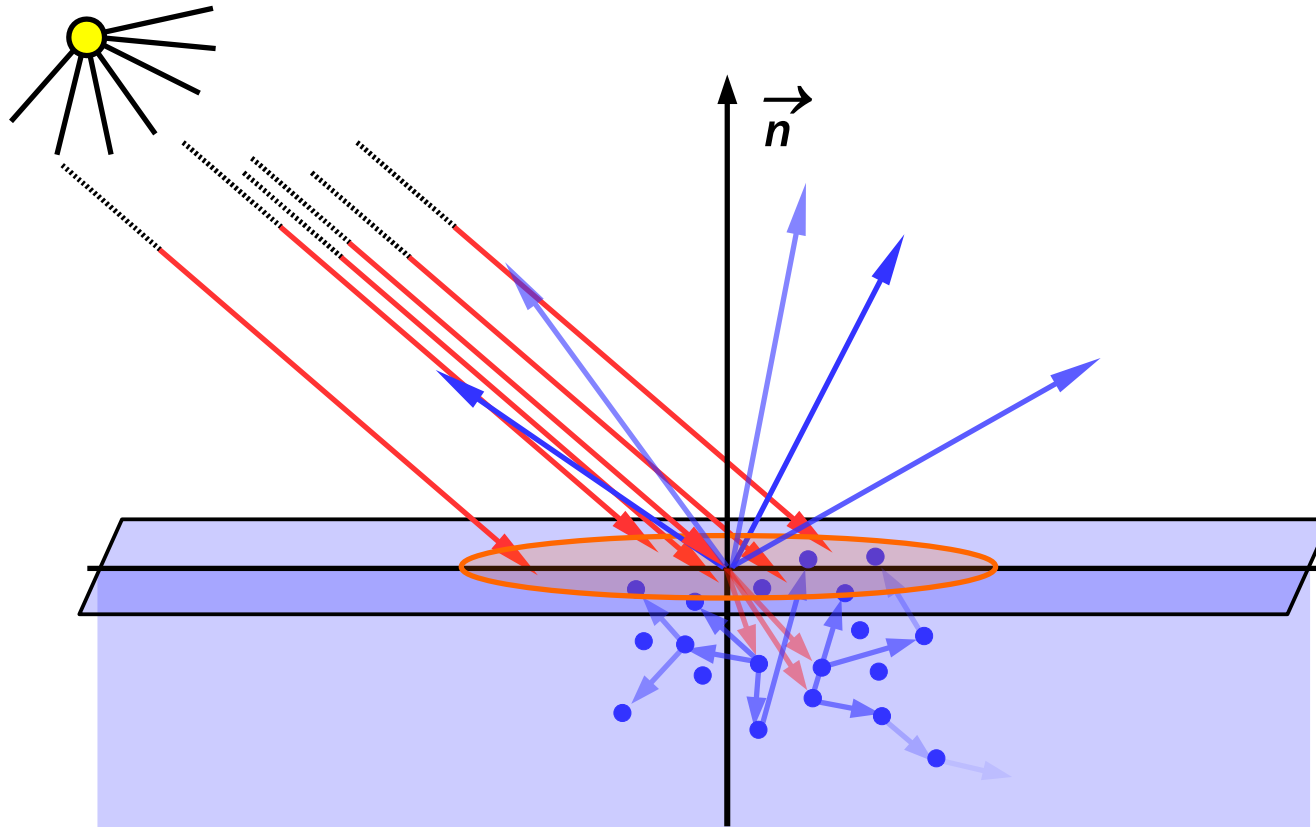


Figure 9: A simulation of subsurface scattering in a marble bust. The marble bust is illuminated from behind and rendered using: (a) the BRDF approximation (in 2 minutes), (b) the BSSRDF approximation (in 5 minutes), and (c) a full Monte Carlo simulation (in 1250 minutes). Notice how the BSSRDF model matches the appearance of the Monte Carlo simulation, yet is significantly faster. The images in (d–f) show the different components of the BSSRDF: (d) single scattering term, (e) diffusion term, and (f) Fresnel term.

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# Ideal sub-surface scattering







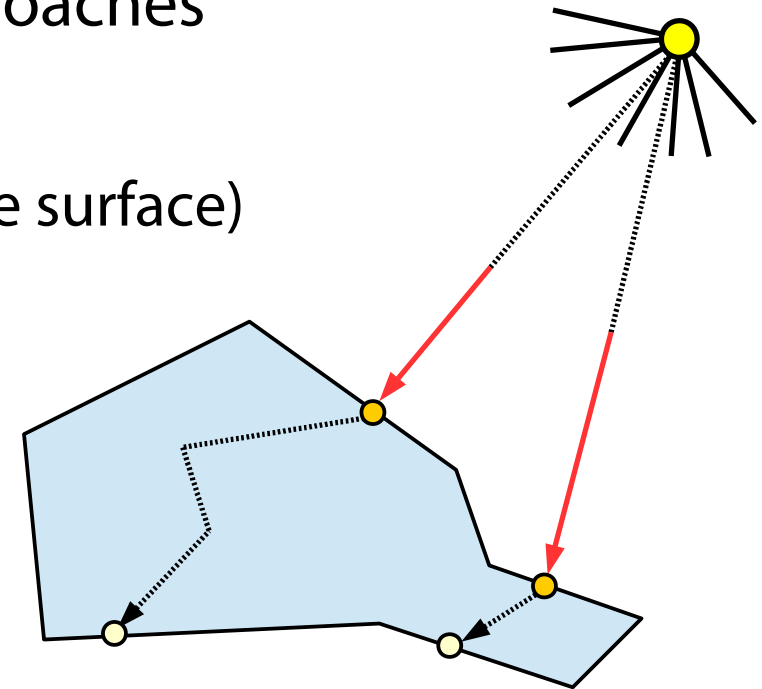
# Sub-surface scattering

Scattering **inside of a 3D volume** should be simulated

- more than single light entry should be considered
- single vs. multiple scattering

**Simplified** and time-optimized approaches

- single scattering only
- 2D models of light distribution ( $\sim$  on the surface)
- fuzzy shadow border (approximation)





# BSSRDF – a glass of milk



(a)



(b)



(c)

Figure 10: A glass of milk: (a) diffuse (BRDF), (b) skim (BSSRDF) and (c) whole (BSSRDF). (b) and (c) are using our measured values. The rendering times are 2 minutes for (a), and 4 minutes for (b) and (c); this includes caustics and global illumination on the marble table and a depth-of-field simulation.

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# BSSRDF – human body

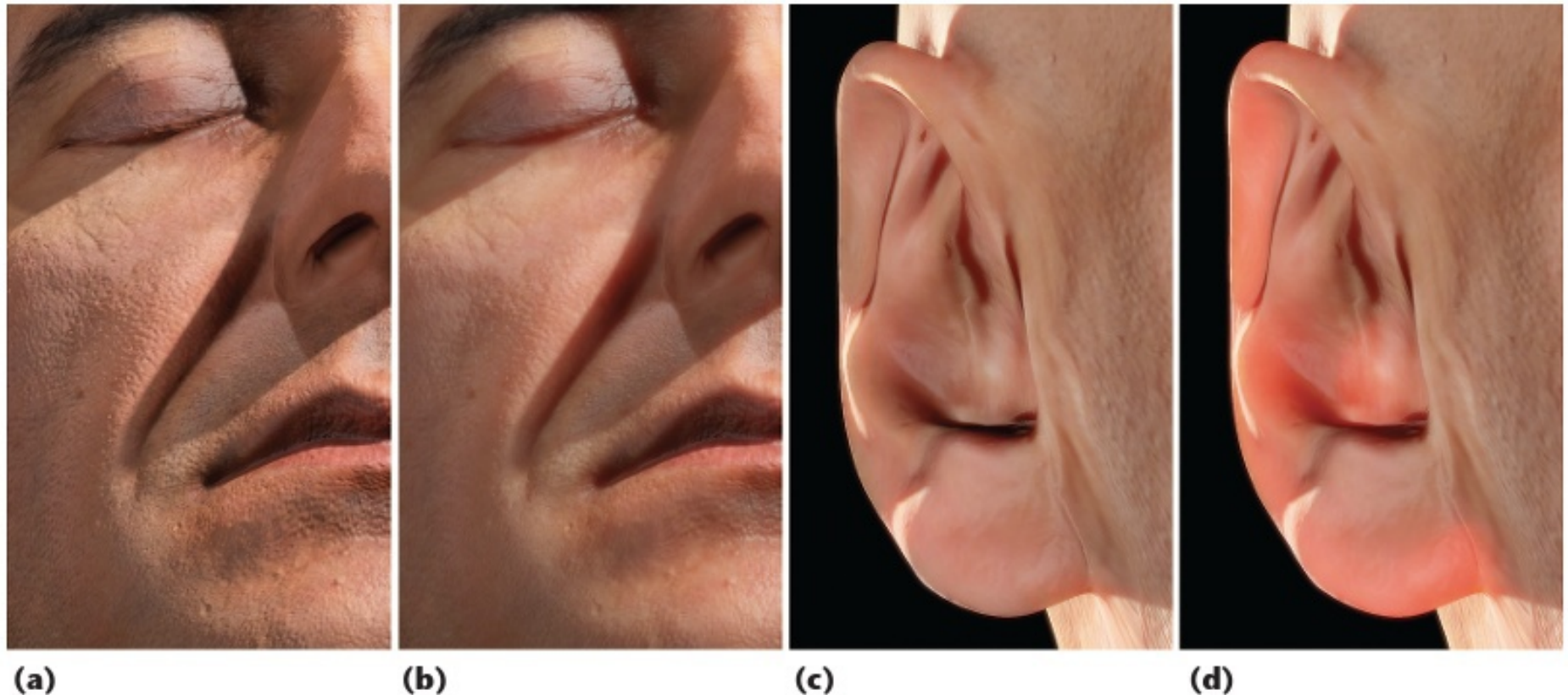


Figure 1. A comparison between (a) ignoring subsurface scattering and (b) accounting for it. The skin's reflectance component is softened from being scattered within the skin. In addition, the figure compares (c) raw screen-space diffusion<sup>1</sup> and (d) screen-space diffusion with transmittance simulation, calculated using the algorithm proposed in this article. Light travels through thin parts of the skin, which the transmittance component accounts for.

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# References I

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**Andrew Glassner: *Principles of Digital Image Synthesis*,  
Morgan Kaufmann, 1995**

**Matt Pharr, Wenzel Jakob, Greg Humphreys: *Physically Based Rendering, 3rd Edition: From Theory To Implementation*,  
Morgan Kaufmann, 2016 (online at <https://www.pbrt.org/>)**

**R. Cook, K. Torrance: *A Reflectance Model for Computer Graphics*,  
ACM Transactions on Graphics, 1982, #1, 7-24**

**Ch. Schlick: *An Inexpensive BRDF Model for Physically-based Rendering*, 1994**

**Lafortune et al.: *Non-linear approximation of reflectance functions*, SIGGRAPH '97**



# References II

**Weidlich A., Wilkie A.: *Arbitrary Layered Micro-Facet Surfaces*, GRAPHITE 2007**

**A. Ozturk et al.: *Linear approximation of BDRF*, C&G 2008**

**Self Shadow tutorials (esp. SIGGRAPH 2012, 2015-2017):**  
<http://blog.selfshadow.com/publications/>

**R. Montes, C. Ureña: *An Overview of BRDF Models*, Technical report, Uni of Granada, 2012**

**S. H. Westin et al.: *A Comparison of Four BRDF Models*, EGSR 2004**

**A. Ngan et al.: *Experimental Validation of Analytical BRDF Models*, SIGGRAPH 2004**



# References III

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**Jensen et al.: *A Practical Model for Subsurface Light Transport*, SIGGRAPH '01**

**Jimenez J., Whelan D., Sundstedt V., Gutierrez: *Real Time Realistic Skin Translucency*, IEEE CG&A, 2010**