matfyz

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Planar test point vs. polygon

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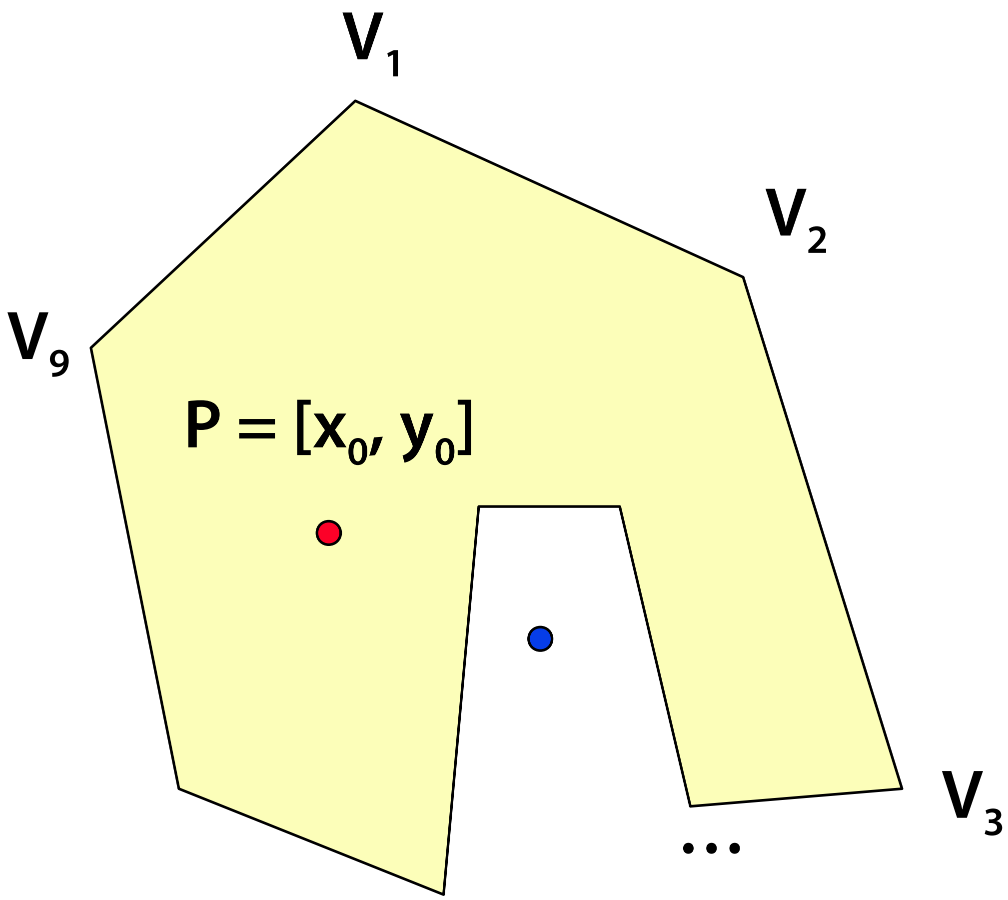
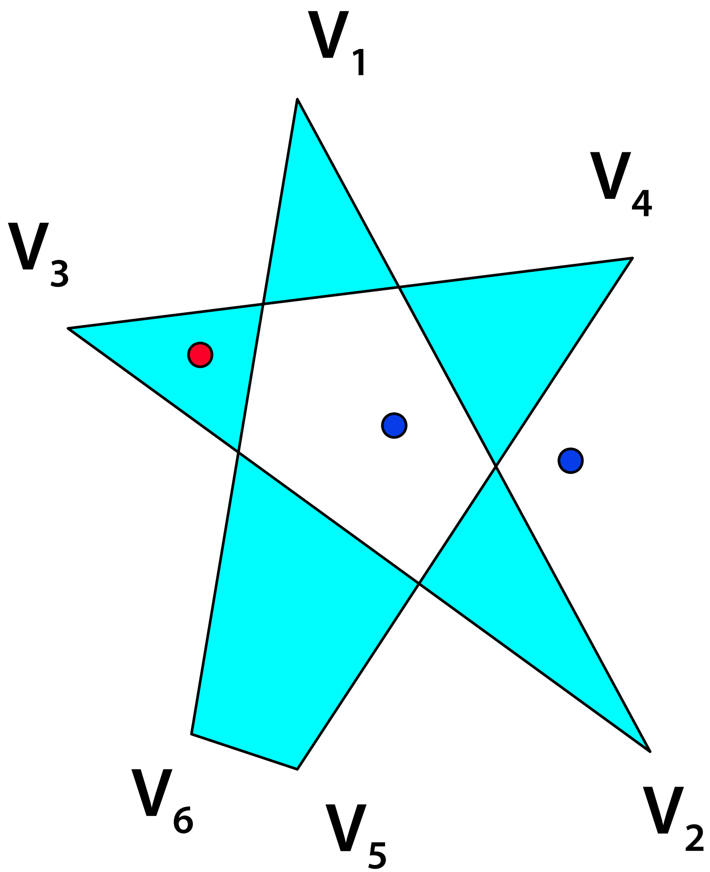
Polygon interior?





$V_i = [x_i, y_i]$

Different definitions

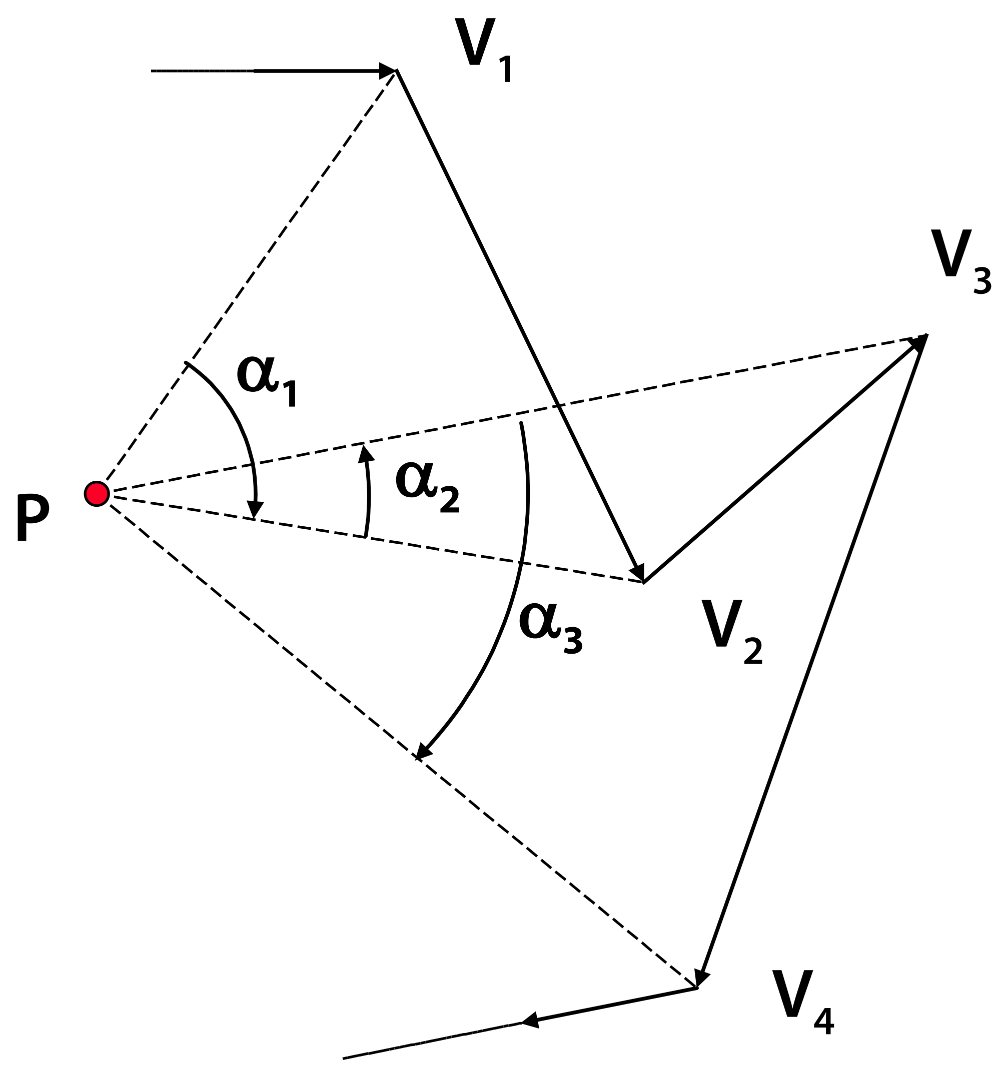
Point **P** lies inside the polygon **[V_1, .. V_M]**, if

❶ it is separated from the outside (infinite component of the plane) by **odd number of borders** (“odd-even rule”, Jordan theorem)

❷ it is separated from the outside by **at least one border** (i.e. not element of the infinite component)

❸ its **”winding number”** with respect to the polygon's outline **W** is **nonzero** (“thread loop + pin”)

Winding number computation



Sum of the oriented angles

$$\Sigma_i \alpha_i = 2\pi \cdot W$$

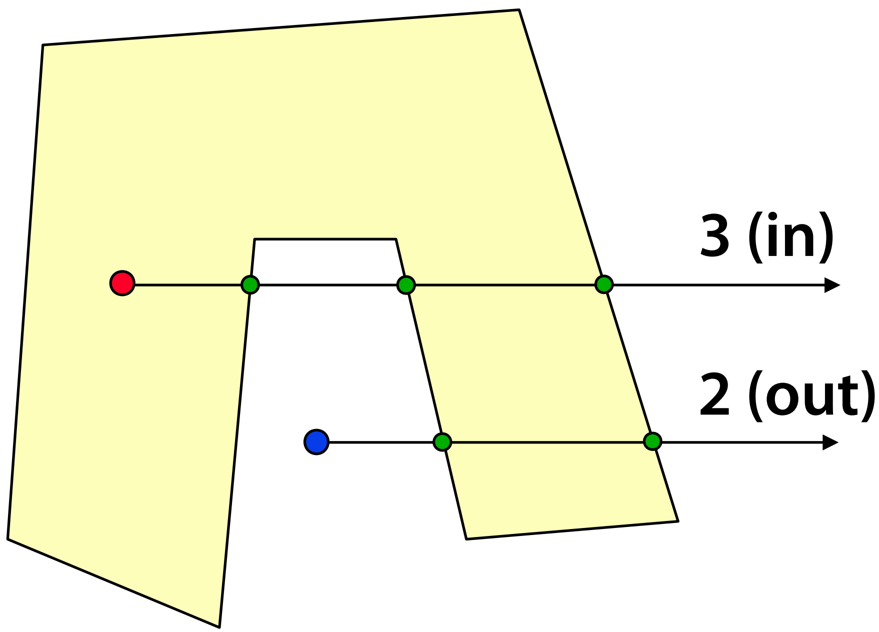
0°, ±360°, ±720°…

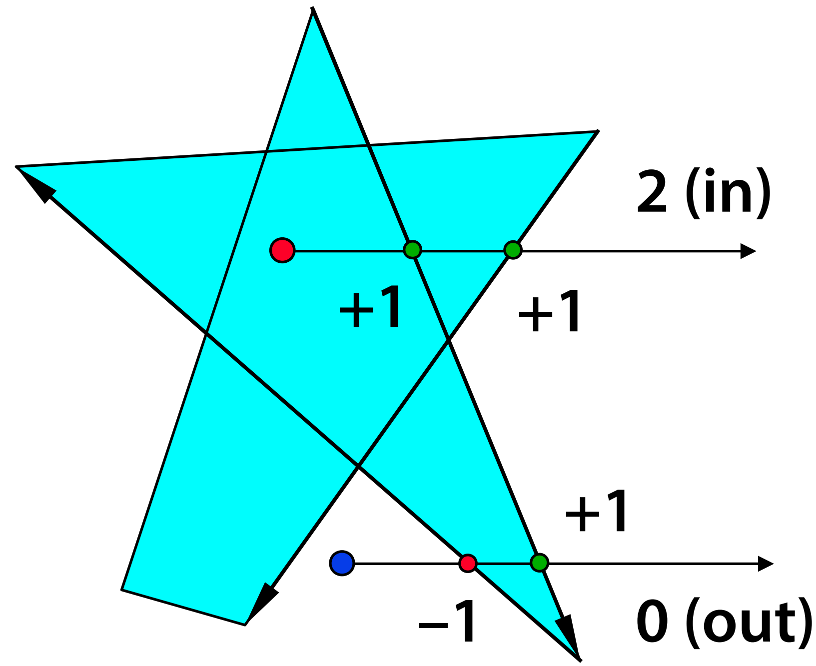
outside **inside**

Efficiency:

table of **arctg(y/x)**

Half line vs. polygon outline

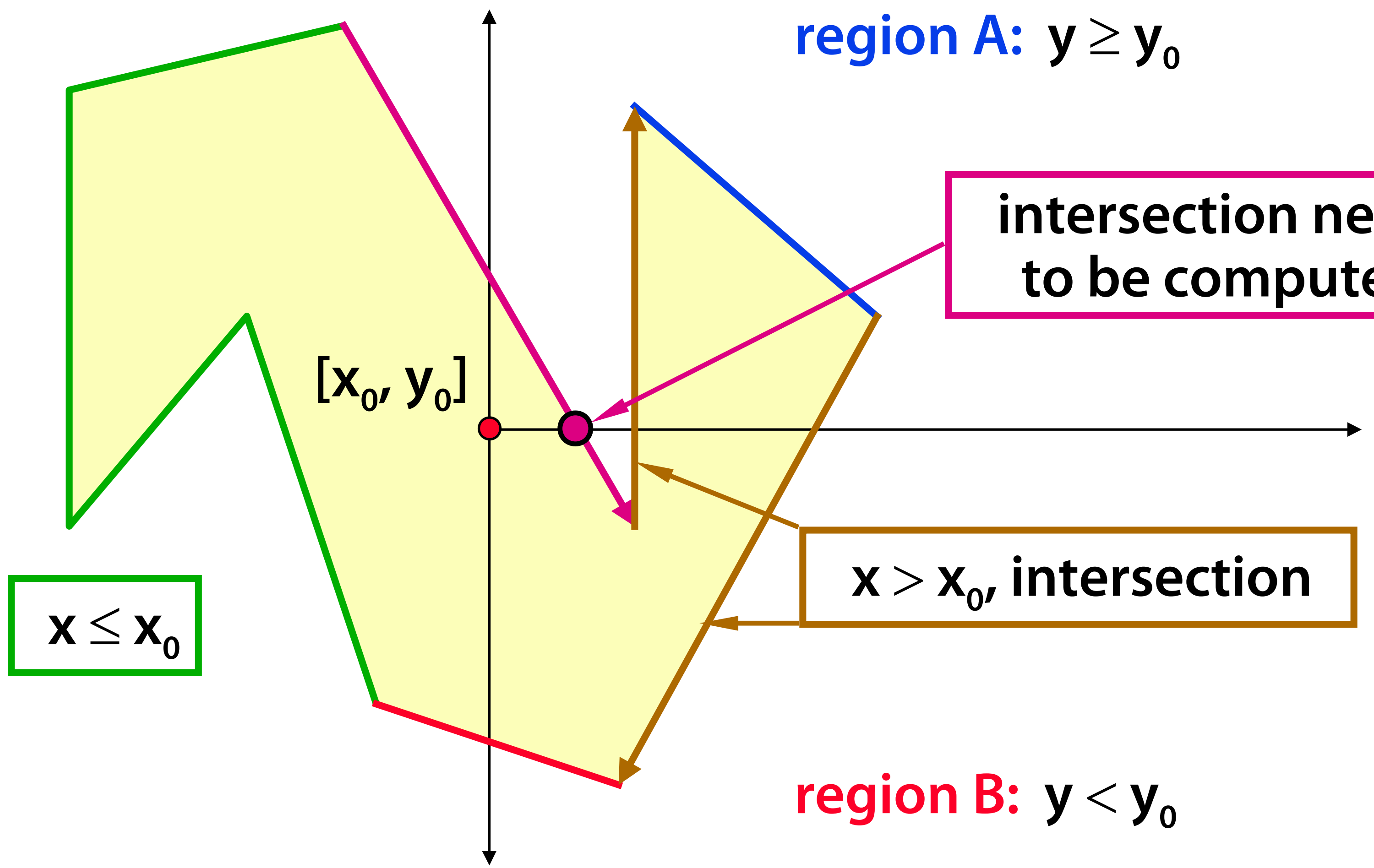




Definition ❶
(non-oriented edges)

Definition ❸
(oriented edges)

Implementation



Implementation

Sequential pass through edges $V_1V_2,\ V_2V_3 \dots V_MV_1$

- every vertex has the flags $x > x_0,\ y \geq y_0$

Trivial negative edges – both vertices have equal boolean value of either condition

- $x \leq x_0$, $y \geq y_0$ or $y < y_0$

Trivial positive edges (intersection exists)

- for both vertices: $x > x_0$
- for exactly one vertex: $y \geq y_0$

Implementation

The rest of the edges are **nontrivial**

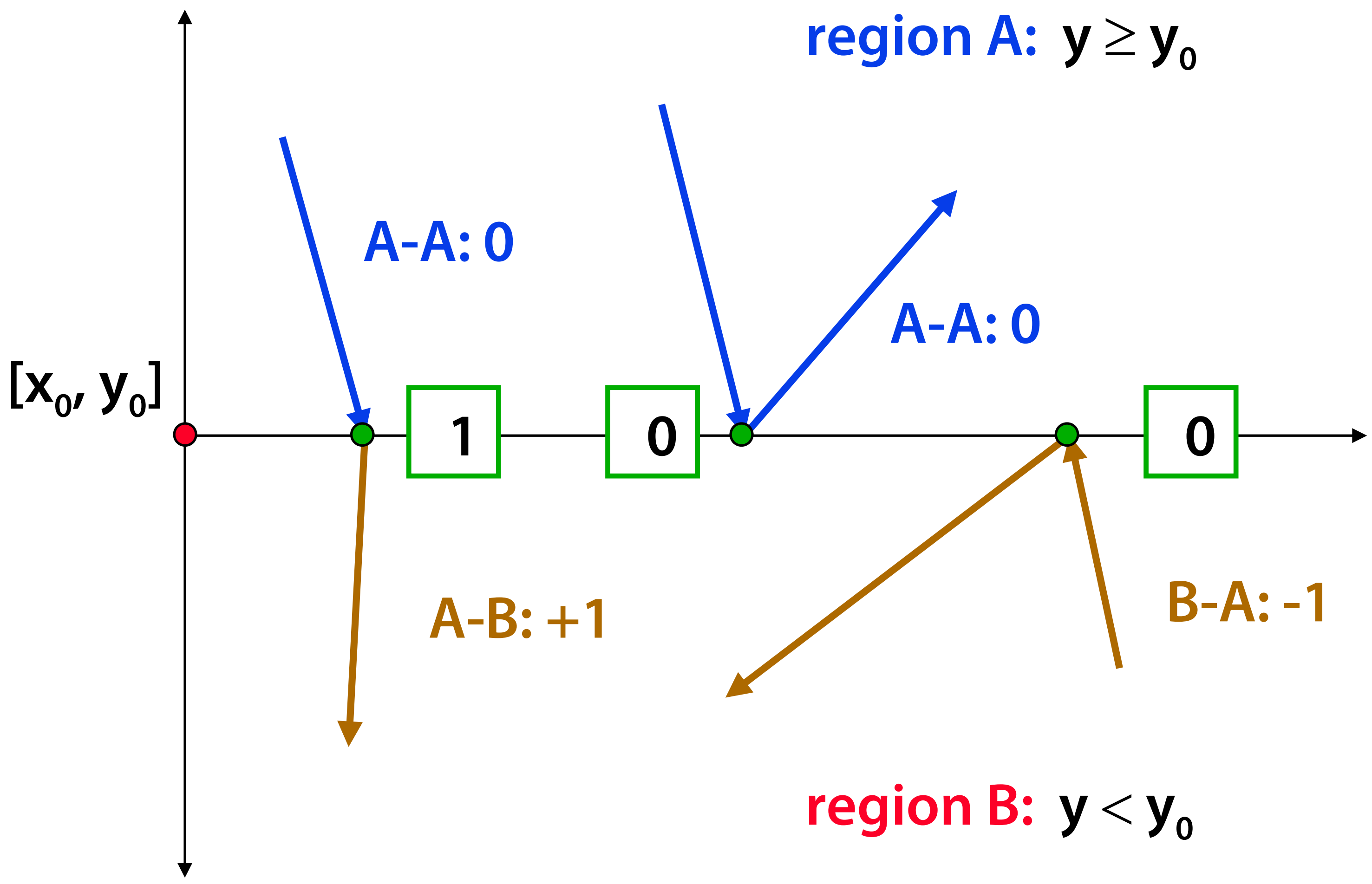
- for exactly one vertex: $\mathbf{x} > \mathbf{x_0}$
- for exactly one vertex: $\mathbf{y} \geq \mathbf{y_0}$
- the **exact intersection** of an edge with the half line $\mathbf{y} = \mathbf{y_0}$ has to be computed

For any **positive edge** (intersection) the contribution is

+1 or **-1** according to edge orientation (definition ❸)

+1 for an unoriented edge (definition ❶)

Special cases



Literature

A. Glassner: ***An Introduction to Ray Tracing***, Academic Press, London 1989, 53-59

J. Foley, A. van Dam, S. Feiner, J. Hughes: ***Computer Graphics, Principles and Practice***, 34